

Minimum-Cost Operation in Heat-Exchanger Networks

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Abstract

This report revises the use of the maximum energy recovery criterion as objective function for real-time optimisation of heat-exchanger networks. Though in general this criterion leads to minimum total heat exchanged in the service units, it is not sufficient to achieve the actual minimum operation cost. This analysis discusses the characteristics of the network structure for which the last statement is applicable, and proposes an alternative performance index to address more directly the final economic objective for which these heat-recovery systems are created. An application example demonstrates the significant differences in operating conditions that may result from using one or the other criterion, something of outmost importance when defining online optimisation for these systems.

Keywords: heat-exchanger networks, optimal operation, utility cost, heat integration

1. Introduction

Marselle et al. (1982) was one of the firsts articles to discuss the optimal operation problem of heat-exchanger networks (HEN), where simultaneous regulation and optimisation are considered. More closely, important results on these topics were presented in Mathisen thesis (1994), Uzturk and Akman (1997), Aguilera and Marchetti (1998) and Glemmestad et al. (1999). Most of these works provide specific representation models of HENs that can be used for searching the convenient operating point attending to the full network capacity under given inlet stream conditions and temperature targets.

When an optimal operation is regarded, it means that at least the following two goals are pursued: i) outlet temperature targets for the process streams, and ii) minimum operating cost. This second goal, however, has been frequently understood as attaining the operation condition that yields the greatest possible heat integration. However, this report objective is to demonstrate and analyse the existence of an additional optimisation space going beyond maximum energy recovery. The presented results show that though maximum heat integration in HENs implies minimum overall heat exchanged in service units, in many cases it is not sufficient for determining minimum operating cost.

The following section describes the structural network conditions that are necessary for the existence of an extra degree of freedom associated to the service units. Then, the third section shows how the optimisation problem must be formulated when minimum utility costs are pursued. The numerical results of running an example problem with

different objective functions are shown in Section 4, and the final section gives the conclusions of this work.

2. Conditions for Extra Degrees of Freedom of the Service Variables

Based on the energy balance of the total network, Aguilera and Marchetti (1998) analysed the degrees of freedom of a HEN as structural possibilities of optimisation once process stream targets and input conditions are fixed. However, once maximum heat integration is achieved in a flexible HEN, there may be additional degrees of freedom for the service variables depending not only on the network structure but also on conditions of temporary operating points. To determine when these characteristics are present, let us analyse first the relationships describing the tasks to be accomplished on each process stream with a temperature target.

To reach the temperature target a hot process stream i has to release the energy Q_i and a cold process stream j has to receive the energy Q_j . These energies are released and received through different process-to-process exchanges q_k and service tasks q_{ci} and q_{hj} along each stream path, namely

$$Q_i = -\sum_{k \in K_i} q_k - q_{c_i}, \quad i \in H, \quad Q_j = \sum_{k \in K_j} q_k + q_{h_j}, \quad j \in C \quad (1)$$

where H stands for all the hot streams, C represents all the cold streams, and K_i and K_j are the subset of heat-exchangers on streams i and j respectively. Because the sum of all the process-to-process heat-exchanger duties represents the same integrated energy in both equations, a simple relationship between cold and hot service units can be obtained

$$\sum_{j \in C} q_{h_j} - \sum_{i \in H} q_{c_i} = \sum_{j \in C} Q_j + \sum_{i \in H} Q_i. \quad (2)$$

Furthermore, notice that for fixed temperature targets and defined inlet conditions, the right side of (2) is constant. Calling global network duty D to this constant, the following expression is reached:

$$\sum_{j \in C} q_{h_j} = \sum_{i \in H} q_{c_i} + D. \quad (3)$$

Let us consider now a performance index typically used when maximum heat integration is required, or equivalently, an index expressing the total heat flow handled by the service units

$$J = \sum_{j \in C} q_{h_j} + \sum_{i \in H} q_{c_i} \quad (4)$$

Combining (3) and (4) gives the following relationship:

$$J = 2 \sum_{i \in H} q_{c_i} + D = 2 \sum_{j \in C} q_{h_j} - D \quad (5)$$

According to (5), for any HEN having n_h active heaters and n_c active coolers, under optimal heat integrating condition, an extra degree of freedom may appear when $n_h \geq 2+n_c$, or $n_c \geq 2+n_h$. For instance, if all the $q_{c_i} = 0$ (or $q_{h_j} = 0$), the index function J is constant despite of how many q_{h_j} (or q_{c_i}) are in the network, how is the connecting

structure, or how is the total service duty distributed among the q_{h_j} (or q_{c_i}). However, performing the same heat duty in different exchanger units (with different UA) implies different utility flow rates. Thus, once the maximum heat integration is attained, different combinations of q_{h_j} (or q_{c_i}) are still possible, which implies different use of utility streams and consequently different utility costs. This extra degree of freedom, which goes beyond heat integration, appears also when a HEN having any number of cooling and heating services goes to temporary operating points where the number of active service units complies with any of the above relationships.

