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Population balance model of heat transfer in gas-solid turbulent fluidization

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Abstract

A population balance model is presented for describing heat transfer processes in gas-solid turbulent fluidized beds. In the model, the gas and particle transport is described by a cells-in-series with back-flow model, while the particle-particle and particle-wall heat transfers are modeled as collisional random events, characterized by collision frequencies and random variables with probability density functions determined on interval [0,1]. An infinite hierarchy of moment equations is derived from the population balance equations, which can be closed at any order of moments. The properties of the model and the effects of process parameters are examined by numerical experimentation.

Keywords: Turbulent fluidization, Heat transfer, Population balance model, Moment equation model, Simulation

1. Introduction

The turbulent fluidization is characterized by low amplitudes of pressure fluctuations and favorable gas-solids contacting. In gas-solid turbulent fluidized beds the solids hold-ups are also high, typically 25-35 % by volume [1], thus, because of intensive motion of particles, particle-particle and particle-surface collisions appear to play significant role in controlling the thermal characteristics of the bed.

For modeling and simulation of collisional heat transfer processes in gas-solid systems, an Eulerian-Lagrangian approach, with Lagrangian tracking for the particle phase [2-5], and a recently developed population balance model [6-9] have been applied.

The population balance equation is a widely used tool in modelling the disperse systems of process engineering [10], describing a number of fluid-particle and particle-particle interactions. This equation was extended by Lakatos *et al.* [7] with terms to describe also direct exchange processes of extensive quantities, such as mass and heat between the disperse elements as well as between the disperse elements and solid surfaces by collisional interactions [8,9].

The aim of the present paper is to develop a population balance model for describing also the spatial distributions of the gas and particle temperatures in turbulent fluidized beds.

2. Population balance model

The axial dispersion model is commonly applied to describe the dispersion of gas and solids mixing in turbulent fluidized beds [11,12]. Axial mixing can be characterized by the axial dispersion and the backmixing coefficients which can be related to each other by the variance of the residence time distributions. Here we apply the cells-in-series with backflow model for both the void and emulsion phases as it is shown in Fig.1 where the heat transfer resistance of the gas in the emulsion phase is added to the gas-particle heat transfer.

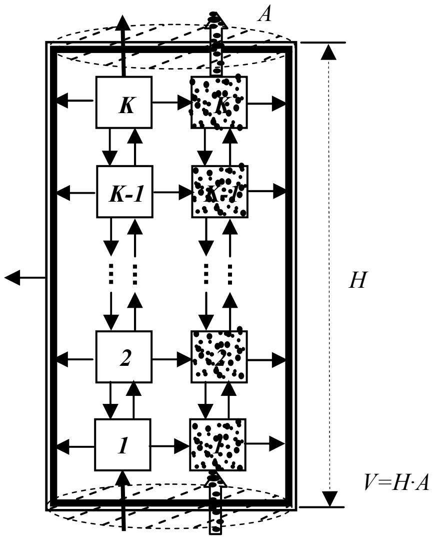

Fig.1. Two-phase model of the bed

In this system five interphase thermal processes are considered: fluid-particle, fluid-wall, particle-particle, particle-wall and wall-environment. Because of intensive motion of particles, the particle-particle and particle-wall heat transfers occur through the interparticle and particle-wall collisions.

The main assumptions concerning the system are as follows:

1) The particles are of constant size and are not changed during the process; 2) The system is operated under steady state hydrodynamic conditions, and the influence of thermal changes on the hydrodynamics is negligible. 3) Heat trans-

fer between the gas and particles, wall and gas, as well as the wall and environment are continuous processes, characterized by the heat transfer coefficients h_{pf}, h_{wf} and h_{we} respectively. 4) Interparticle heat transfer occurs by collisions, and is described by the random variable $\xi_1 \in [0,1]$ with probability density function b_1. 5) The particle-wall heat transfer also occurs by collisions that is characterized by the random variable $\xi_2 \in [0,1]$ with probability density function b_2. 6) There is no heat source inside the particles. 7) The heat transfer by radiation is negligible. 8) The temperature of the wall is homogeneous.

Let $n_k(T_p,t)$ denote the population density function for the k^{th} cell, k=1,2…K, by means of which $n_k(T_p,t)\mathrm{d}T_p$ provides the number of particles from interval $(T_p, T_p+\mathrm{d}T_p)$ in a unit volume of the cell at time t. If $T_{g;k}(t)$ denotes the gas temperature in the k^{th} cell and $T_w(t)$ stands for the temperature of the wall, then the population balance model is formed by the following equations.

