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A Design and Scheduling RTN Continuous-time Formulation

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Abstract

This paper presents a general mathematical formulation for the simultaneous design and scheduling of multipurpose plants. The formulation is based on the Resource Task Network process representation, uses a periodic, uniform time grid, continuous-time representation and originates mixed integer nonlinear programs (MINLPs) or mixed integer linear programs (MILPs), depending on the type of tasks and objective function being considered. Its performance is illustrated through the solution of two batch-plant example problems that have been examined in the literature.

Keywords: Resource Task Network, Uniform-time grid, Event points

1. Introduction

Multipurpose plants are general purpose facilities where a variety of products can be produced by sharing the available plant resources (raw materials, equipment, utilities and manpower) properly in time. The production of a particular product involves a sequence of operations that can be batch, semi-continuous or continuous in nature, where a particular unit is usually suitable for more than a single operation. As a consequence, multipurpose plants are more flexible and suitable for the production of small quantities of high value-added products with short life cycles, the current trend of consumers' demands in today's competitive global market. These same special characteristics however, introduce extra degrees of complexity into the design and operation of such plants. In particular, it is not possible to design a plant without considering how it will be operated, neither it is possible to schedule all the required operations without knowing the plant configuration. Hence, design and scheduling must be considered simultaneously to avoid over or under design.

Several authors have addressed the design and scheduling problem of batch plants. Few, however, have based the mathematical formulations on general process representations. Barbosa-Póvoa and Macchietto (1994) presented a discrete-time formulation that uses the maximal State Task Network (mSTN). Examples of continuous-time formulations

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are the work of Lin and Floudas (2001) for the STN and of Castro et al. (2004) for the RTN, with both assuming a short-term mode of operation.

This work follows that of Castro et al. (2004) but now a periodic mode of operation is assumed. In addition, both batch and continuous tasks can now be handled. However, to avoid generating MINLPs, only equipment items characterized by size (not processing rate) will be considered. Thus, the two example problems chosen to illustrate the performance of the formulation involve the design and scheduling of batch plants.

2. Fundamental Concepts

In the proposed formulation, the time horizon of interest (H) is divided into a fixed number of time intervals/slots. The interval boundaries are called event points (set T) and their exact location (T_t), as well as the cycle time (H) is unknown a priori. A single time grid keeps track of all events taking place (see Figure 1). In periodic problems, the beginning and end of the time horizon are the exact same event point. The wrap-around operator defined in eq 1 is used to overcome the problem of modelling the execution of tasks that span across two cycles and to facilitate the formulation of some constraints.

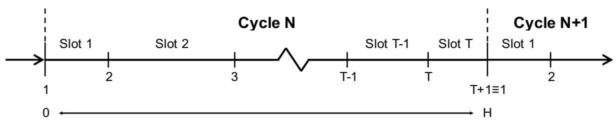

Figure 1. Uniform time grid for periodic scheduling problems

$$\Omega(t) = t - |T| \; if \; t > |T|, \Omega(t) = t \; if \; 1 \le t \le |T|, \Omega(t) = t + |T| \; if \; t < 1 \qquad (1)$$

The quality of the solution returned depends greatly on the number of event points considered, |T|. For MILP models, the returned solution is a global optimum solution only if the pre-specified number of |T| is sufficient and does not act as a hidden constraint. The search for the global optimum usually involves solving the problem for different values of |T| until no increment is found in the objective function.

To account for the fact that the majority of process tasks does not span across the whole time horizon, the number of event points allowed between the beginning and end of a task must not exceed Δt. A global value is assumed for all tasks belonging to the same type (e.g. batch, continuous, storage), with the requirement that Δt must equal 1 for continuous and storage tasks. Again, the use of an exceedingly low Δt value can also work as a hidden model constraint, so a similar procedure to that of |T| should be used.

