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A Rigorous MINLP for the Simultaneous Scheduling and Operation of Multiproduct Pipeline Systems

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Abstract

The problem addressed in this work is based on a system that is composed by a refinery that transfers multiple products through a single pipeline connected to several depots. The depots must supply local consumer markets with large amounts of oil derivatives. The objective of this work is to develop a rigorous formulation for pipeline scheduling. This novel formulation for pipeline scheduling uses a continuous time representation and also considers a rigorous hydraulic approach for pipeline operations. The MINLP formulation is compared to a hydraulic linear formulation that parameterizes flow and yield rates. The results show that the MINLP presents better solutions and computational performance.

Keywords: pipeline, scheduling, hydraulic, continuous time representation.

1. Introduction

Pipelines transfer large amounts of different petroleum types and their products at a lower cost than any other transportation mode. Pipeline scheduling is a challenging operation due to their multiproduct nature and to the large amounts of energy required. Westerlund et al. (1994) solve the optimal pump configuration that consists of a set of series and parallel pumps. Van den Heever and Grossmann (2003) show a hydrogen supply chain network where optimal flow rates are obtained from a detailed hydraulic representation. Neiro and Pinto (2004) and Rejowski Jr. and Pinto (2004) present mixed-integer models for planning and scheduling of pipeline operations, respectively.

The objective of this work is to develop a hydraulic formulation for pipeline scheduling. The formulation is based on a continuous time representation that handles variable flow rates in the pipeline.

2. Problem Description

A refinery must transfer *P* petroleum products to *D* depots connected to a single pipeline. The depots have to satisfy product demands determined by local consumer

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markets. Furthermore, several booster station configurations may be utilized for pumping the products along the time horizon (Figure 1).

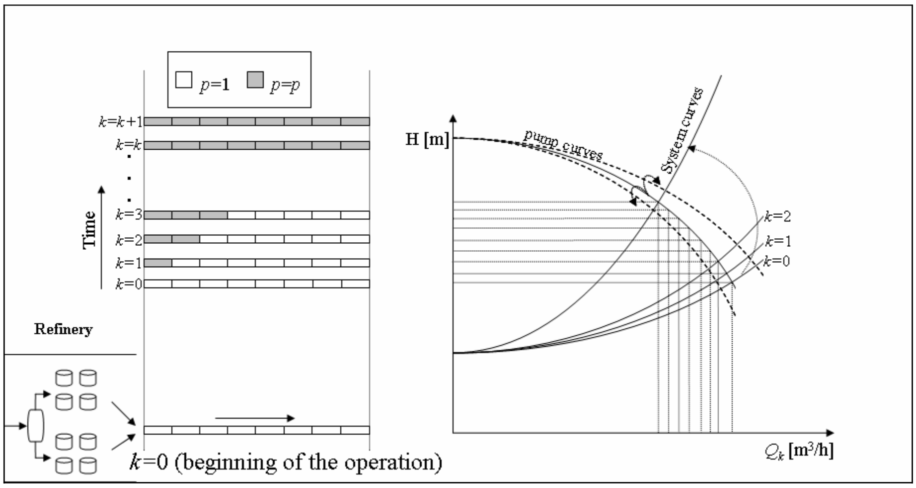

Figure 1 - Distribution pipeline system

The refinery stores its products in dedicated tanks and these load their derivatives at predetermined amounts and initial and end time instants along the time horizon. Whenever the pipeline receives any product from the refinery, the same amount leaves it and feeds the depots at the same time in order to satisfy volume and flow rate constraints. The depots have to control their inventory levels and fulfill product demands. Due to demand variations, the pipeline operation is in most cases intermittent.

Figure 1 also shows the system and the pump operating curves. The hydraulic behavior depends on the sequencing of products and their allocation inside the pipeline, the flow rate variations, the topographical profile of the pipeline and diameter variations. The pump curves operating depend on the pump types, adjustable speeds, and represent the energy that is provided by the booster stations to transfer the products. Moreover, the pipeline operating point is given by the intersection of these two curves.

