16th European Symposium on Computer Aided Process Engineering
and 9th International Symposium on Process Systems Engineering
W. Marquardt, C. Pantelides (Editors)



# Disturbance propagation and rejection models for water allocation network

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## Abstract

In this paper, a system modeling methodology for characterizing disturbance propagation is proposed to predict the worst scenario to occur in a water allocation network during synthesis. Then a model-based design procedure for suitable mixing for disturbance rejection is introduced. A case study is given to show the efficacy of the methodology.

**Keywords**: Water allocation network; process modeling and design; disturbance; propagation and rejection.

## 1. Introduction

When a water allocation network is highly integrated, the resultant plant will be structurally more interacted among process units. If a water network is improperly designed, its operation may be unstable or even uncontrollable regardless of the advances of control techniques (Yan and Huang, 2002). Up to now, all the work about optimal water allocation network is based on designing the network for nominal operation conditions and no work on operational aspect, for example, flexibility or controllability, of water networks has been reported.

On the other hand, in the synthesis of mass exchanger networks, some methodologies for system flexibility or controllability analysis have been proposed (Papalexandri and Pistikopoulos, 1994; Yan and Huang, 2002; Yang et al, 1999; Karafyllis and Kokossis, 2002).

A water allocation network, although sometimes at some extent can be treated as a mass exchange network, has some different features from a mass exchange network as follows. (1) The quality of water may not be affected by a target pollutant, but characterized by some other parameters, e.g., PH, hardness, and turbidity (Huang, et al, 1999). (2) Not all water-using processes can be described with the available models for mass exchangers, for examples, filtration, centrifugal separation (Huang, et al, 1999). (3) When designing or analyzing a water network, people are more interested in the parameters about water rather than about process streams. Although the original disturbances come from some process streams, the target parameters to be controlled are usually some inlet and outlet contaminant concentrations of water streams.

This paper is mainly focuses on the development of an analytical tool for evaluating structural disturbance propagation and disturbance rejection in a water allocation network.

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## 2. Process structural related disturbance propagation model

*2.1. Unit based model*

In a water system, the original disturbance usually comes from some process streams. Then the disturbances will be propagated to some water streams with which the process streams transfer contaminants. Further, the water discharged from these processes will propagate disturbance to the downstream processes.

When designing or analyzing a water network, people are more interested in the parameters about water streams rather than about process streams. Therefore, the disturbance propagation in process streams should be translated to water streams.

Any disturbance from a process stream will lead to a variation of the mass load to be transferred to the water stream in the process. For example, the variation of contaminants mass transfer load for a mass transfer water-using unit is as follows:

$$\delta m = \delta M_P (C_P^{IN} - C_P^{OUT}) + M_P(\delta C_P^{IN} - \delta C_P^{OUT}) \tag{1}$$

Therefore, when a disturbance from the process stream is given and the process is specified, the variation of contaminants mass transfer load, δm, can be calculated. In this way, any disturbance original from a process stream can be expressed by the corresponding variation in the mass transfer load of contaminants in the process, which in turn is the original disturbance of the water allocation network.

Then from the contaminant balance, the disturbance propagating to the water outlet concentration from $\delta m$ will be the second term in the right side of Equation (2).

Then if the water from the disturbed process is reused in the downstream processes, it will affect the water inlet concentration in the process so that the disturbance will be propagated in the whole water network.

When there is $\delta C_W^{IN}$ in the inlet concentration of water of a process, it will influence the operation of the process and cause disturbances of the outlet concentration of the water stream. The variation of the outlet concentration caused only by the variation of the inlet concentration is the same as the variation of inlet concentration of the water stream.

Therefore, the outlet concentration disturbance of water in a process caused by the variation of the contaminant transfer load and the variation of the inlet concentration can be expressed as

$$\delta C_W^{OUT} = \delta C_W^{IN} + \delta m / M_W \tag{2}$$

In a water allocation network, addition to water-using processes, there are some water mixers and water splitters (Yang et al, 1999). For a mixer, the fluctuation of flow rate and that of concentration of the output stream after mixing can be calculated as follows.

$$\delta C_m^{OUT} = \sum_{i=1}^{n} M_{mi}^{IN} \delta C_{mi}^{IN} / M_m^{OUT} \tag{3}$$

For a stream split to n branches, the fluctuation of the outlet concentration of each branch is the same as that of the stream before splitting.

*2.2. System disturbance model*

To identify an optimal water allocation network, a superstructure for the network is to be used, in which the output water stream from any process can be partially or entirely reused by other processes (Savelski and Bagajewicz, 2001). Therefore, at the inlet of every water-using process, there is a mixer and at the outlet of every water-using process, there is a splitter. If there are N water-using units and one freshwater stream, then there are N potential water reuse streams. Then, the mass balance at the inlet of each water-using process unit should be considered as follows.

$$M_{Wi} = \sum_{j=0}^{N} M_{j \to i} \tag{4}$$

$$M_{Wi} C_i^{IN} = \sum_{j=1}^{N} M_{j \to i} C_j^{OUT} \tag{5}$$

The target parameters to be controlled are usually some inlet and outlet concentrations of water streams. Because every inlet water stream is the outlet water stream from a certain mixer, the fluctuations of the target inlet concentrations about all N processes can be predicted by the following model.

$$\delta C_i^{IN} = \sum_{j=1}^{n} M_{j \to i} \delta C_j^{OUT} / M_{Wi} \tag{6}$$

$\delta C^{OUT}$ can be determined by

$$\delta C_j^{OUT} = \delta C_j^{IN} + \frac{1}{M_{wi}} \delta m_i \tag{7}$$

For new design, such superstructure will be set up, while for existing network, the network structure is specified, but the concentration variation will be determined by using the above equations.