The RTN process representation regards all processes as bipartite graphs comprising two types of nodes: resources (set R) and tasks (set I), the latter being operations that transform a certain set of resources into another set. Two variables are used to characterize the instance of task i starting at event point t and ending at t'. The binary variable $\overline{N}_{i,t,t'}$ identifies the occurrence of the task, while $\overline{\xi}_{i,t,t'}$ (positive continuous variable) gives the total amount of material processed between t and t'. The other required variables are the excess resource variables $R_{r,t}$, the existence variables for equipment resources ($R_r^0 \le 1 \, \forall r \in R^{EQ}$), the net consumption/production over the cycle Δ_r (for raw-materials, $r \in R^{RM}$ or final products, $r \in R^{FP}$), the capacity V_r ($r \in R^{BE}$) or the

processing rate ρ_r ($r \in R^{CE}$) of equipment resource r (depending on whether the equipment processes batch or storage tasks, I^b or I^s, or continuous tasks, I^c).
The task processing time is assumed to be given by a constant (α_i) plus a term proportional to the amount of material being processed (β_i). The amounts of each resource consumed/produced at the start/end of a task are assumed to be proportional to the binary ($\mu_{r,i} / \bar{\mu}_{r,i}$) and/or continuous ($\nu_{r,i} / \bar{\nu}_{r,i}$) extents of the task. The former parameters are usually linked with equipment resources, while the latter are typically linked with material resources. The other required parameter (sf_i) allows the amount processed by task i to be lower than the minimum design capacity of the equipment. Note that a 1:1 correspondence is assumed between tasks and equipment resources.

$$\sum_{r \in R} \mu_{r,i} = \sum_{r \in R} \bar{\mu}_{r,i} = 1 \; \forall i \in I \tag{2}$$

3. Mathematical Formulation

The objective function considered (eq 3) minimizes the capital cost of units, which consists of a fixed term ($\tilde{\alpha}_r$) plus a term ($\tilde{\beta}_r$) proportional to the size or processing rate of the unit. Other performance criteria can also be incorporated. In particular, one can maximize the profit of the plant for product demands between given upper and lower bounds. In the following constraints, R^{TC} represents all equipment resources with the exception of storage tanks. Note that batch tasks producing materials subject to zero-wait policies (I^{ZW}) or continuous tasks subject to a predefined minimum rate (I^{EMR}) require two additional set of constraints (eqs 6 and 7).

$$\min \sum_{r \in R^{EQ}} \left(\tilde{\alpha}_r R_r^0 + \tilde{\beta}_r V_r \Big|_{r \in R^{BE}} + \tilde{\beta}_r \rho_r \Big|_{r \in R^{CE}} \right) \tag{3}$$

$$T_{t'} - T_t \geq \sum_{i \in I^b} \bar{\mu}_{r,i} (\alpha_i \bar{N}_{i,t,t'} + \beta_i \bar{\xi}_{i,t,t'}) + \left[\sum_{i \in I^c} \frac{\bar{\mu}_{r,i} \bar{\xi}_{i,t,t'}}{\rho_r^{\max}} \right] \Big|_{t'=t+1} \forall r \in R^{TC}, t, t' \in T, t < t' \leq \Delta t + t \tag{4}$$

$$T_t - T_{t'} \leq H - \sum_{i \in I^b} \bar{\mu}_{r,i} (\alpha_i \bar{N}_{i,t,t'} + \beta_i \bar{\xi}_{i,t,t'}) - \left[\sum_{i \in I^c} \frac{\bar{\mu}_{r,i} \bar{\xi}_{i,t,t'}}{\rho_r^{\max}} \right] \Big|_{t'=t+1-|T|} \forall r \in R^{TC}, t, t' \leq \Delta t + t - |T| \tag{5}$$

$$\begin{aligned} T_{t'} - T_t \leq\ & H^{\max} \Big(1 - \sum_{i \in I^b, I^{ZW}} \bar{\mu}_{r,i} \bar{N}_{i,t,t'} - \sum_{i \in I^c, I^{EMR}} \bar{\mu}_{r,i} \bar{N}_{i,t,t'} \Big|_{t'=t+1} \Big) + \\ & \sum_{i \in I^b, I^{ZW}} \bar{\mu}_{r,i} (\alpha_i \bar{N}_{i,t,t'} + \beta_i \bar{\xi}_{i,t,t'}) + \left[\sum_{i \in I^c, I^{EMR}} \frac{\bar{\mu}_{r,i} \bar{\xi}_{i,t,t'}}{\rho_r^{\min}} \right] \Big|_{t'=t+1} \forall r \in R^{TC}, t, t < t' \leq \Delta t + t \end{aligned} \tag{6}$$