3. Optimization Model

The MINLP formulation is based on the previously MILP developed by Rejowski Jr. and Pinto (2004), which was however based on a discrete time formulation. The objective function is shown by equation (1).

$$
\begin{aligned}
\text{Min } C = & \sum_{k=1}^{K}\left[\sum_{p=1}^{P} CER_p.VR_{p,k} + \sum_{p=1}^{P}\sum_{d=1}^{D} CED_{p,d}.VD_{p,d,k}\right].\Delta_k \\
& + \sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{k=1}^{K} CP_k.OPW_{d,k}.\Delta_k + \sum_{p=1}^{P}\sum_{p'=}^{P}\sum_{d=1}^{D}\sum_{k=1}^{K} CONTACT_{p,p'}.TY_{p,p',k}^{d}
\end{aligned}
\qquad (1)
$$

The terms in brackets represent the inventory costs at the refinery and at the depots that take into account the variable time interval duration (Δ_k). The next term accounts for the pumping costs, that are composed by the overall power consumption by the booster stations ($OPW_{d,k}$) and the time interval that the pipeline remains in operation (Δ_k). The last term corresponds to costs associated to product interface formation (Rejowski Jr. and Pinto, 2004).

3.1 Temporal and refinery constraints

The refinery sends the products to its tanks in lots (i) with predefined amounts and initial and end instants, as shown in Figure 2. The temporal constraints must satisfy the operational time horizon (H), according to equation (2) and their initial (Ti_i) and end instants (Tf_i) along the transfer operation, as in equations (3) and (4), respectively. The mass balances at the refinery are given by (5) and their volumes are bounded in (6).

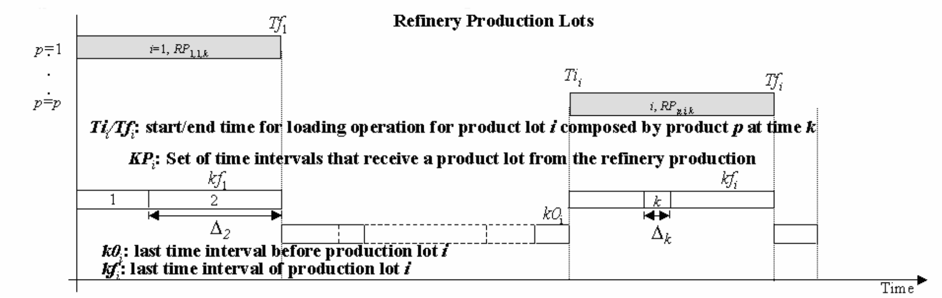

Figure 2 – Production lots and continuous time representation

$$\sum_{k=1}^{K} \Delta_k = H \tag{2}$$

$$\sum_{k \le k0_i} \Delta_k = Ti_i \qquad \forall i \tag{3}$$

$$\sum_{k \le kf_i} \Delta_k = Tf_i \qquad \forall i \tag{4}$$

$$VR_{p,k} = VRZERO_p + \sum_{i=1}^{I} \sum_{\substack{k' \le k \\ k' \in KP_i}} \left(RP_{p,i,k'} \times \Delta_k - XR_{p,k'} \times U_d \right) \qquad \forall p,\ d=1,\ k \tag{5}$$

$$VRMIN_{p,k} \le VR_{p,k} \le VRMAX_{p,k} \qquad \forall p,\ k \tag{6}$$

3.2 Pipeline scheduling and depot constraints

The idea of this approach is to assign logical variables $XV^{d}_{p,l,k}$ to each (*product, depot, pack, time*). The pipeline operation can be expressed by disjunction (7) (Rejowski Jr. and Pinto, 2004) that is composed by two additional terms that relate the flow rate variations and the time interval durations. This linear disjunction can be transformed into mixed-integer constraints. Boolean variable $Y_{d,k}$ controls the pipeline segment operation and is expressed as logical 0-1 variable $XS_{d,k}$ in mixed-integer form.