Population balance equations:

$$\begin{aligned}
&\frac{\partial n_k(T_p,t)}{\partial t} - \frac{\partial [K_p(T_{g;k}(t)-T_p)n_k(T_p,t)]}{\partial T_p} = \frac{(1+S_kR)q}{V_k} n_{k-1}(T_p,t) + \frac{Rq}{V_k} n_{k+1}(T_p,t) - \\
&-\frac{(1+Z_kR)q}{V_k} n_k(T_p,t) - k_{2k} n_k(T_p,t) + k_{2k}\int_0^1 n_k\left(\frac{T_p - p_w z T_w}{1-p_w z}, t\right) b_2(z)\frac{1}{1-p_w z}dz + \\
&- k_{1k} n_k(T_p,t) + \frac{k_{1k}}{M_{0,k}} \int_{T_{p\min}}^{T_{p\max}} \int_0^1 n_k\left(\frac{2(T_p - S)}{z} + S, t\right) n_k(S,t) b_1(z)\frac{2}{z} dz dS, \quad k=1,2...K,\ t>0
\end{aligned} \tag{1}$$

subject to the initial conditions $n_k(T_p,0)=n_0(T_p), k=1,2...K$. In Eqs (1), $n_0(T_p,t)=n_{in}(T_p,t)$, $n_{K+1}(T_p,t)\equiv 0$, and the auxiliary symbols, introduced for the sake of shortness, are: $S_1=0$, $S_K=1$, $Z_1=Z_K=1$, $S_l=1$, $Z_l=2$, and $l=2,...,K-1$. Further, q is the volumetric flow rate, V_k is the volume of the k^{th} cell, $V_k=V/K$, V is the volume of the bed, and R denotes the back-flow coefficient.

The factors $p_2 = m_p c_p/(m_p c_p + m_w c_w)$ and $p_1 = m_w c_w/(m_p c_p + m_w c_w)$ characterize the ratios of particle-wall heat capacities where m and c denote, respectively, mass and specific heat, while the indices p and w regard the particle and the wall.

The second term on the left hand side of Eq.(1) describes the gas-particle heat transfer with coefficient K_p, while on the right hand side: the first three terms represent the transport of particles between the cells, the next two terms describe the collisional wall-particles heat transfer with collision frequencies k_{2k}, and the last two terms describe the collisional particle-particle heat transfer with collision frequencies k_{1k}.

The axial inhomogeneity of the solids hold-up in Eq.(1) is represented by the variation of the solids concentration given, in principle, by the total number of particles $M_{0,k}$ in the k^{th} cell, defined as

$$M_{0,k} = \frac{6(1-\varepsilon_k)V_k}{\pi d_p^3} = \int_{T_{p\min}}^{T_{p\max}} n_k(T_p,t)dT_p \tag{2}$$

where ε_k is the void fraction in the k^{th} cell, and d_p denotes the particle diameter. Here, the axial voidage distribution is modeled by means of the balance equations

$$\frac{d\varepsilon_1(t)}{dt} = \frac{q}{V_1}\varepsilon_{in}(t) + \frac{Rq}{V_1}\varepsilon_2(t) - \frac{(1+R)q}{V_1}\varepsilon_1(t) + f_1\left(\varepsilon_{1,meas}\right)$$
$$\frac{d\varepsilon_k(t)}{dt} = \frac{(1+R)q}{V_k}\varepsilon_{k-1}(t) + \frac{Rq}{V_k}\varepsilon_{k+1}(t) - \frac{(1+2R)q}{V_k}\varepsilon_k(t) + f_k\left(\varepsilon_{k,meas}\right), \; k = 2...K-1$$
$$\frac{d\varepsilon_K(t)}{dt} = \frac{(1+R)q}{V_K}\varepsilon_{K-1}(t) - \frac{(1+R)q}{V_K}\varepsilon_K(t) + f_K\left(\varepsilon_{K,meas}\right) \tag{3}$$

where the source terms f_k are to be obtained by fitting those to the measured $\varepsilon_{k,meas}$ voidage distribution data [1,13]. Based on the voidage distribution, variation of the collision frequencies can also be estimated [14].

