European Symposium on Computer Aided Process Engineering – 15
L. Puigjaner and A. Espuña (Editors)

Back-off Application for Dynamic Optimisation and Control of Nonlinear Processes

Silvina I. Biagiola[a], Alberto Bandoni[b] and José L. Figueroa[a*]

[a]Departamento de Ingeniería Eléctrica y de Computadoras, Universidad Nacional del Sur-CONICET
Av. Alem 1253, 8000 Bahía Blanca, Argentina

[b]Planta Piloto de Ingeniería Química-CONICET
La Carrindanga Km.7; 8000 Bahía Blanca, Argentina

Abstract

The operating point of a process plant is obtained through optimisation of an objective function subject to certain constraints. Typically, the resultant point lies in the boundary of the operative region. Therefore, in the presence of disturbances, the constraints may be violated and the process may be obliged to operate in an infeasible region.

The goal of this paper is to present an efficient back-off algorithm to determine the operating point which guarantees feasibility even under disturbances effects. The proposed method is especially oriented to diminish the computational time associated with nonlinear processes. For this purpose, the nonlinear process is approximated by means of a piecewise linear (PWL) model which allows a substantial computational time reduction. Finally, an example is dealt with to show the application of the proposed approach. For this purpose, a nonlinear steam generating process is modelled, optimised and controlled.

Keywords: back-off, canonical piecewise linear approximations, dynamic optimisation, process control

1. Introduction

Applications of nonlinear optimisation problems have become very common in the process industries, especially in the area of process operations. To achieve the optimal design and operation of a process, one seeks the best design and operation which will result in a maximum profit. Therefore, in a first design stage, the operating point is usually determined to maximize (or minimize) an objective function. This cost function is normally a weighted combination of various utility costs, material costs, production costs and penalties of environmental emissions. The process model and constraints describe the interrelationship of the key variables. These constraints define a feasibility set for the possible operating points, and in most cases, the optimal operating point lies in the boundary of the set. Therefore, in the presence of disturbances, the current operating point (i.e. optimum point) might exceed some constraint value for the changed conditions. This would lead to undesirable or even unsafe operation of the plant. To avoid operating the process in an infeasible region (Figueroa et al., 1996), it is possible

[*] Author to whom correspondence should be addressed: figueroa@uns..edu.ar

to move the operation point away from the one determined in the optimisation level and to oblige the new point to satisfy the constraints under the disturbed operation. The movement of the operation point due to the likely effect of disturbances is referred to as back-off, and was originally calculated from the desire for evaluating and comparing control strategies on the economical basis (Perkins and Walsh, 1994).

Because in the process industry there is a great number of strongly nonlinear processes, the accomplishment of the back-off algorithm may become very slow or even impossible to perform. To avoid this obstacle, the optimisation problem can be turned into a linear one. In the present paper, we propose to use a canonical piecewise linear (PWL) approximation (Julián et al., 1999) to the problem. This formulation allows the systematic description of any nonlinear function. Then, the whole procedure, i.e. the dynamic back-off analysis, the steady-state calculations, the closed-loop output predictions, are accomplished on the basis of the PWL representation of the process. The main advantages of the proposed approach are the problem simplification and the substantial calculation-time reduction.

In a following stage, a controller is designed to regulate the behaviour of the plant around the designed steady-state value. The underlying idea is that the controller provides perfect control, so that the plant remains very close to its nominal operating points against disturbances and parameter variations and uncertainties on the plant characteristics.

Finally, an example is presented to show the application of the introduced method. For this purpose, a nonlinear steam generating process is modelled, optimised and controlled. Computer simulations are developed for showing the performance of the optimised and controlled nonlinear process. The results obtained with the exact nonlinear approach and the approximating PWL one, are compared.

2. Back-off problem formulation

The mathematical formulation of the closed-loop back-off problem can be posed as follows:

$$F_{obj}[u_s, x_0, d_0] \tag{1}$$

subject to

$g[u_s, x(t), d] \leq 0,$

$f[\dot{x}(t), u_s, x(t), d] = 0,$

$x(0) = x_0,$

$u_s^L \leq u_s \leq u_s^U$

$x^L \leq x \leq x^U$

where $x(t)$ are the process states, u_s are the optimisation variables; $d(t)$ are the disturbances, f are the state equations of the process and g is the set of algebraic inequalities. It is assumed that vector d belongs to a set D which contains all the possible disturbances: $D=\{d: d^L \leq d \leq d^H\}$ where d^L and d^H are the extremes of the set.

In this formulation, the model includes the complete description of the process and the controller information.
Usually, the back-off technique is used to achieve the process operation feasibility, even under unmeasurable disturbances influence. The solution of this problem implies the solution of a dynamic optimisation under uncertainties and the method by Figueroa et al. (1996) can be followed.