$$\left[\begin{array}{ll} \multicolumn{2}{c}{Y_{d,k}} \\ XV^{d}_{p,l,k} = XT_{p,d,k} & \forall p, l=1 \\ XV^{d}_{p,l,k} = XV^{d}_{p,l-1,k-1} & \forall p, l=2,\dots,L_d \\ XD_{p,d,k} + XT_{p,d+1,k} = XV^{d}_{p,L_d,k-1} & \forall p \\ \Delta_k = U_d/(v_k.A) & \\ v_k^{LO} \le v_k \le v_k^{UP} & \end{array}\right] \vee \left[\begin{array}{ll} \multicolumn{2}{c}{Y_{d,k}} \\ XV^{d}_{p,l,k} = XV^{d}_{p,l,k-1} & \forall p, l=1 \\ XV^{d}_{p,l,k} = XV^{d}_{p,l,k-1} & \forall p, l=2,\dots,L_d \\ XD_{p,d,k} + XT_{p,d+1,k} = 0 & \forall p \\ \Delta^{LO}_{STOP} \le \Delta_k \le \Delta^{UP}_{STOP} & \\ v_k = 0 & \end{array}\right]$$

$$\forall d>1,\ k=2,\dots,K \qquad (7)$$

If $Y_{d,k}$ is true, the first pack of the segment takes the feed variable values ($XT_{p,d,k}$ or $XR_{p,k}$ for d=1). The products in the pipeline are displaced from pack l-1 to pack l. The product stored at pack L_d can be sent to depot d ($XD_{p,d,k}$) or to the next segment ($XT_{p,d+1,k}$). The velocity limits (v_k) must be respected when the pipeline operates and its time interval (Δ_k) is equal to the ratio between the pack size (U_d) and the flow rate used at time k. If the segment remains idle, $Y_{d,k}$ is false and all packs keep their contents. Interfaces are detected when the first two packs from the segments store different products. Moreover, any pipeline segment can only be stopped if it does not contain any interfaces. The depot constraints are similar to the ones of the refinery and demands must be met at the end of the operation. Finally, Rejowski Jr. and Pinto (2004) propose integer cuts that relate demands and the initial inventory at the depots and in the segments.

The temporal variables are disaggregated into two parcels. The first one (Δ_k^1) regards the pipeline operation, whereas the second (Δ_k^2) considers the time that the pipeline remains idle. Equations (9) and (10) state that the time that the pipeline operates at interval k must be equal to the ratio between the pipeline volume pack (U_d) and its operational flow rate ($v_k.A$). Equation (11) imposes zero values to Δ_k^1 whenever the pipeline remains idle. The product velocities must take positive values when the pipeline is operative, according to (12).

$$\Delta_k = \Delta_k^1 + \Delta_k^2 \qquad \forall\, k \qquad (8)$$

$$\Delta_k^1.(v_k.A) = U_d.XS_{d,k} \qquad \forall d,\, k \qquad (9)$$

$$\Delta_k^1 \le U_d.XS_{d,k}\Big/\left(v_k^{LO}.A\right) \qquad \forall d,\, k \qquad (10)$$

$$\Delta^{LO}_{STOP}.\left[1 - XS_{d,k}\right] \le \Delta_k^2 \le \Delta^{UP}_{STOP}.\left[1 - XS_{d,k}\right] \qquad \forall d,\, k \qquad (11)$$

$$v_k^{LO}.XS_{d,k} \le v_k \le v_k^{UP}.XS_{d,k} \qquad \forall d,\, k \qquad (12)$$