## 3. Disturbance rejection model with optimal mixing

Mixing some water at lower concentration with the disturbed inlet water stream to a strict controlled process can easily reject the disturbance. Then source disturbances ($\delta m_i$) can be rejected through manipulating mixings ($\delta M_{Wi}$) so that the fluctuations of the target inlet concentrations can be within a certain range. Mathematically, their relationship can be described as

$$\delta C_i^{IN}{}' = \delta C_i^{IN} - \varepsilon_i \tag{8}$$

$$\delta C_i^{OUT}{}' = \delta C_i^{OUT} - \Delta_i \tag{9}$$

$$(M_{wi} + \delta M_{wi}) \cdot (C_i^{OUT}{}' - C_i^{IN}{}') = m_i + \delta m_i \tag{10}$$

$$\delta M_i = \sum_{\substack{j=0 \\ j \neq i}}^{N} \delta M_{j \to i} \tag{11}$$

The objective functions of the design are the minimum freshwater consumption at normal operation conditions and the minimum freshwater addition for control at disturbances. For the control of an existing network, the objective function is minimum freshwater addition for control at disturbances. Therefore, the optimization problem is a NLP programming.

## 4. Case study

Table 1 gives the limiting water data, the mass load of contaminants, introduced source disturbances, and the permissible target concentration ranges.

Table 1 Water data

| Process No. | $m$ (kg/h) | $C^{IN}_{Max}$ (ppm) | $C^{OUT}_{Max}$ (ppm) | $\delta m$ (kg/h) | $\delta C^{IN}_{Max}$ (ppm) | $\delta C^{OUT}_{Max}$ (ppm) |
|---|---|---|---|---|---|---|
| 1 | 2.0 | 25 | 80 | 0.2 | 1 | 5 |
| 2 | 2.88 | 25 | 90 | 0.29 | 1 | 6 |
| 3 | 4.0 | 25 | 200 | 0.4 | 2 | 10 |
| 4 | 3.0 | 50 | 100 | 0.3 | 4 | 6 |
| 5 | 30.0 | 50 | 800 | 3 | 5 | - |
| 6 | 5.0 | 400 | 800 | - | - | - |
| 7 | 2.0 | 200 | 600 | - | 40 | 20 |
| 8 | 1.0 | 0 | 100 | 0.1 | 0 | 4 |
| 9 | 20.0 | 50 | 300 | 2 | 4 | 20 |
| 10 | 6.5 | 150 | 300 | 0.65 | 10 | 20 |

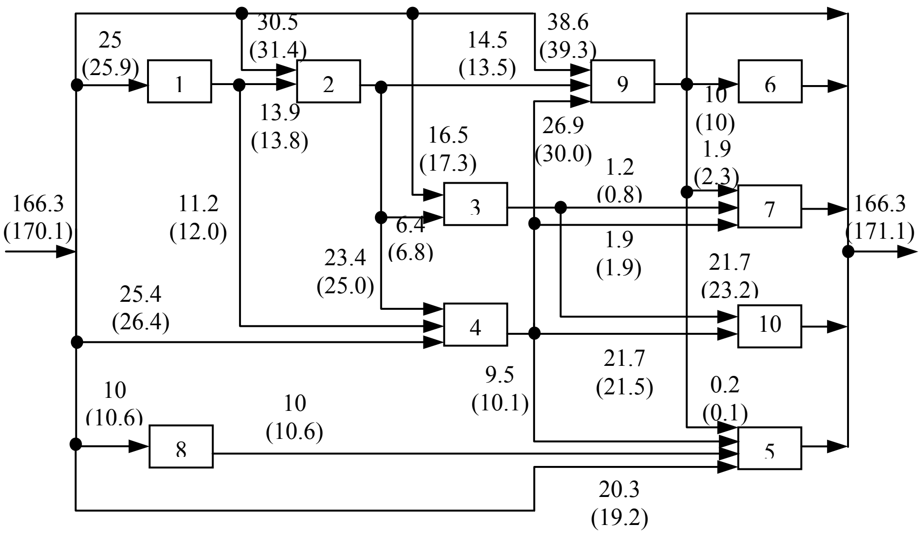


Fig. 1 Solution A

Two network design alternatives are available, both of which reach the freshwater target under normal operation conditions. Solution A is synthesized by Savelski and Bagajewicz (2001), as shown in Fig. 1, and Solution B by the authors featured with simplest structure (Fig. 2). In both the figures, the numbers outside the brackets are the water flowrates at normal operation conditions, and the numbers inside the brackets are the water flowrates at the worst disturbance scenarios.