3. Piecewise linear approach

In the back-off formulation there is a significant computational complexity associated to the solution of the nonlinear problem. Hence, it is a bottleneck in the on-line application of the back-off algorithm to many nonlinear processes. For this reason, the use of a PWL description to approximate the nonlinear model of the process appears as an appealing strategy to reduce the computational time.
A PWL function $f{:}D \subset \Re^{m_o} \to \Re^1$, with D a compact set, is defined as follows:

Definition: Let the domain D be divided into a set of regions or convex polyhedrons $\Re^{(i)}$, $i \in \{1,\ldots,k_1\}$ such that $D =_{i=1}^{k_1} \Re^{(i)}$. This partition is performed through a set of hyperplanes $\left(H := \left\{H_i \subset D; i \in \{1,\ldots,k_1\}\right\}\right)$ with dimension (m_o-1). They are also called limits $[H_i = \{v \subset \Re^{m_o} : \pi_i(v) := \alpha_i^{'} v - \beta_i = 0\}]$, where $i \in \left\{1,\ldots h\right\}, \alpha_i \in \Re^{m_o}$ and $\beta_i \in \Re^1 \; \forall i$. Hence, a PWL function f is defined by the local functions $[f^{(i)}(v) = J^{(i)} v + w^{(i)}]$, with $J^{(i)} \in \Re^{1 \times m_0}$ the Jacobian of the region $\Re^{(i)}, w^{(i)} \in \Re^l$ and $f(v) = f^{(i)}(v)$ for any $v \in \Re^{(i)}$.

Theorem: The components of the PWL functions of Λ calculated by Julián (1999) are a base of the linear vectorial space $PWL_H[D]$.

Then, every PWL function $f \in PWL_H[D]$ can be uniquely represented (Julián *et al.*, 1999) as a linear combination of the elements in Λ as $[E^T \Lambda(v) = f(v)]$. A relevant property is that, under certain conditions (Lussón Cervantes *et al.*, 2001 and references therein), there exists the inverse function $[E^T \Lambda]^{-1}$ that performs the inverse transform to obtain v from $f(v)$.
This aspect is very important and must be taken into account for evaluating possible models of the static nonlinearity, because when the model is to be used for control purpose, the function must be invertible (Norquay *et al.*, 1998).

3.1 Approximation using PWL functions

As described above, the PWL functions have shown to be an appealing approach for modeling and analysis of nonlinear processes. The nonlinearities in the differential equations that describe the process dynamics can be approximated using a piecewise linear representation. In such a way, the operative region can be divided into several domains that constitute the *simpleces*. Hence, in each domain there is a linear model which approximates the real one. The operation point can change making the system leave a simplex to enter another one. In such a case, the linear approximating model changes. This model can be represented as follows:

$$\dot{x} = \mathrm{A}^{i}_{PWL}\ x + \mathrm{B}^{i}_{PWL}\bar{u} \tag{2}$$

where A_{PWL} and B_{PWL} are the model parameters that stand for the simpleces *i* and $\bar{u}$ is an extended vector which includes the control inputs and the disturbances *d*:

$$\bar{u} = [u'\ \ d'1]' \tag{3}$$

The representation given in Eq.(2) is dealt with as a linear system. Therefore, the system's output can be calculated using the solution for linear dynamics.

4. Simulation example

To illustrate the algorithm proposed above, the 200 MW drum type boiler of a steam-generating unit is considered. Ray and Majumder (1983) developed the following model for this nonlinear unit:

$$\frac{dP}{dt} = -1.9310^{-3}SP^{1/8} + 0.014524F - 7.3610^{-4}Wc + 1.2110^{-3}L \tag{4}$$

$$\frac{dS}{dt} = 10cvP^{1/2} - 0.785716S \tag{5}$$

$$\frac{dL}{dt} = 0.00863Wc + 0.002F + 0.463cv - 610^{-6}P^{2} - 0.00914L - 8.210^{-5}L^{2} - 0.007328S \tag{6}$$

The pressure *P*[Kg/cm^2], the steam flow to the turbine *S*[Kg/sec] and the drum level *L*[cm] are the state variables of the process which is disturbed by the feed water temperature *Te*[˚K] and the control valve setting (*cv*). To keep *P* and *L* in their setpoints, both fuel *F*[Kg/sec] and water Wc[Kg/sec] flows are manipulated. The operation constrains due to the physical process features are:

$120 \le \mathrm{P} \le 190$

$90 \le \mathrm{S}$

$40 \le \mathrm{L} \le 80$

$30 \le \mathrm{F} \le 50$

$150 \le \mathrm{Wc} \le 240$

and the bounds for the probable disturbances are:

$$0.280 \le \mathrm{Te} \le 0.320$$
$$0.700 \le \mathrm{cv} \le 0.900$$

The following objective function is proposed in order to minimize the operating cost and to maximize the steam production:

$$F_{obj} = -0.6S - 0.5P + 0.8F + 0.1Wc \tag{7}$$

To provide the back-off algorithm a linear representation of the process, the nonlinear plant was approximated using a PWL description. For this purpose, the nonlinearities $SP^{1/8}$, $cv\sqrt{P}$, P^2 and L^2 in Eqs. (4-6) were replaced by piecewise linear functions.