$$\begin{aligned} T_t - T_{t'} \geq\ & H - H^{\max} \Big(1 - \sum_{i \in I^b, I^{ZW}} \bar{\mu}_{r,i} \bar{N}_{i,t,t'} - \sum_{i \in I^c, I^{EMR}} \bar{\mu}_{r,i} \bar{N}_{i,t,t'} \Big|_{t'=t+1-|T|} \Big) - \\ & \sum_{i \in I^b, I^{ZW}} \bar{\mu}_{r,i} (\alpha_i \bar{N}_{i,t,t'} + \beta_i \bar{\xi}_{i,t,t'}) - \left[\sum_{i \in I^c, I^{EMR}} \frac{\bar{\mu}_{r,i} \bar{\xi}_{i,t,t'}}{\rho_r^{\min}} \right] \Big|_{t'=t+1-|T|} \forall r \in R^{TC}, t, t' \leq \Delta t + t - |T| \end{aligned} \tag{7}$$

$$T_1 = 0 \wedge H^{\min} \leq H \leq H^{\max} \wedge T_t \leq H^{\max} \; \forall t \in T \tag{8}$$

$$
\begin{aligned}
R_{r,t} = {} & R_{r,\Omega(t-1)} + \sum_{i \in I^b} \left[\sum_{\substack{t' \in T \\ t<t' \le \Delta t + t \vee t' \le \Delta t + t - |T|}} (\mu_{r,i} \overline{N}_{i,t,t'} + \nu_{r,i} \overline{\xi}_{i,t,t'}) + \sum_{\substack{t' \in T \\ t - \Delta t \le t' < t \vee t' \ge t - \Delta t + |T|}} (\overline{\mu}_{r,i} \overline{N}_{i,t',t} + \overline{\nu}_{r,i} \overline{\xi}_{i,t',t}) \right] + \\
& \sum_{i \in I^s \vee i \in I^c} \left[\mu_{r,i} \overline{N}_{i,t,\Omega(t+1)} + \overline{\mu}_{r,i} \overline{N}_{i,\Omega(t-1),t} + (\nu_{r,i} \overline{\xi}_{i,t,\Omega(t+1)} + \overline{\nu}_{r,i} \overline{\xi}_{i,\Omega(t-1),t}) \Big|_{i \in I^s} \right] + \sum_{i \in I^c} \lambda_{r,i} \overline{\xi}_{i,\Omega(t-1),t} + \\
& \sum_{i \in I^t} \left[(\mu_{r,i} + \overline{\mu}_{r,i}) N_{i,t} + (\nu_{r,i} + \overline{\nu}_{r,i}) \xi_{i,t} \right] + \Delta_r \Big|_{t=1, r \in R^{RM}} - \Delta_r \Big|_{t=1, r \in R^{FP}} \ \forall r \in R, t \in T
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
R_{r,T} = {} & R_r^0 + \sum_{i \in I^b} \sum_{t \in T} \Big(\sum_{\substack{t' \in T \\ t<t' \le \Delta t + t \vee t' \le \Delta t + t - |T|}} \mu_{r,i} \overline{N}_{i,t,t'} + \sum_{\substack{t' \in T \\ t<t' \le \Delta t + t}} \overline{\mu}_{r,i} \overline{N}_{i,t,t'} \Big) + \\
& \sum_{i \in I^c \vee i \in I^s} \sum_{t \in T} (\mu_{r,i} \overline{N}_{i,t,\Omega(t+1)} + \overline{\mu}_{r,i} \overline{N}_{i,t,t+1}) \ \forall r \in R^{EQ}
\end{aligned} \tag{10}
$$

$$
R_r^{\min} \le R_{r,t} \le R_r^{\max} \ \forall r \in R, t \in T \wedge \Delta_r = d_r^{\max} H \ \forall r \in R^{FP} \tag{11}
$$

$$
\overline{N}_{i,t,t'} sf_i \sum_{r \in R} \overline{\mu}_{r,i} V_r^{\min} \le \overline{\xi}_{i,t,t'} \le \overline{N}_{i,t,t'} \sum_{r \in R} \overline{\mu}_{r,i} V_r^{\max} \ \forall i \in I^b, t, t < t' \le \Delta t + t \vee t' \le \Delta t + t - |T| \tag{12}
$$

$$
\overline{N}_{i,t,\Omega(t+1)} \Delta T^{\min} \sum_{r \in R} \overline{\mu}_{r,i} \rho_r^{\min} \le \overline{\xi}_{i,t,\Omega(t+1)} \le \overline{N}_{i,t,\Omega(t+1)} H^{\max} \sum_{r \in R} \overline{\mu}_{r,i} \rho_r^{\max} \ \forall i \in I^c, t \in T \tag{13}
$$

