European Symposium on Computer Aided Process Engineering – 15
L. Puigjaner and A. Espuña (Editors)

Improving of Wavelets Filtering Approaches

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Abstract

In this work, some simple strategies for signals filtering and estimation with wavelets are presented. Firstly, it is studied the adequacy of some type of wavelet for filtering. Then, it is proposed a strategy to determine the best decomposition level and, then, to improve wavelet filtering accuracy. Some known benchmark signals are used to validate the performance of the proposed methods and their comparison with some existing approaches. The results obtained expand the applicability and reliability of existing filtering schemes with wavelets and propose some useful alternative to do it.

Keywords: Data rectification, Wavelets, Depth of Decomposition.

1. Introduction

Measured process signals are very important to support a wide number of engineering tasks with a critical impact on the global operation of the plant. Otherwise, these measurements inherently contain noise originating from different sources. Hence, data filtering is a critical step in the operation and control of any chemical plant.

Over the last decades, numerous techniques have been proposed for filtering or data rectification. If a process model is available, data reconciliation may be used. If it is not the case, but measurements are redundant, rectification based on an empirical process model derived from data may be proved. However for cases without model or redundancy in measurements the option is the use of univariate filters. These methods are the most widely used in the chemical and process industry (Bakshi, 1997) and include EWMA, median filters and so on. Most recently, because the multiscale nature of process data, wavelets have been proposed for data rectification. In this work, we are focusing on developments for this category of data filtering.

Wavelets are families of mathematical functions which are capable of decomposing any signal, y(t), into its contributions in different regions of the time-scale space such as:

$$y(t) = \sum_{\upsilon \in Z} c_{L,\upsilon} \cdot \phi_{L,\upsilon}(t) + \sum_{l=1}^{L} \sum_{\upsilon \in Z} d_{l,\upsilon} \cdot \psi_{l,\upsilon}(t) \tag{1}$$

Each term at right of the equation represent a decompose part of the original signal. $c_{L,\upsilon}$ are the approximation coefficients, $d_{l,\upsilon}$ are the detail or wavelets coefficients, $\phi_{l,\upsilon}$ represents scale functions, $\psi_{l,\upsilon}$ represents wavelet functions, l is the scale factor, υ is the translation factor and L is the coarsest scale, normally called the decomposition level.

The above decomposition has been shown as very useful for filtering and signal trend estimation (Donoho et al, 1995, Bakhtazad et al, 1999). In these applications, any measured variable signal, y(t), is assumed to be the result of:

$$y(t) = x(t) + e(t) \tag{2}$$

Where *x(t)* is the vector of true process variables and *e(t)* is the associated measurement error (noise). Then, the basic idea to estimate *x(t)* (filtering of *y(t)* and extracting the true trend) with wavelets is as follows (1) Decompose the raw signal by using wavelets (equation 1); (2) Remove wavelets coefficient below a certain threshold value β (thresholding step); (3) Reconstruct the processed signal using the inverse of the wavelet used.

The above procedure (Waveshrink method) was the first method proposed for filtering with wavelets (Donoho and Johnston, 1995). Other methods have also been proposed. In all cases, they are variations or extensions of the Waveshrink method and it remains as the more popular strategy for filtering. A practical difficulty encountered in the application of Waveshrink, consists on how to select the decomposition level L. As it is highlighted by Nounou (1999), thresholding of $d_{l,v}$ under high values of L may result in the elimination of important features of the signal, whereas thresholding under low values of L may not eliminate enough noise. Additionally, wavelets Daubechies (*dbN*) are commonly adopted for different filtering schemes (Doymaz et al, 2001; Addison, 2002) because their very good capabilities at representing polynomial behaviours within a signal. However, the choice of different *dbN* can slowly affect the quality of filtering (Nounou, 1999). In general, the choice *dbN* vary between authors and no rules of what to select exists.

In this work, an empirical study of filtering with wavelets is presented. Firstly, it is explored the ability of some popular wavelets for filtering. Then, it is proposed a strategy to determine the best decomposition level.