A simple controller structure to keep P and L in their desired values was used in both cases: the nonlinear model and the PWL approximation. For these purposes, two PI controllers were designed and their parameters were set to: K=300, K_c = 1, T_{int} = 50 for the pressure regulator and K=100, K_c=1, T_{int} = 70 for the level regulator, respectively.

Table 1 shows the simulation results for both cases: the real nonlinear process and its PWL approximation.

Table 1. Optimisation results: Nonlinear vs. PWL

	F_{obj}	F	W_c	Calculation time[sec]
Nonlinear model	131.01	40.2318	158.4186	6840.6
PWL model	131.17	40.2319	159.4192	156.4

Figures 1-4 illustrate the performance of the proposed algorithm when applied to the nonlinear process and the PWL approximation. The simulations were accomplished using MATLAB 5.3 optimisation Toolbox. A solver to deal with the algebraic equations was embedded in the optimisation program used to carry out the simulations. The algorithm was performed on a 550 MHz Pentium III processor.

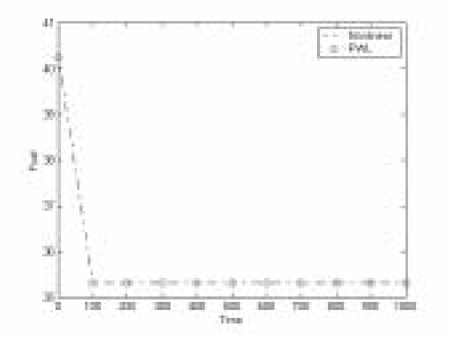
Figure 1. Fuel flow

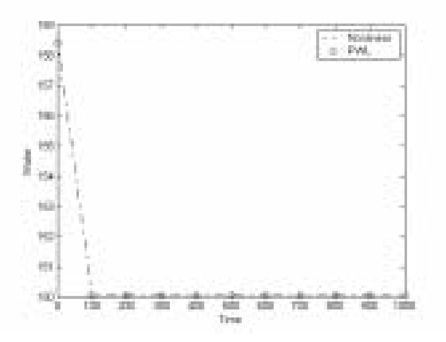
Figure 2. Water flow

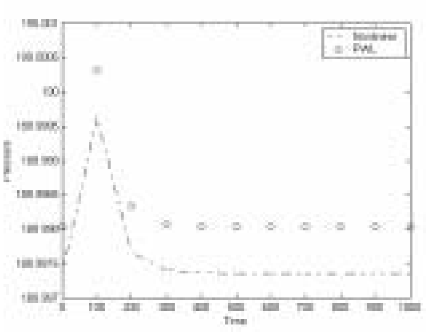

Figure 3. Pressure

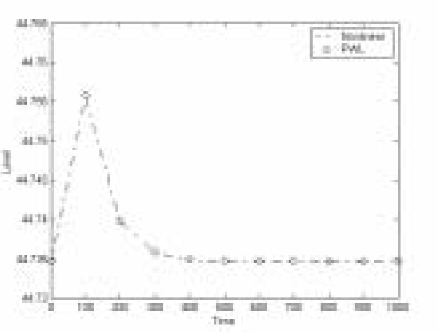

Figure 4. Level

5. Conclusions

In this work, the problem of dynamic simulation and optimisation was tackled in the back-off formulation. The proposed approach is generic and it can be applied to any nonlinear process. The calculation algorithm involves approximating the real nonlinear dynamics by means of a piecewise linear model. This modelling approach allows finding the process response by using the solution for discrete-time linear systems. The primary benefit of this method is a substantial reduction of the computational time.

References

Figueroa, J.L., P.A. Bahri, J.A. Bandoni and J.A. Romagnoli, 1996, Comp. Chem. Eng. 453, 20(4).

Figueroa, J.L. and A. Desages, 2003, Optimal Control Applications and Methods 103, 24.

Julián, P., 1999, High level canonical piecewise linear representation: Theory and applications. Doctoral thesis, Universidad Nacional del Sur, Argentina.

Julián, P., A. Desages and O. Agamennoni, 1999, IEEE Trans. Circuits Syst. 463, I(46).

Perkins, J.D. and S.P.K. Walsh, 1994, Optimization as a Tool for Design/Control Integration. In Interactions Between Design and Process Control, Ed. J.D. Zafirou, Baltimore, Maryland, USA, 1-10.

Lussón Cervantes, A., O. Agamennoni and J.L. Figueroa, 2003, Journal of Process Control, 655, 13.

Ray, K.S. and D. Dutta Majumder, 1983, Proceedings of the International Conference on Systems, Man and Cybernetics 705, I.

Acknowledgements

This work was financially supported by the CONICET and the Universidad Nacional del Sur, Bahía Blanca, Argentina.