$$
\overline{N}_{i,t,\Omega(t+1)} sf_i \sum_{r \in R} \overline{\mu}_{r,i} V_r^{\min} \le \overline{\xi}_{i,t,\Omega(t+1)} \le \overline{N}_{i,t,\Omega(t+1)} \sum_{r \in R} \overline{\mu}_{r,i} V_r^{\max} \ \forall i \in I^s, t \in T \tag{14}
$$

$$
N_{i,t} sf_i \sum_{r \in R} \overline{\mu}_{r,i} V_r^{\min} \le \xi_{i,t} \le N_{i,t} \sum_{r \in R} \overline{\mu}_{r,i} V_r^{\max} \ \forall i \in I^t, t \in T \tag{15}
$$

$$
\overline{\xi}_{i,t,t'} \le \sum_{r \in R} \overline{\mu}_{r,i} V_r \ \forall i \in I^b, t \in T, t' \in T, t < t' \le \Delta t + t \vee t' \le \Delta t + t - |T| \tag{16}
$$

$$
\overline{\xi}_{i,t,\Omega(t+1)} \le (T_{t+1}\Big|_{t \ne |T|} + H\Big|_{t=|T|} - T_t) \sum_{r \in R} \overline{\mu}_{r,i} \rho_r \ \forall i \in I^c, t \in T \tag{17}
$$

$$
\overline{\xi}_{i,t,\Omega(t+1)} \le \sum_{r \in R} \overline{\mu}_{r,i} V_r \ \forall i \in I^s, t \in T \tag{18}
$$

$$
\xi_{i,t} \le \sum_{r \in R} \overline{\mu}_{r,i} V_r \ \forall i \in I^t, t \in T \tag{19}
$$

$$
V_r^{\min} R_r^0 \le V_r \le V_r^{\max} R_r^0 \ \forall r \in R^{EQ}, r \in R^{BE} \tag{20}
$$

$$
\rho_r^{\min} R_r^0 \le \rho_r \le \rho_r^{\max} R_r^0 \ \forall r \in R^{EQ}, r \in R^{CE} \tag{21}
$$

4. Case Studies

Two example problems taken from Barbosa-Póvoa and Macchietto (1994) are used to illustrate the capabilities of the proposed formulation. The resulting MILPs were solved to optimality on a Pentium IV-3.4 GHz machine, running Windows XP Professional, by the commercial solver GAMS/CPLEX 8.1.

In the first example (EX1) two raw-materials (S1 and S2) are transformed into two final products (S5 and S6) through a sequence of processing steps. A maximum of four different equipment items (R1-R4) can be used, together with a storage vessel (V4) to store the intermediate material S4. The complete RTN representation of the process is given in Figure 2 while the resource data is given in Table 1. The maximum demand of S5 and S6 is equal to 8.333 t/h. It is assumed that the cycle time can vary between 6 and 10 h and that all intermediates are subject to ZW policies. As can be seen from Table 2, the problem is solved rather rapidly despite its relatively high integrality gap, and features an optimal cycle time of 6 h. The optimal schedule, together with the required capacities of the chosen equipment units (R1, R2 and V4), is given in Figure 4.

The second example (EX2) involves three raw-materials (FeedA-FeedC), 5 intermediates, 2 final products (P1, P2), 4 main equipment units (Heater, R1, R2 and Still) and 4 storage vessels (V4-V7). The RTN representation of the process is given in Figure 3, while the problem data is given in Table 3. The maximum demand of P1 is set to 3.3703, while that of P2 is set to 6.5 t/h. It is assumed that the cycle time lies between 5 and 15 h and that ZW policies apply. The optimal solution is given in Figure 5 and features a plant worth k\$ 400.87 with an optimal cycle time of 6.994 h. The plant consists of four pieces of equipment, where R2 and V6 are designed at their minimum capacities (70 and 10 tons, respectively). These values are still higher than the maximum amount of material being processed by all tasks taking place in those vessels (58.933 and 5.051, respectively) so there is still some unused capacity.

