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Measurement of Plant Flexibility

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Abstract

Operational flexibility is an important consideration when designing and operating a chemical plant. Very often, flexibility is concerned with the problem of insuring feasible steady-state operation over a variety of operating uncertainties. To quantify how flexible a process is, various metrics have been developed. Grossmann and his co-workers[1,2] first introduced the flexibility index (FI_G) which quantifies the smallest percentage of the uncertain parameters' expected deviation that the process can handle. Another metric named resilience index (*RI*) was adopted by Saboo et al.[3] However, these two measurements require identification of the nominal point which must locate within the feasible region. In addition, these measurements just consider the critical uncertainty. This may cause serious flexibility under-estimation or neglect the ability of the process to handle other process uncertainties. To solve this problem, Pistikopoulos and Mazzuchi[4] proposed the stochastic flexibility (*SF*). Although *SF* accounts for the chance that the process can operate feasibly, the probability distribution of all the uncertain parameters may not be available at the design stage. Even though the probability distributions are obtainable, the calculation of *SF* is usually tedious.

This paper aims at introducing a new process flexibility metric. Flexibility is reckoned as the size of the feasible space (S_f) in which the uncertain parameters can be feasibly handled. A new flexibility index (FI_V) is defined as the size ratio of S_f to the overall space (S_o) bounded by the uncertain parameters' expected limits. A process with $FI_V = 1$ indicates that it can run feasibly with all the expected values of the uncertain parameters. Alternatively, a process with $FI_V = 0$ implies that none of the expected uncertainties can be handled feasibly. Unlike FI_G, FI_V measures the entire space where all uncertain parameters can be

freely moved. This paper will provide an algorithm used for estimating S_f and the newly defined FI_V. Examples will be presented to demonstrate the applications of this new FI_V and compare this with the other flexibility measurements.

Keywords

Chemical plant flexibility; Flexibility index; Process design; Uncertainty; Algorithms

1. Introduction

With the swift changes in chemical process status, resilience becomes more and more important in chemical plant design. Resilience can be classified into two types: static or dynamic.[5] Dynamic resilience concerns with the ability of the process to tackle transient changes of the process while static resilience refers to the ability of the process to handle different steady state changes. In this study, the discussion is focused on the static resilience. The ability of the plant to handle process parameter changes on steady state operation is regarded as process flexibility. It is necessary to quantify process flexibility so as to measure and compare the flexibility level of designed processes. Therefore, various metrics, such as FI_G, RI and SF, have been developed.

2. Problem Statement, background

Although various flexibility metrics have been developed to quantify chemical process flexibility at steady state operation, these metrics are either incomprehensive or too complicate to compute. Sometimes, their determinations require information that is not readily available. It is necessary to develop another metric to solve these shortfalls. This paper aims at introducing a new flexibility index which can provide comprehensive and important flexibility information for process design and its computation is facile.

3. New Flexibility Index

In order to clearly illustrate this new flexibility index, systems with two uncertain parameters is used in this paper. This flexibility evaluation can be extended to chemical systems with multi-uncertain parameters.

3.1. Definition

The uncertain space (θ_1 and θ_2) of a chemical process is shown in Fig. 1. The constraint boundary separates the uncertain space into the constrained space (S_c)

and the infeasible space. The expected upper and lower limits of the uncertain parameters (θ_{iU} and θ_{iL}) bound the whole space (S_o). The communal space ($S_e \cup S_o$) is the feasible space, S_f. All combinations of the uncertain parameters in S_f are within their expected limits and can be operated without violating the process constraints. The flexibility index (FI_V) is defined as the size ratio of S_f to S_o. Therefore, FI_V equals to the area ratio (A_f / A_o) in this *2-D* case. Larger the FI_V indicates that a larger portion of S_f is within S_o, and thus, a higher chance that the process can be feasibly operated as the uncertain parameters alter. Although the feasible convex hull ratio (*FCHR*)[6,7], which evaluates process feasibility, bears definition similar to FI_V, the *FCHR* research focused on the tedious computation and iteration steps to enhance evaluation accuracy. This current work proposes a simple scheme to estimate process flexibility with acceptable accuracy with relatively simpler computation. In addition, the auxiliary vectors can also indicate the appropriate directions of design changes to enhance process flexibility.