The computed target concentration ranges of each process for each solution are summarized in Table 2. It can be found that both of the solution alternatives cannot meet the control requirements.

The freshwater consumption target under disturbances can be determined as 170.1, that is, the minimum freshwater increment, $\Delta M_{W0}$=3.8.

The next step is to control the fluctuation within the permissible range by mixing some water with lower concentrations. No new pipe is allowed. The adjustment schemes of the two alternatives are also shown in Fig. 1 and 2, respectively.

It can be found solutions A can reach the freshwater target but Solution B cannot. Therefore, from the freshwater consumption point of view, solutions A is better.

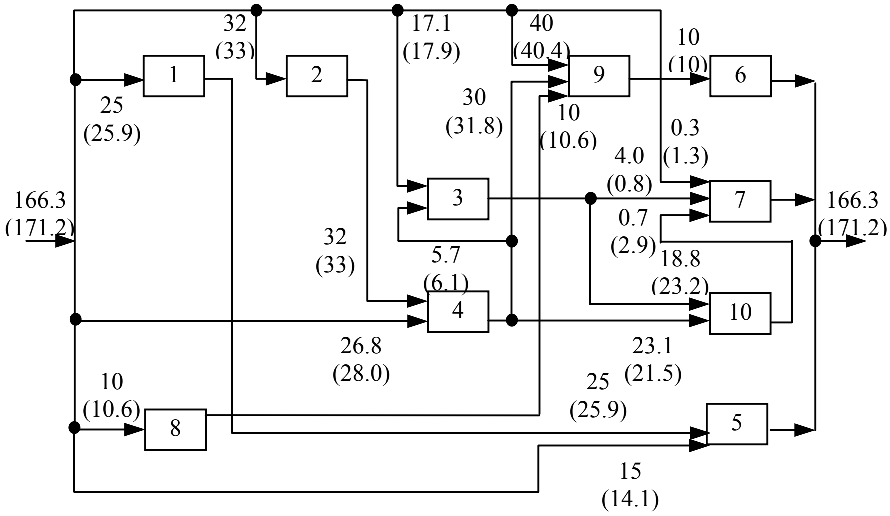


Fig. 2 Solution B

Table 2 Comparison of the target concentration fluctuation

| $\delta C$ | $\delta C_1^{IN}$ | $\delta C_2^{IN}$ | $\delta C_3^{IN}$ | $\delta C_4^{IN}$ | $\delta C_5^{IN}$ | $\delta C_7^{IN}$ | $\delta C_9^{IN}$ | $\delta C_{10}^{IN}$ |
|---|---|---|---|---|---|---|---|---|
| A | 0 | ≤3 | ≤3 | ≤5 | ≤5 | ≤20 | ≤5 | ≤15 |
| B | 0 | 0 | ≤2 | ≤4 | ≤5 | ≤20 | ≤5 | ≤9 |
| $\delta C$ | $\delta C_1^{OUT}$ | $\delta C_2^{OUT}$ | $\delta C_3^{OUT}$ | $\delta C_4^{OUT}$ | $\delta C_8^{OUT}$ | $\delta C_9^{OUT}$ | $\delta C_{10}^{OUT}$ | $\Delta M_{W0}$ |
| A | ≤8 | ≤9 | ≤20 | ≤10 | ≤10 | ≤30 | ≤30 | ≤3.8 |
| B | ≤8 | ≤9 | ≤20 | ≤9 | ≤10 | ≤30 | ≤29 | ≤4.9 |

## 5. Conclusion

Disturbance propagation can be explored by the system disturbance propagation model introduced in this paper, so as to predict the controlled concentration fluctuations. Total disturbance rejection at the level of structure can be achieved by optimal mixing. In this paper, the disturbance rejection model with optimal mixing is also introduced.

The objective function in the methodology proposed in this paper is freshwater consumption both under normal operation conditions and under disturbances. When considering capital cost and control complex, a model can be established easily based on the model in this paper.

## Acknowledgment

This work is supported by National Natural Science Foundation of China (20376066 and 20436040), and the Major State Basic Research Development Program of China (2003CB214500). The authors are also grateful to the U.S. ACS-PRF for the Summer Research Fellowship as part of a grant to Prof. Yinlun Huang at Wayne State University.

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## Nomenclature

$C$ = *concentrations of contaminant*
$M$ = *mass flowrate*
$m$ = *mass load of contaminant.*
$N$ = *number of water-using processes in the system*
$\Delta$ = *control correction vector for outlet concentration*
$\delta$ = *change*
$\varepsilon$ = *control correction vector for inlet concentration*

**Superscripts**

$IN$ = *inlet*
$OUT$ = *outlet*
‘ = *after controlled by mixing*

**Subscripts**

$i$ = *process i*
$j \rightarrow i$ = *from process j to process i,*
$m$ = *mixer*
$Max$ = *control precision requirement*
$P$ = *process stream*
$W$ = *water*
$0$ = *freshwater.*