3. The Optimal HEN Operation Problem

Let us assume that the HEN structure and the heat exchanger areas are completely defined for a given case problem, where there are enough degrees of freedom as to perform steady-state optimisation. Assume also that all process stream targets are known, and that the convenient control structure for temperature regulation has been already defined. When the desired condition actually focus on minimum operating cost, the optimal solution can be obtained by solving the following minimization problem:

$$\min_{w_{c_i}, w_{h_j}} \left\{ \sum_i c_{c_i} w_{c_i} + \sum_j c_{h_j} w_{h_j} \right\} \quad i \in H, j \in C, \quad (6)$$

subject to

$$-\sum_{k \in K_i} q_k - q_{c_i}(w_{c_i}) = Q_i \quad i \in H, \quad (7)$$

$$\sum_{k \in K_j} q_k + q_{h_j}(w_{h_j}) = Q_j \quad j \in C, \quad (8)$$

$$q_k \leq e_k^o L_k^o \quad k \in \{1, ne + s\}, \quad (9)$$

$$-q_k \leq 0 \quad k \in \{1, ne + s\}, \quad (10)$$

$$q_{c_i}(w_{c_i}) = e_{c_i}(w_{c_i}) L_{c_i}(w_{c_i}) \quad i \in H, \quad (11)$$

$$q_{h_j}(w_{h_j}) = e_{h_j}(w_{h_j}) L_{h_j} \quad j \in C, \quad (12)$$

where ne stands for the total number of process-to-process heat-exchangers, s is the total number of serviced units, w_{c_i} stands for the cold utility flow rate of the service unit on the hot stream i , and w_{h_j} is the hot utility flow-rate of the service unit on the cold stream j . The supra index o in (9) indicates fully open control valve or fully closed bypass; the functions e and L are defined in the Appendix. Note that the utility costs per unit mass (c_{c_i} and c_{h_j}) are included in the performance index, and that (11) and (12) are non-linear equations determining w_{c_i} and w_{h_j} at the optimal operation condition.

If the network structure does not include stream splits or multiple bypasses (bypass to more than one unit), excellent initialisations for the above NLP problem are obtained by first solving the associate LP for maximum heat integration; this LP uses heat flows as exclusive optimising variables and excludes the equality constraints (11) and (12).

4. Example Problem

Figure 1 shows the sketch of a HEN composed by two process-to-process exchangers and three service units connected by one hot and two cold process streams. Table 1 gives the stream conditions, and Table 2 shows the UA values for two network designs A and B. The factor F_t is assumed 1.0 in all the units, and the utility cost c_{ci} and c_{hj} are also set to 1.

The first row in Table 3 shows the results obtained by solving the above optimisation problem when the design A is used. Though the problem of maximizing the utility expenses J makes no sense from the operating point of view, when this maximum is subject to the same energy recovery obtained before, an important reference solution is determined. For instance, the result in the second row of Table 3 is obtained by including the constraint $q_1 + q_2 = 115$, or equivalently $q_{s1} + q_{s2} + q_{s3} = 60$, in the problem formulation. Thus, the results in rows 1 and 2 become the extremes of a set of infinite solutions maintaining the same energy recovery, but showing different levels of utility expenses. Any of these solutions in the set could be reached by minimizing the total service heat duty depending on the initialisation or the numerical method used to find the minimum.

A similar numerical experience is repeated for design B, where the results show a greater cost function difference than in case A (see rows 3 and 4). Analysing the slack variables when there is an extra degree of freedom, it was noted that each extreme operation condition is associated with an active constraint (9) related to different units located in the path connecting S2 and S3.

Notice that with the stream conditions in Table 1, the solutions avoid using the cooler S1, creating the condition for an extra degree of freedom. However, when the output temperature of stream H1 is set to 50 °C, the cooler is activated and the extra degree of freedom is lost; see rows 5 and 6 where both, the $\min J$ and the constrained $\max J$ problems give the same solution for each design. Nevertheless, even though the designs A and B have the same total exchange area, they show different operation cost.

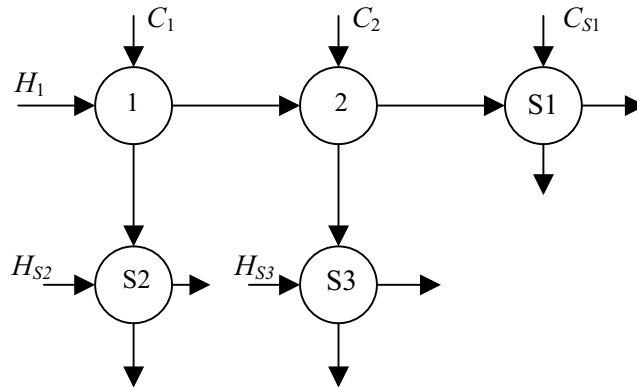


Figure 1: General structure of the HEN example

Table 1. Nominal process and service stream conditions

	H_1	C_1	C_2	C_{S1}	H_{S2}	H_{S3}
<i>wc</i>	1.0	1.5	0.5	1.0	1.0	1.0
Inlet temp. ($^{\circ}\text{C}$)	190	80	20	15	200	200
Outlet temp. ($^{\circ}\text{C}$)	75	160	130	--	--	--

Table 2. UA values used in the above network

	q_1	q_2	q_{s1}	q_{s2}	q_{s3}
UA (case A)	5.0	2.0	3.0	3.0	1.0
UA (case B)	5.0	2.0	3.0	1.0	3.0

Table 3. Effect of the extra degree of freedom on the operation cost.