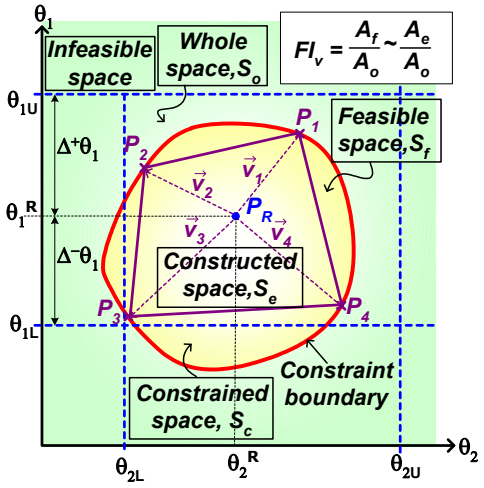

Figure 1 FI_V determination in system with *2* uncertain parameters.

3.2. FI_V Determination

To determine FI_V, the size of S_f and S_o have to be computed. As the expected limits of the uncertain parameters are usually independent, $A_o = (\theta_{1U}-\theta_{1L})\times(\theta_{2U}-\theta_{2L})$. However, the calculation of A_f may not be strict forward and exact A_f may not be easily obtained. An estimation method of A_f is suggested. The approach is demonstrated in Fig. 1. A reference point, P_R is picked within the feasible region. Various auxiliary vectors, $\vec{v}_j$ can branch outwards from P_R until they intercept with the S_f or S_o boundaries at interception points $P_j(\theta_{1j},\theta_{2j})$.These points can be used to generate a new constructed space, S_e. With careful selection on $\vec{v}_j$ directions, the S_e, whose area (A_e) determination is less difficult,

can be obtained. With A_e determined, the FI_V can then be computed by A_e / A_o. The general computation algorithm is shown below. For different $\overrightarrow{v_j}$ selection schemes, extra constraints can be imposed to restrict the variables, x_j and y_j.

objective function: $\max V_e(x_j, y_j)$ by varying x_j, y_j

constraints: $g_k(\theta_{1j}, \theta_{2j}) \le 0$ and $0 \le x_j, y_j \le 1$

such that
$$\theta_{1j} = v_{1j} x_j \Delta_{1j} + \theta_1^R \quad , \quad \Delta_{ij} = \begin{cases} \Delta^+\theta_i & (v_{ij} = 1) \\ \Delta^-\theta_i & (v_{ij} = -1) \end{cases}$$
$$\theta_{2j} = v_{2j} y_j \Delta_{2j} + \theta_2^R$$

where $V_e(x_j, y_j)$ = *generalized S_e volume function*
$g_k(\theta_{1j}, \theta_{2j})$ = *k-th constraint function evaluated at P_j.*
x_j, y_j = *variables giving the pointing direction of vector j*
v_{1j}, v_{2j} = *auxiliary vector direction indices at P_j (equal to 1 or -1)*

3.3. Example 1

A space with two uncertain parameters (δ_1 and δ_2) is shown in Fig. 2. The three blue solid lines are the process constraints. The expected upper and lower limits of both δ_1 and δ_2 are *1* and *-1* respectively. Thus, S_o is a square region and its area is *4*. S_f is the region bounded by the constraint boundaries within S_o. To determine A_f, S_e has to be constructed. The origin of the space is picked as P_R and four $\overrightarrow{v_j}$ selection schemes are employed. Various S_e are sketched in Fig. 2. By radiating the vectors into the four quadrants, S_{e1} (outlined in red) which is a rectangle with maximized area can be obtained. On the other hand, the vectors can run along the principal axes to generate a quadrilateral S_{e2} (outlined in purple) with its polygon diagonals parallel to the principal axes. The third approach allows the vectors pointing *45°* from the principal axes. The corresponding S_{e3} is shown in the figure outlined in orange. In the last scheme, the directions of the vectors are optimized to point towards the four quadrants with area of S_{e4} being maximized. The estimated values of A_e and the corresponding FI_V are summarized in the figure. As shown, S_{e4} generated from the last scheme is the largest and it gives the closest estimate of the A_f and FI_V.