3.3 Flow rate and hydraulic constraints

The hydraulic model is described by disjunction (13). If $Y_{d,k}$ is true, the friction factor ($f_{d,l,k}$) follows equation (13a), where logical 0-1 variable $XV^{d}_{p,l,k}$ is multiplied by an exponential that depends on the product physical properties. The friction losses for each pack l take into account the friction factor, the pipeline internal diameter (Din), the pack extension ($LEXT_l$) and the pipeline flow rate, according to (13b). The yield rates (η_k) used by the booster stations are represented by an m^{th} order polynomial that depends on the velocity. The booster stations have several stages (N) that can be activated during the operation from binary variable ($NS_{n,k}^{d}$) and their power consumption ($PW_{n,k}^{d}$) must satisfy lower and upper bounds, according to (13d). Finally, the energy balance over the pipeline is given by (13e) and the overall power consumption from (1) is given in (13f). If the pipeline remains idle, the friction factor and losses, the power consumed by the booster stations and the yield rates equal to zero, as shown by (13g) to (13k).

$$
\left[\begin{array}{lr}
\multicolumn{2}{c}{Y_{d,k}} \\
f_{d,l,k} = \sum_{p} XV_{p,l,k}^{d} \cdot \left(a_p \cdot v_k^{-b_p}\right) \quad \forall l \le L_d & (13a) \\
lwf_{d,l,k} = LEXT_{d,l} \cdot f_{d,l,k} \cdot v_k^2 / 2.Din \quad \forall l \le L_d & (13b) \\
\eta_k = c_m \cdot \left(v_k\right)^m + c_{m-1} \cdot \left(v_k\right)^{m-1} + \ldots + c_{m-n} \cdot \left(v_k\right)^{m-n} + \ldots + c_0 & (13c) \\
PW_k^{LO} . NS_{d,k} \le PW_{n,k}^{d} \le PW_{d,k}^{UP} . NS_{d,k} \quad \forall n & (13d) \\
\left[\sum_{l=0}^{L_d} g.\rho_p.(z_{d,l} - z_{d,l-1}) + \sum_{l=1}^{L_d} lwf_{d,l,k}\right] v_k A = \sum_{n=1}^{N} PW_{n,k}^{d} & (13e) \\
OPW_{d,k} = \sum_{n=1}^{N} PW_{n,k}^{d} \Big/ \eta_k & (13f)
\end{array}\right]
\vee
\left[\begin{array}{lr}
\multicolumn{2}{c}{\neg Y_{d,k}} \\
f_{d,l,k} = 0 \quad \forall l \le L_d & (13g) \\
lwf_{d,l,k} = 0 \quad \forall l \le L_d & (13h) \\
\eta_k = 0 & (13i) \\
PW_{n,k}^{d} = 0 \quad \forall n & (13j) \\
OPW_{d,k} = 0 & (13k)
\end{array}\right]
\quad \forall d, k
$$

4. Examples

This section addresses two examples that are composed by a depot. In both examples the proposed MINLP (M1) is compared with an MILP proposed by Rejowski Jr. and Pinto (2003) (M2). In M2, the pipeline operates with fixed flow and yield rates and uses a discrete time representation. Data for this example are given in table 1. GAMS/CPLEX/DICOPT++ (Brooke et al., 2000) was used to implement and solve the models. Examples E1 and E2 consider a 100 and 150 hours time horizon, respectively.

Table 1 – Data for Example

Number of Time Intervals (K)					
Formulation		*E1* (H=100 hours)		*E2* (H=150 hours)	
MINLP (M1)		14		15	
MILP (Rejowski Jr. and Pinto, 2003) (M2)		20 (5 hours each)		30 (5 hours each)	
Production Profile		50 first hours (*p*=1, *p*=2)	50 last hours (*p*=3, *p*=4)	100 first hours (*p*=1, *p*=2)	50 last hours (*p*=3, *p*=4)
Product	CER_p [\$/m^3.h]	$CED_{p,d}$ [\$/m^3.h]	$RP_{p,i,k}$ [x10^{-2} m^3/h]	$CONTACT_{p,p'}$ [x10^{-2} \$]	
Gasoline (1)	0.010	0.0240	2.5	(1,2)/(1,3)/(1,4) 100/150/120	
Diesel Oil (2)	0.009	0.0250	2.5	(2,1)/(2,3)/(2,4) 100/150/120	
LPG (3)	0.005	0.0240	1.5	(3,1)/(3,2)/(3,4) 150/X/X	
Jet fuel (4)	0.007	0.0250	1.5	(4,1)/(4,2)/(4,3) 120/X/X	
Product	$VRZERO_p$ [x10^{-2} m^3]	$VDZERO_{p,d}$ [x10^{-2} m^3]	$XVZERO_{p,d,l}$	$DEM_{p,d}$ [x10^{-2} m^3] E1/E2	
Gasoline (1)	1,000	150	0	170/150	
Diesel Oil (2)	1,050	200	l=1,2,3,4,5,6,7,8	100/100	
LPG (3)	400	200	0	10/110	
Jet fuel (4)	700	200	0	50/50	