Case	q_1	q_2	q_{s1}	q_{s2}	q_{s3}	x_{s1}	x_{s2}	x_{s3}	J
A – min J	102.07	12.93	0.0	17.93	42.07	0.0	0.345	0.32	0.665
A – max J	67.44	47.56	0.0	52.56	7.44	0.0	0.757	0.088	0.845
B – min J	102.07	12.93	0.0	17.93	42.07	0.0	0.39	0.274	0.664
B – max J	85.98	29.02	0.0	34.02	25.98	0.0	1.0	0.213	1.213
A – min/max J	102.07	31.50	6.43	17.93	23.50	0.155	0.345	0.208	0.533
B – min/max J	102.07	31.50	6.43	17.93	23.50	0.155	0.390	0.201	0.591

5. Conclusions

Any HEN having n_h active heaters and n_c active coolers, which operates under optimal heat integration, may have an extra degree of freedom when $n_h \geq n_c + 2$, or when $n_c \geq n_h + 2$. The reference to active service units emphasizes that this additional optimisation space may turn on and off depending on the operating condition. The immediate consequence of this finding is that the problem formulation for HEN online optimisation has to be adapted to effectively address the minimum utility cost objective. In other words, the extra degree of freedom means that optimising service costs by directly minimising expenses associated to utility flow rates may yield additional benefits as compared to maximising heat-integration. Besides, minimising expenses associated to utility flow rates implies maximising heat-integration. Furthermore, these results also show that HEN designs based on maximum energy recovery combined with minimum total-exchange-area cost should be revised when operation cost is the main goal.

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Appendix: Model of a Heat Exchanger in a Network

A convenient way of writing the main equations modeling the stationary condition of a single heat exchanger is obtained from the steady energy balance and the constitutive equation for the heat transferred, namely

$$q = w_i c_i (T_i^0 - T_i) = w_j c_j (T_j - T_j^0) = UA(F_T \Delta T_{ml}) \quad (A1)$$

In this expression, the supra index ⁰ stands for inlet conditions, A is the heat-exchanger area, U is the overall heat transfer coefficient, and F_T is a factor correcting the logarithmic mean temperature difference ΔT_{ml} to account for deviations from pure counter-current pattern. Algebraic rearranges combining the above equalities show the convenience of defining the following parameters:

$$R = \frac{w_j c_j}{w_i c_i} = \frac{(T_i^0 - T_i)}{(T_j - T_j^0)}, \quad N_{TU} = \frac{UA}{w_j c_j} \quad \text{and} \quad B = \exp\{N_{TU} F_T (R - 1)\} = \frac{(T_i^0 - T_j)}{(T_j - T_j^0)}. \quad (A2)$$

These three parameters help the computation of the heat-exchanger efficiency defined by

$$e = \frac{1 - B}{1 - RB} = \frac{(T_j - T_j^0)}{(T_i^0 - T_j^0)} \quad \text{for } R \neq B \neq 1, \quad \text{or} \quad e = \frac{N_{TU}}{1 + N_{TU}} \quad \text{for } R = B = 1 \quad (A3)$$

Combining Eqs. A1 to A3 allows writing the heat flow rate as

$$q = q(w_i, w_j) = e(w_i, w_j) L(w_j), \quad \text{with} \quad L(w_j) = w_j c_j (T_i^0 - T_j^0) \quad (A4)$$

where $L(w_j)$ may be interpreted as the virtual amount of exchangeable energy and the efficiency $e(w_i, w_j)$ defines the amount of heat transferred when a finite exchange area is available between the two fluids. Thus, the equations representing any heat-exchanger k being part of a network at the stream match (i, j) are similar to Eqs. (A4), the main difference is that the heat-exchanger inlet temperatures T_i^0 and T_j^0 are the results of previous exchanges, namely

$$T_i^0(k) = T_i^{in} - \frac{1}{w_i^{in} c_i} \sum_{l_i} q_{l_i}, \quad l_i \in pre_i(k), \quad i \in H \quad (A5)$$

$$T_j^0(k) = T_j^{in} + \frac{1}{w_j^{in} c_j} \sum_{l_j} q_{l_j}, \quad l_j \in pre_j(k), \quad j \in C \quad (A6)$$

where the superscript *in* stands for inlet to the network, and $pre_i(k)$ and $pre_j(k)$ represent exchanges previous to k . Notice that w_i or w_j may be different from the nominal inlet stream flow rate because of the control valve position. Thus, the heat duty upper bound is determined by (A4) for fully open control valve or fully closed bypass, namely

$$q \leq q^o = e(w_i^{in}, w_j^{in}) L(w_j^{in}) = e^o L^o. \quad (A7)$$