3.4. Example 2

The flow problem suggested by Swaney and Grossmann[2] and Floudas et al.[8] is used as an example. With the design parameters fixed, the constraint boundary can be obtained and plotted in the liquid flowrate, m and desired pressure, P_D uncertain space. The FI_G is found to be *0.62* while the *RI* is *1.00*. To create S_e, the nominal point (m^N= *10*kg/s and P_D^N = *800* kPa) is picked as P_R.

The values of FI_V of this design obtained from the four $\vec{v}_j$ selection schemes previously described in Example 1 are summarized in Table 1.

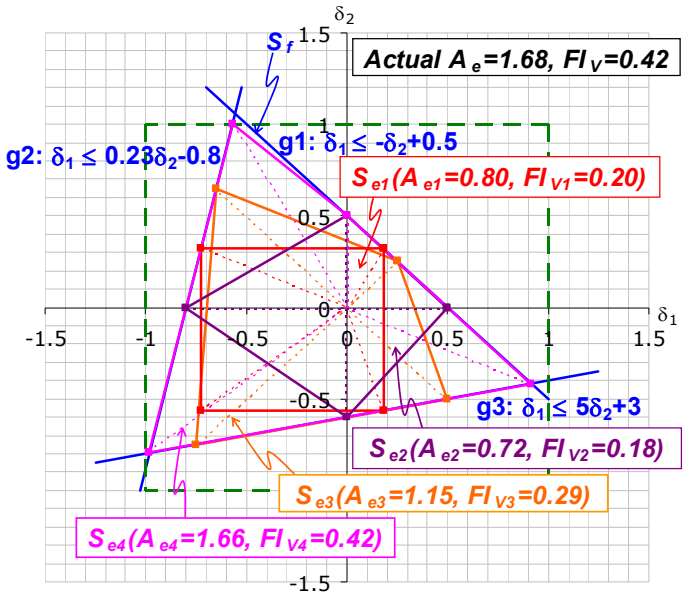

Figure 2 FI_V determination for Example 1.

Table 1 Result summary for Example 2

	Actual	Scheme 1	Scheme 2	Scheme 3	Scheme 4
A_e, kg.kPa/s	5130	4265	2625	4696	4757
FI_V	0.98	0.81	0.50	0.89	0.91

3.5. Results & discussions

Since FI_G only accounts the critical uncertainty, it may seriously under-estimate the true degree of process flexibility. Determination of FI_V allows comprehensive evaluation of the whole S_f and therefore, it can provide better flexibility analysis. The small FI_G in Example 2 may lead to improper decision to input additional investments. However, FI_V indicates that the existing process can already handle *98%* of the expected conditions. Additional investment may not be necessary if such level of FI_V is acceptable. In case excess investment is needed, the direction of $\vec{v}_j$ may also provide valuable information for feasible process improvements. Moreover, as shown, the different $\vec{v}_j$ selection schemes give different S_e and FI_V values. The importance of $\vec{v}_j$ selection schemes may

not be obvious in system with two uncertain parameters since the area calculations are well documented. However, with the dimensionality increases, calculation of the hyper-volume with irregular shape will be less obvious. It would be better to pick schemes that can create S_e with simple or regular shape to ease the determination. Alternatively, when V_e is unavailable, other objective functions can be employed and size of S_e can be determined separately. Balance between computation effort and evaluation accuracy should be struck.

As long as a point inside S_f (not necessarily the nominal point) can be identified, S_e inscribable in S_f can be created and FI_V determination can be performed. The position of P_R can be optimized to give a closer A_f approximate. When P_R in the second $\overrightarrow{v_j}$ selection scheme in Example 1 is optimized, a closer FI_V (*=0.39*) is obtained. Furthermore, if the joint probability distribution within S_o is evenly distributed and its cumulative probability equals to *1*. The joint probability at each point in S_o equals $1/A_o$ and SF is A_f/A_o which is equivalent to FI_V. Since the probability distributions may not be available, a uniform probability distribution can always be a reliable guess and FI_V can measure the chance of feasible operation in this case. Determination of FI_V is relatively facile compared with that of SF, thus FI_V measurement is generally more applicable.

4. Conclusions

A new flexibility index is proposed. This metric provides valuable information on the ability of a process to handle steady state operation as the process parameters alter. Its determination approach is facile and easy to handle. It is generally applicable to most process constraints and systems with multi-uncertain parameters.

Acknowledgements

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