2. Analysing the performance of wavelets for filtering

It was conducted an experiment based on using different *dbN* and different L values within the Waveshrink scheme. The experiments were organised as follows:

- Typical signals from literature were used (Doymaz et al, 2001). They originally contain 1024 observations. In the experiments, they were used in the following intervals: (1) Blocks signal or S1 from 451 to 627; (2) Blocks signal or S2 from 620 to 965; (3) HeaviSine Signal or S3 from 251 to 820; Doppler signal or S4 from 35 to 550.
- All signals were contaminated with random errors of $\boldsymbol{N}(0,0.5)$ (see figure 2).
- Daubechies from *db1* to *db9* were applied for each signal.
- Constant L values (from 2 to 9) were used for each *dbN* and for each Signal.
- Each combination (Signal-*dbN*-*L*) was applied on-line and according to the On Line rectification (OLMS) and Boundary Corrected Translation Invariant (BCTI) schemes (Nounou et al, 1999)
- For the waveshrink, soft thresholding was used and the threshold, β, was determined by the visushrink rule (Nounou et al, 1999).

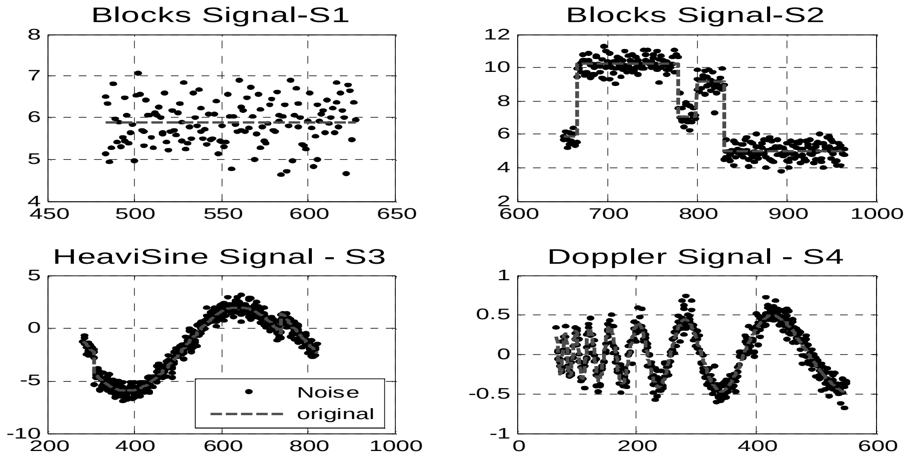

Figure 2. Reference Signals for experiments.

Mean Square Error (mse) between the filtered signal and the original signal (without noise) is computed on windows of 32 observations length and each time after 16 observations are filtered. This results in 8 consecutive intervals with mse computed for signal S1, 18 consecutive intervals for signal S2, 32 consecutive intervals for signal S3 and 29 consecutive intervals for signal S4. Then, the frequency of each *L* value that leads to the low mse in each interval is computed. Similarly, the frequency of each *dbN* filter that leads to the low mse in each interval is computed. These frequencies are shown in table 1.

Table 1. Frequency of L values and dbN filters as leadings to the best estimation of different signals.

	Frequencies for *L* values							
	OLMS				**BCTI**			
	S_1	**S_2**	**S_3**	**S_4**	**S_1**	**S_2**	**S_3**	**S_4**
L=2	0	**5**	2	4	0	**5**	2	4
L=3	0	0	2	**10**	0	1	**9**	**9**
L=4	1	**6**	**15**	**11**	1	**6**	**13**	**10**
L=5	1	0	**10**	3	**7**	4	**8**	5
L=6	**5**	3	3	0	0	0	0	0
L=7	0	2	0	1	0	0	0	1
L=8	0	0	0	0	0	0	0	0
L=9	1	2	0	0	0	2	0	0

	Frequencies for *dbN*							
	OLMS				**BCTI**			
	S_1	**S_2**	**S_3**	**S_4**	**S_1**	**S_2**	**S_3**	**S_4**
db_1	**6**	**8**	**13**	5	**8**	**11**	**20**	2
db_2	0	5	1	**4**	0	1	3	**6**
db_3	0	2	1	5	0	0	0	1
db_4	0	0	6	2	0	1	3	4
db_5	0	0	1	3	0	2	2	0
db_6	1	1	0	0	0	2	2	3
db_7	1	1	5	3	0	1	1	4
db_8	0	0	1	**6**	0	0	1	**7**
db_9	0	1	4	1	0	0	0	2