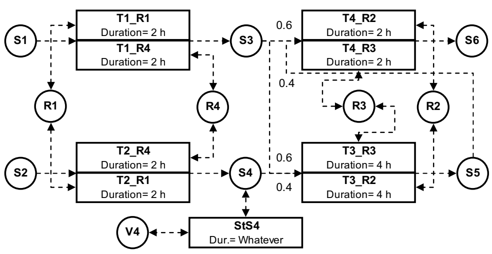

Figure 2. RTN representation for EX1

Table 1. Problem data for EX1

Resource	$V_r^{\min} / V_r^{\max}$	$\tilde{\alpha}_r / \tilde{\beta}_r$
R1	40/70	150/0.5
R2	70/120	120/0.2
R3	70/120	121/0.2
R4	40/70	151/0.5
V4	10/60	20/0.2

Table 2. Computational statistics

Problem	|T|	I.V.	C.V.	C.	Obj rel. MILP	Obj MILP	H (h)	Nodes	CPUs
EX1	5	85	246	409	202.3	330.6	6	2300	3.5
EX2	6	240	716	1220	149.5	400.9	7.00	42353	149

Table 3. Problem data for EX2

Resource	Heater	R1	R2	Still	V4	V5	V6	V7
$V_r^{\min} / V_r^{\max}$	20/50	50/70	70/70	50/80	10/30	10/60	10/70	50/100
$\tilde{\alpha}_r / \tilde{\beta}_r$	100/0.2	150/0.5	120/-	150/0.3	30/0.1	15/0.1	10/0.1	20/0.2

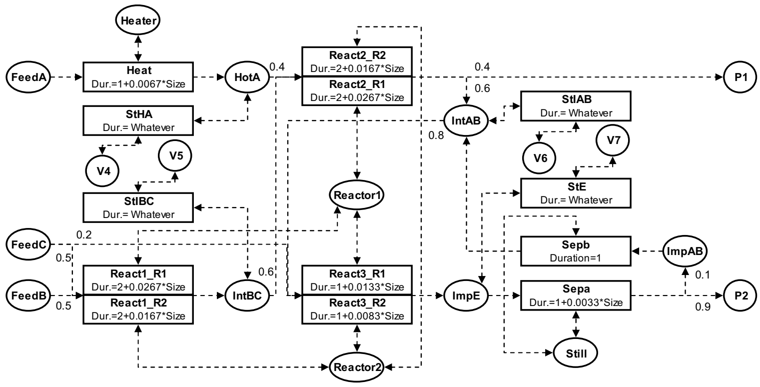

Figure 3. RTN representation for EX2

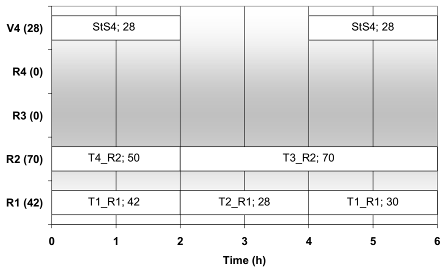

Figure 4. Optimal solution for EX1

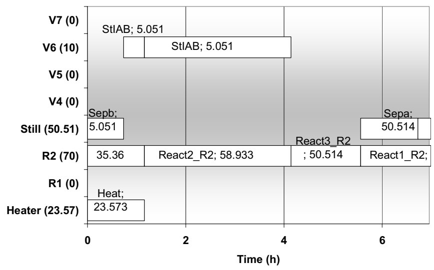

Figure 5. Optimal solution for EX2

5. Conclusions

This paper presents a new continuous-time formulation for the simultaneous design and scheduling of multipurpose plants featuring batch and/or continuous tasks with fixed or variable duration. The formulation employs the Resource Task Network process representation making it entirely general and directly applicable to virtually any process. The efficiency of the proposed formulation was tested by solving two well known example problems. The results show that by solving a single model, one is informed of which equipments to buy, their size, the optimal cycle time and operating schedule. Although this is rather convenient, bear in mind that with current computers only small problems can be handled by this continuous-time formulation. Thus, there is still need for improvement and further developments should be expected in the forthcoming years.

References

Barbosa-Póvoa, A., Macchietto, S. (1994). Detailed Design of Multipurpose Batch Plants. Comp. Chem. Eng., 18, 1013.

Castro, P., Barbosa-Póvoa, A., Novais, A. (2004). Design and Scheduling of Multipurpose Plants using a RTN Continuous-time Formulation. In Proceedings of the Sixth Conference on Foundations of Computer-Aided Process Design, Cache Publications: Austin, 2004; p 431.

Lin, X., Floudas, C. (2001). Design, Synthesis and Scheduling of Multipurpose Batch Plants via an Effective Continuous-time Formulation. Comp. Chem. Eng., 25, 665.