By using the voidage distribution, the balance equation for the gas temperature takes the form

$$\frac{dT_{g,k}(t)}{dt} = \frac{\varepsilon_{k-1}(1+S_kR)q}{\varepsilon_k V_k}T_{g,k-1}(t) + \frac{\varepsilon_{k+1}Rq}{\varepsilon_k V_k}T_{g,k+1}(t) - \frac{(1+Z_kR)q}{V_k}T_{g,k}(t) -$$
$$- \int_{T_{p\min}}^{T_{p\max}} K_{gp}\left(T_{g,k}(t) - T_p(t)\right)n_k(T_p,t)dT_p - K_{wg}(T_{g,k}(t) - T_{w,k}(t)), \quad k = 1,2...K, \; t > 0 \tag{4}$$

while the balance equation for the wall becomes

$$\frac{dT_w(t)}{dt} = \sum_{k=1}^{K} K_{wg}\left(T_{g,k}(t) - T_w(t)\right) - K_{we}\left(T_w(t) - T_e\right) -$$
$$- k_{2k}\sum_{k=1}^{K}\int_{T_{p\min}}^{T_{p\max}}\int_0^1 p_2\left(T_w(t) - T_p\right)n_k\left(T_p,t\right)z\, b_2(z)\, dz\, dT_p \qquad k = 1,2...K, \; t > 0 \tag{5}$$

In Eq.(5) the environment temperature T_e is kept constant.

3. Simulation results and discussion

An important point of applying the population balance model is the solution of the population balance equation. A number of methods have been developed for that purpose [10,15-18] but, since the moment equations induced by Eq.(1) can be closed at any order of moments [9], the set of equations (1)-(5) was solved by applying a second order moment equation reduction of the population balance equation (1), written for the first three leading moments of the temperature of particle population [8,9]

$$M_{Ik}(t) = \int_0^1 T_p^I n_k(T_p,t)dT_p \;, \; I = 0,1,2 \;, k = 1,2,...K \tag{6}$$

which are necessary for a basic characterization of the temperature distribution of particles. The zero order moments $M_{0,k}$ provide the total numbers of particles, by means of which the solids concentrations can also be computed. The mean temperatures of particles are expressed as $m_{1,k} = M_{1,k} / M_{0,k}$, while the temperature distributions arising in the individual cells are characterized by the variances $\sigma_k^2 = M_{2,k} / M_{0,k} - (M_{1,k} / M_{0,k})^2$. The program developed in MATLAB can handle arbitrary number of cells, and the resulted set of ordinary differential equations is solved by means of an ode solver of MATLAB. The results to be presented here were obtained for 8 cells using the same constitutive parameters and expressions as given in detail in [9]. The gas input temperature was 180°C, the inlet feed of particles was a mixture of temperatures 20°C and 60°C, while the environment temperature was kept 20°C.

Fig.2 presents the variation of the gas temperature and the mean temperature of particle population as a function of the cell number for different back-flow coefficients of particles and plug flow conditions for gas. It is seen that equalization of the gas temperature and mean temperature of particles becomes completed already in the second and third cells, i.e. at the lower part of the bed, although when the back-mixing of particles is large, some temperature gradient arises in the upper part of the bed.

Backmixing of the particulate phase affects also the temperature distribution of particles significantly, as it is illustrated in Fig.3, presenting the variance of temperature of the particle population as a function of the cell sequence. As the back-flow ratio increases the temperature distribution of particles remains inhomogeneous even at the outlet of the bed. Since the axial voidage distribution is characterized by an increase of gas volume concentration therefore the intensity of collisional events and, as a consequence, their contribution to temperature homogenization may be reduced significantly in the upper part of the bed.

