

IMC Design of Cascade Control

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Abstract

Cascade control is one of the most successful methods for enhancing single-loop performance. However, the literature about synthesis methods for designing and tuning cascade control systems appears to be rather limited. In this contribution, a model-based procedure using internal model control (IMC) approach is proposed for synthesizing the controllers transfer functions. The suggested tuning procedure determines the controller filter time constants such to assure robust stability. Simulation examples are provided to demonstrate the goodness of the synthesis method and to compare its performance with those of PID-PID cascade configuration tuned with already accepted rules.

Keywords: Cascade Control, Robust Control, IMC Design, Model Based Control

1. Introduction

The robust process control has received considerable attention in the last twenty years. The IMC structure, as base for the robust controller design, is treated with great detail in Morari and Zafiriou (1989) where robust control is associated with IMC design. When addressing cascade control, the authors mention the utility of this control configuration when the secondary process is dominated by an important uncertainty.

Skogestad and Postlethwaite (1996) analyze different cascade control structures, but they do not present any particular robust synthesis method. The paper of Tan et al. (2000) is one of the few contributions of robustness analysis for series cascade systems, where they propose conventional PID controllers for the inner and outer loops. Lately, Brosilow and Joseph (2002) used IMC design approach employing the M_p parameter and considered both stability and robust performance simultaneously. In this framework, the contribution by Hahn et al. (2002) presents a procedure to obtain the uncertain information in order to design robust IMC controllers.

In this work, the IMC Series Cascade Control structure is studied (see Figure 1). The analysis includes robust stability conditions for tuning both controllers. Finally, the closed-loop performance and robustness of the synthesized system are compared with cascaded PID regulators tuned according to Lee et al. (1998), one of the few systematic tuning rule for cascade systems reported in the literature.

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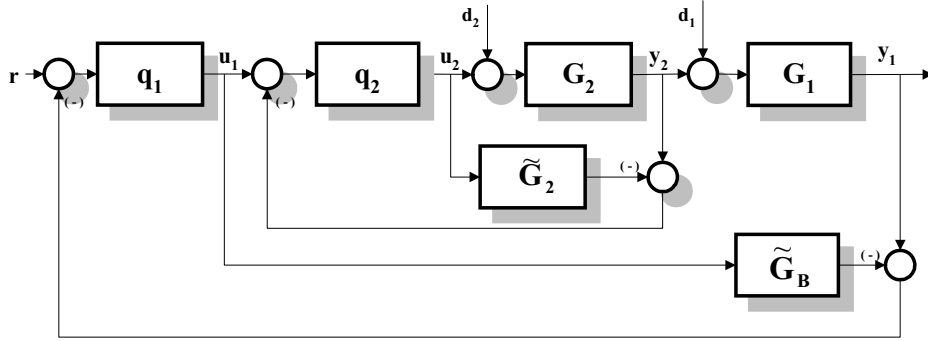


Figure 1. IMC Series Cascade Control Structure

2. IMC Cascade Control Synthesis

2.1 Nominal Performace

The primary disturbance d_1 , and the secondary disturbance d_2 are typically analyzed when dealing with cascade control systems. In particular, the effect of the secondary disturbance on the main output y_1 is considered for synthesizing the inner controller q_2 . Thus, the IMC design proceeds by minimizing the H_2 norm of the y_1 error according to

$$\min_{\tilde{q}_2} \int_0^{\infty} e_1^2 dt = \min_{\tilde{q}_2} \|e_1\|_2^2 = \min_{\tilde{q}_2} \|(1 - G_2 \tilde{q}_2) G_1 d_2\|_2^2 \quad (1)$$

On the other hand, the optimum primary controller q_1 is obtained by minimizing the H_2 norm of the error e_1 , caused by the primary disturbance

$$\min_{\tilde{q}_1} \int_0^{\infty} e_1^2 dt = \min_{\tilde{q}_1} \|e_1\|_2^2 = \min_{\tilde{q}_1} \|(1 - \tilde{q}_1 \tilde{G}_B) d_1\|_2^2 \quad (2)$$

Under nominal conditions (plant equal model), the time constants of the IMC filters should be chosen as small as possible. To avoid excessive noise amplification, the filter parameter should be chosen so that the controller high frequency gain is not greater than β times its low frequency gain. This criterion can be expressed as

$$\beta = \sup_{\forall \omega} \frac{|q(j\omega)|}{|q(0)|} \quad (3)$$

where $q(s)$ is the transfer function of the IMC controller. Brosilow and Joseph (2002) proposed $\max \beta = 20$, however, factors between 5 and 20 are encountered in practice. In this work, $\beta = 10$ is adopted, which follows the standard industrial practice of limiting the high-frequency gain of PID controllers. If the controller does not have complex poles in the left-half plane, equation (2) can be transformed in the limit:

$$\beta = \lim_{\omega \rightarrow \infty} \frac{|q(j\omega)|}{|q(0)|} \quad (4)$$

2.2 Robust Stability

In order to evaluate the robust stability conditions a multiplicative description of the uncertainty is assumed. Thus, two families of models with uncertain parameters are defined as:

$$\Pi_i = \left\{ G_i : \left| \frac{G_i(j\omega) - \tilde{G}_i(j\omega)}{\tilde{G}_i(j\omega)} \right| = |\ell_i(j\omega)| \leq \ell_{m_i}(\omega) \right\} \quad i = 1, 2 \quad (5)$$

where in each family set, $\tilde{G}_i$ is the nominal model, $\ell_i(j\omega)$ is the multiplicative uncertainty, and $\ell_{m_i}(\omega)$ stands for the largest module.