Table 2 shows the objective function values as well as the computational performance for both examples and models.

Table 2 – Results for the proposed example

Example – Model	E1.M1	E1M2	E2.M1	E2.M2
Relaxed Solution (C)	26,042.2	8,372.3	34,927.2	10,326.01
Solution Found (C)	26,060.2	38,247.5	45,865.1	103,756.8
Gap (%)	-	0.000	-	0.8100
CPU Time [s]	197.6	3,544.7	532.9	54,900.0
Number of Equations	2,286	5,296	2,448	7,936
Number of Variables	1,537	3,217	1,647	4,827

The MILP presented by Rejowski Jr. and Pinto (2003) (M2) has several limitations. Firstly, it requires a large number of time intervals to synchronise the loading and unloading tasks. Consequently, a much larger model is created that increases CPU times. Furthermore, model M1 demands fewer time intervals for the operation and allows variations in the flow and yield rates within the time horizon.

Another important aspect of formulation MDH besides its fixed time interval duration is that it can also provide flow rates that result in low yield rates for the booster stations during the horizon due to the tasks synchronisation at the refinery, as shown in Figure 2. Model E1.M1 found a solution that is 46.8% higher than model E1.M2, whereas model E2.M1 found a solution with total costs 126% higher than model E2.M2. These factors result in formulations with fewer time intervals and with better objective function values for this challenging problem. Figure 3 shows the yield and flow rates for examples E2.M1 and E2.M1. Note that in E2.M1 the pipeline remains idle in the first 50 hours, whereas in E2.M2 it does not operate at the end of the operation.

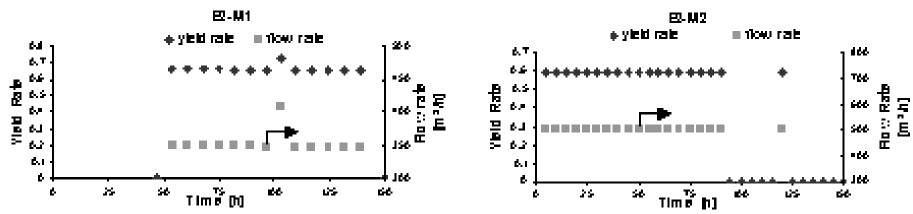

Figure 3 – Yield and flow rates for E2.M1 and E2.M2

5. Conclusions

This work addresses the simultaneous multiproduct pipeline scheduling and hydraulic operation. The resulting MINLP showed better results than a previous MILP.

References

Brooke, A., Kendrick, D., Meeraus, A. A., 2000, The Scientific Press. Redwood City, USA.

Neiro, S., Pinto, J. M., 2004, Comp. & Chem. Eng., 28, 871.

Rejowski Jr.,R., Pinto, J.M., 2003, AIChE Annual National Meeting [446y], San Francisco, USA.

Rejowski Jr., R. & Pinto, J.M., 2004, Comp. & Chem. Eng., 28, 1511.

Van den Heever, S.A.; Grossmann, I.E., 2003, Comp. & Chem. Eng., 27, 1813.

Westerlund, T., Petterson, F., Grossmann, I.E., 1994, Comp. & Chem. Eng., 18, 845.

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