Transients of the gas temperature and mean temperature of the particle population are presented in Fig.4. These plots illustrate well that the heat transfer induced changes are characterized by much smaller time constants than those

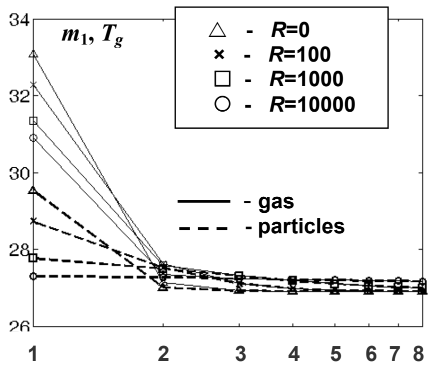

Fig.2. Variation of the mean temperature of particle population m_1 and the gas temperature T_g along cell sequence

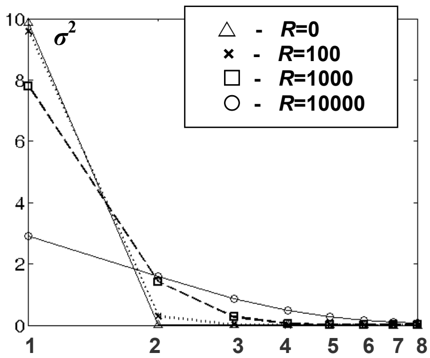

Fig.3. Variance of the temperature distribution of particle population along the cell sequence

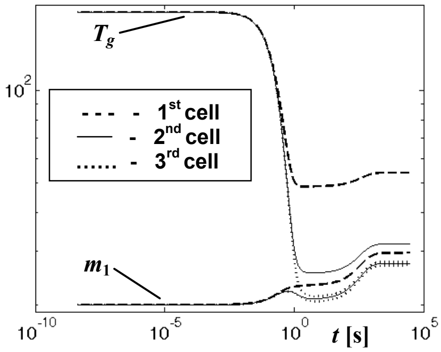

Fig.4. Transients of the gas temperature T_g and mean temperature of particles m_1 in the first three cells of the 8-cell compartment model for R=10

caused by the mass transport and backmixing of particles, predicting some difficulties in developing control systems for turbulent fluidized beds. Here, processes were plotted only up to the third cell since practically all the remaining transients are covered by the third cell processes.

References

1. H.T. Bi, N. Ellis, A. Abba and J.R. Grace, Chem. Eng. Sci., 55 (2000) 4789.
2. P. Boulet, S. Moissette, R. Andreaux, B. Osterlé, Int. J. Heat Fluid Flow, 21 (2000) 381.
3. Z. Mansoori, M. Saffar-Avval, H. Basirat-Tabrizi, G. Ahmadi, S. Lain, Int. J. Heat Fluid Flow Transfer, 23 (2002) 792.
4. V. Chagras, B. Osterlé, P. Boulet, Int. J. Heat Mass Transfer, 48 (2005) 1649.
5. Z. Mansoori, M. Saffar-Avval, H. Basirat-Tabrizi, B. Dabir, G. Ahmadi, Powder Technology 159 (2005) 35.
6. Cs. Mihálykó, B.G. Lakatos, A. Matejdesz, T. Blickle, Int. J. Heat Mass Transfer, 47 (2004) 1325.
7. B.G. Lakatos, Cs. Mihálykó, T. Blickle, Chem. Eng. Sci., 61 (2006) 54.
8. Z. Süle, Cs. Mihálykó, B.G. Lakatos, Comp-Aided Chem. Eng., 21A, Elsevier, Amsterdam, 2006, pp. 589-594.
9. B.G. Lakatos, Z. Süle, Cs. Mihálykó, Int. J. Heat Mass Trans. (submitted)
10. D. Ramkrishna, Population Balances. Theory and Applications to Particulate Systems in Engineering. Academic Press, San Diego, 2000.
11. M. Foka, J. Chaouki, C. Guy, D. Klvana, Chem. Eng. Sci., 55 (1996) 713.
12. F. Wei, S. Lin, G. Yang, Chem. Eng. Techn., 16 (1993) 109.
13. J.C. Scouten, R.C. Zijerveld, C.M. van den Bleek, Chem. Eng. Sci., 54 (1999) 2103.
14. C. You, H. Zhao, Y.C. Haiying, H. Qi, X. Xu, Int. J. Multiphase Flow, 30 (2004) 1121.
15. N.V. Mantzaris, P. Daoutidis, F. Srienc, Comp. Chem. Eng., 25 (2001) 1411-1440.
16. S. Motz, A. Mitrovic, E.D. Gilles, Chem. Eng. Sci., 57 (2002) 4329-4344.
17. D.L. Marchisio, J. Pikturna, R.O. Fox, R.D. Virgil, A.A. Barresi, AIChE Journal, 49 (2003) 1266-1276.
18. M. Kostoglou, A.J. Karabelas, AIChE Journal, 50 (2004) 1746-1759.