Analysing the frequency for *dbN* it can be noted that *db1* is particularly appropriate for signals like *S1* in both OLMS and BCTI applications. For signals *S2* and *S3 db1* is also useful for BCTI. In the OLMS case some other filters are also frequents for *S2* (*db2*) and *S3* (*db4-db7*). It is noted that these lasts occurrences corresponds to intervals where abrupt changes are at the end of the data window in case of S2 and for intervals with slow trends in case of S3. So, *db1* may be more appropriate for stationary patterns (as in S1) or for dealing with abrupt changes (see discontinuities in signals S2 and S3). In case of S4 the pattern of the curve is continuously changing through fast and smooth patterns and many filters occurs at different intervals. Only is noted a slow tendency of more occurrences of *dbN* with even values of *N* (particularly *db2* and *db8*). So, slow and changing patterns as in S4 may be best treated with *db2* or *db8* filters.

Now, for the frequencies of *L* values it is shown that every signal tends to be handled around bands of *L* values (L=2-L=4 for S2, L=4-L=5 for S3 and L=3-L=4 for S4) but a pattern is more difficult to establish than in the case of *dbN*.

3. Optimal Depth of Wavelet's Decomposition

Here, it is explored a simple strategy to deal with L. To do this, consider the curve, $y(t)$, that is shown in figure 3 (labelled as measured). Several approximations, A_L, of $y(t)$ are calculated through equation 3 and varying the scale L from 1 to nL. Then, several powers P_L, between $y(t)$ and each one of the A_L are calculated. Also, variations of power, ΔP_L, from one scale to another, are computed. Now, by plotting the successive values of ΔP_L, one or more minima will be detected (see figure 4). The first minimum is identified and the associated L is labelled as L_m. It is observed that at L_m, the associated A_L shows the closest behaviour to the original signal (see figure 3). Therefore, an optimal L can be set as the one corresponding to the first minimum reached in ΔP_L.

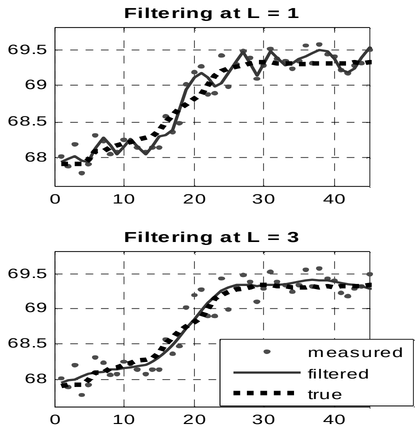

Figure 3. Decomposition of the Jumps signal wit db8. From L=1 to L=4.

Mathematically, the different steps required for this optimal depth determination can be set as follows:

1- The, $c_{L,v}$ and $d_{l,v}$ at various scales l are obtained with wavelets as in equation (1).

2- The approximations A_L, at each scale L is reconstructed through:

$$A_L = \sum_{l=1}^{L} c_{l,v} \cdot \phi_{l,v}(t) \tag{3}$$

3 - The power P_L, at each scale L (da_L), is computed as follows:

$$P_L(da_L) = \sum_{l=1}^{L} \left| y(t) - A_L \right|^2 = \sum_{l=1}^{L} \left| da_L \right|^2 \qquad (4)$$

4. The variation of P_L is computed as follows:

$$\Delta P = P_l(da) - P_{l-1}(da) \qquad (5)$$

The optimal scale L_m that corresponds to the first minimum of ΔP_L is identified.

5. At L_m a first thresholding, based on setting to zero all the $d_{l,\upsilon}$ in scales greater than L_m, is performed. Then, a second thresholding over remaining coefficients is performed through WaveShrink. The first thresholding gives the appropriate L and the second thresholding eliminates coefficients related to noise in scales less or equal than L_m.

6. The de-noised signal is obtained by taking the inverse of the wavelets used.

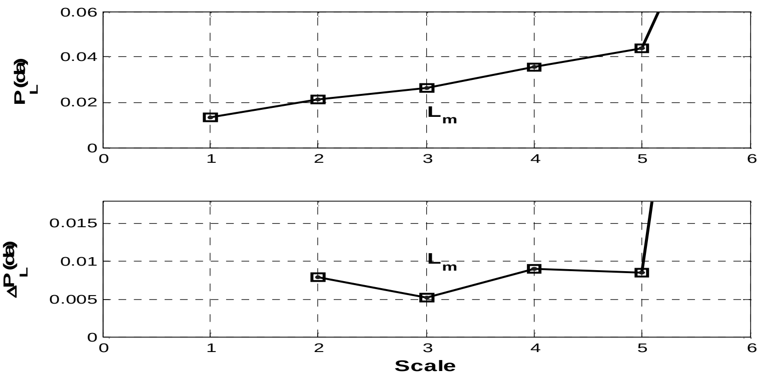

Figure 4. Determination of the optimal level L_m.