The robust stability condition (Skogestad and Postlethwaite, 1996), for the inner loop is

$$|\tilde{T}_2(j\omega)| = |q_2(j\omega) \tilde{G}_2(j\omega)| < \frac{1}{\ell_{m_2}(\omega)} \quad \forall \omega \quad (6)$$

where $\tilde{T}_2$ is the complementary sensibility function under nominal conditions.

The primary controller observes the dynamics the transfer function composed by the inner loop and the primary plant connected in series, namely

$$G_B = \frac{q_2 G_2}{1 + q_2(G_2 - \tilde{G}_2)} G_1 \quad (7)$$

Thus, the controller q_1 must stabilize the following set of equivalent plants

$$\Pi_B = \left\{ G_B : \left| \frac{G_B(j\omega) - \tilde{G}_B(j\omega)}{\tilde{G}_B(j\omega)} \right| = |\ell_B(j\omega)| \leq \ell_{m_B}(\omega) \right\} \quad (8)$$

where

$$|\ell_B(j\omega)| = \frac{|\ell_1(j\omega)\ell_2(j\omega) - q_2(j\omega) \tilde{G}_2(j\omega) \ell_2(j\omega)|}{|1 + q_2(j\omega) \tilde{G}_2(j\omega) \ell_2(j\omega)|} \quad (9)$$

Finally, the robust stability condition for the primary loop may be expressed as

$$|\tilde{T}_1(j\omega)| = |q_1(j\omega) \tilde{G}_B(j\omega)| = |q_1(j\omega) q_2(j\omega) \tilde{G}_2(j\omega) \tilde{G}_1(j\omega)| < \frac{1}{\ell_{m_B}(\omega)} \quad \forall \omega \quad (10)$$

3. Controller Synthesis for Low-Order Plant Models

A frequent simplified dynamic characterization of chemical processes consists of a first-order-lag plus dead-time transfer function. In this way, most of the systems can be represented with enough accuracy for controller tuning (Shinsky, 1996). Thus, this low-order modeling is used for both, the primary and secondary plants

$$\tilde{G}_1(s) = \frac{K_1 \exp(-\theta_1 s)}{(\tau_1 s + 1)} \quad \tilde{G}_2(s) = \frac{K_2 \exp(-\theta_2 s)}{(\tau_2 s + 1)} \quad (11)$$

3.1 Nominal Performance

When a unit step is considered as secondary disturbance d_2 , the ideal controller resulting from equation (1) is

$$\tilde{q}_2 = \frac{(\tau_2 s + 1)(\alpha \tau_1 s + 1)}{K_2} \quad , \quad \alpha = 1 - \exp\left(-\frac{\theta_2}{\tau_1}\right) \quad (12)$$

Note that the synthesized controller contains dynamic parameters from both inner and outer plants. Thus, the realizable controller is obtained including a second order filter

$$q_2 = \tilde{q}_2 F_2 = \frac{1}{K_2} \frac{(\tau_2 s + 1)(\alpha \tau_1 s + 1)}{(\lambda_2 s + 1)^2} \quad (13)$$

where the IMC filter time constant λ_2 that verifies equation (3) is

$$\lambda_2^{NP} = \sqrt{\frac{\tau_2 \alpha \tau_1}{\beta}} \quad (15)$$

Similarly, assuming a unit step in the primary disturbance d_1 , and including a second order IMC filter in equation (2), the primary controller results

$$q_1 = \tilde{q}_1 F_1 = \frac{1}{K_1} \frac{(\tau_1 s + 1)}{(\alpha \tau_1 s + 1)} \frac{(\lambda_2 s + 1)^2}{(\lambda_1 s + 1)^2} \quad (16)$$

Finally, the noise amplification constraint in equation (3) implies

$$\lambda_1^{NP} = \lambda_2 \frac{1}{\sqrt{\beta \alpha}} \quad (17)$$

3.2 Robust Stability

When the structure of the secondary transfer function $\tilde{G}_2$, and the synthesized secondary controller q_2 are considered, the condition (6) for robust stability becomes

$$\frac{\sqrt{(\alpha \tau_1 \omega)^2 + 1}}{(\lambda_2^{ER} \omega)^2 + 1} \leq \frac{1}{\ell m_2(\omega)} \quad \forall \omega \quad (18)$$

Besides, the minimum value for the IMC filter time-constant λ_1^{ER} , that assures robust stability is obtained from equation (10)

$$\frac{1}{(\lambda_1^{ER} \omega)^2 + 1} \leq \frac{1}{\ell m_B(\omega)} \quad \forall \omega \quad (19)$$

Although it is not explicit, tuning the primary controller depends on the secondary controller through the global uncertainty ℓ_B .