In the above procedure, computing approximations until nL with values between [10, 12] is sufficient to identify L_m.

The signals used in section 2 are used for testing the above proposed approach. The experiments were organised as follows:

- The proposed strategy is applied on-line and for both OLMS and BCTI schemes.
- MSE between the filtered and the true signal are computed locally (on data windows as in section 2) and globally (over the entire set of each processed signal).

Figures 5 shows the best fifteen estimations (expressed as global MSE values) that were obtained with the proposed approach (labelled as LevaShrink) and for the WaveShrink strategy. It is shown that, in general, the proposed approach can compete in estimation accuracy with WaveShrink for both OLMS and BCTI schemes. Only the signal S1 presents considerable differences with traditional WaveShrink but for the first estimation it is comparable in accuracy with WaveShrink. It is also shown some cases where LevaShrink gives best accuracy (lower mse in some plots of figure 5). This is possible because at each time the level tuned is adapted to the current pattern of the trend which is more appropriate than setting a same L for all times as it is the case for WaveShrink. Then, the LevaShrink method can be an advantageous alternative use for filtering with wavelets. The advantage of use LevaShrink is to avoid the offline analysis of each signal for setting of appropriate values of *L*.

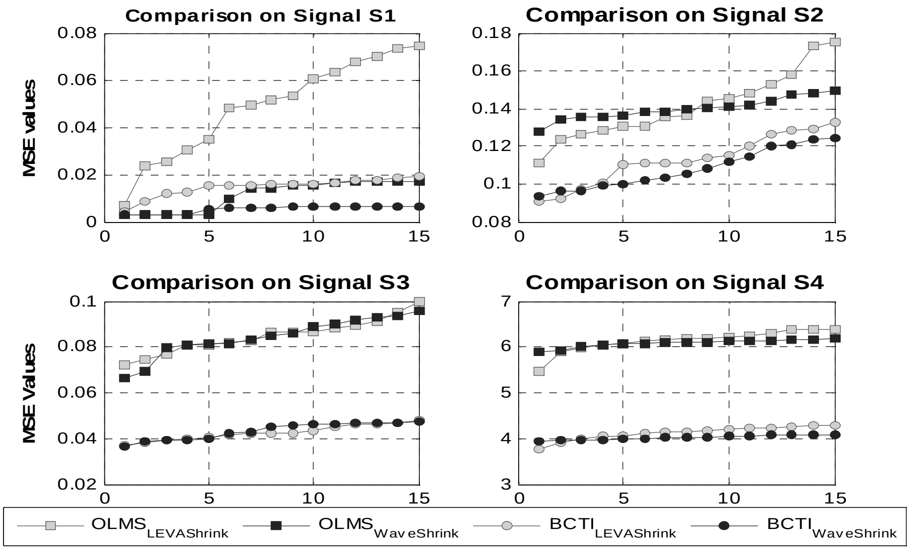

Figure 5.Comparison of LevaShrink with traditional WaveShrink.

4. Conclusions

Wavelet filtering schemes have been studied. By the way of experiments it has been show the adequacy of *db1* for signals with stationary and/or abrupt change patterns, particularly under BCTI schemes. By other hand, wavelets like *db2*-*db8* may be more appropriate for dealing with signals with smooth changing patterns. It has also been show that appropriate level values can be very variables from one type of signal pattern to another. Then, the proposed approach can deal with this issue by identifying at each time the required level. Finally, further improvements and extended comparisons with other existing approaches and for different case studies will be made in future works.

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Acknowledgements

Financial support received from the “Generalitat de Catalunya” (a FI research grant to Tona, R. V.), from “Ministerio de Ciencia y Tecnología” (project DPI2002-00856), and from the European Community (projects GIRD-CT-2000-00318) are fully appreciated.