4. Controllers Tuning

Notice that the IMC concept allows the controller transfer function synthesis, namely, it determines the controller form. Then, the filter time constant is the tuning parameter used to obtain stability and the desired performance. The proposed tuning approach is:

$$\lambda_i = \max(\lambda_i^{NP}, \phi \lambda_i^{RS}) \quad i = 1, 2 \quad (13)$$

The factor ϕ must be greater than 1 to assure an appropriate damping of the controlled variable response; its role is similar to the gain margin. Several simulated cases show that ϕ values between 1.2 and 1.5 produce satisfactory results.

Notice that the secondary controller must also satisfy the robust stability condition, because if the primary controller is set to manual operation, the inner loop has to remain stable.

5. Simulation Study

A large number of numerical cases have been simulated to test the proposed synthesis procedure. Because of space reasons however, only two examples with opposite dynamic plant characteristics are presented. The parameters used in these examples are shown in Table 1. Case A is a typical cascade control system with secondary dynamics faster than the primary dynamics. On the contrary, case B is an example where the use of cascade control is not recommended because the inner dynamics are slower than the primary one (Shinsky, 1996).

Table 1. Plants parameters of the simulated cases.

Parameter	Case A		Case B	
	Primary	Secondary	Primary	Secondary
K - Gain	1.000	1.000	1.000	1.000
τ - Time constant	0.490	0.176	0.116	0.547
θ - Dead time	0.245	0.088	0.058	0.273

Figure 2 shows the transient responses for a unit-step change in the secondary disturbance at time 0, and unit-step change in the primary disturbance at time 10.

In order to establish a fair evaluation, a conventional cascade control system with PID controllers tuned according to Lee et al. (1998) is considered. This is one of the few systematic tuning rule reported in the literature for cascade PID controllers. The referenced tuning however produces unstable responses in Case B and with others similar dynamics. Consequently, the controller parameters must be readjusted to achieve a reasonable damping in the controlled response.

Changes in dead-times of about ± 30 % regarding the nominal model are considered to test the proposed tuning procedure. This uncertainty datum is used basically to find the bounds $\ell_{m_1}(\omega)$ and $\ell_{m_2}(\omega)$.

Figure 2 shows the simulation results where two transient responses to changes in both the primary and the secondary disturbances are presented. The upper plots give the responses under nominal conditions, while the lower ones correspond to plants with extreme dead-time values. Notice that cascade IMC gives always better responses under both, nominal and perturbed plants conditions. Furthermore, it yields good performance to both, secondary and primary disturbances, when using on plants like Case B, despite of its usually unfavorable conditions for cascade control. The control effort of cascade IMC remains inside acceptable limits (Marlin, 1995).

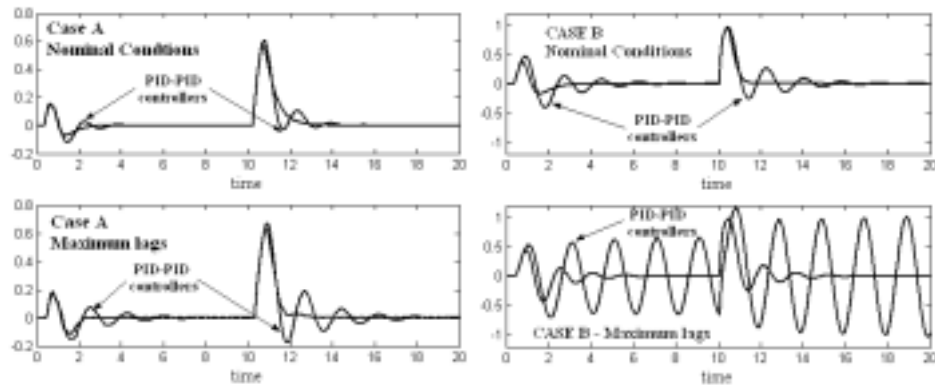


Figure 2. Closed-loop responses for cases A and B, to step changes in d_2 ($t = 0$) and d_1 ($t = 10$)

6. Conclusions

This contribution revises the synthesis of cascade controllers with IMC structure, and provides a tuning procedure that accounts for nominal performance and robust stability. The approach is developed using low-order models with available estimated limits for the uncertainties; in particular, first-order lags and dead times transfer functions are used to represent both the primary and the secondary plant dynamics. The convenience of tuning the secondary controller first is confirmed by analytical relationships leading to robust stability. A large number of simulation results confirm that IMC cascade control yields better performance than the conventional cascade structure where PID-PID controllers are adjusted following standard procedures.

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