

Effect of pricing, advertisement and competition in multi-site capacity planning

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Abstract

The multi-site capacity planning problem with a long term time horizon, with transportation constraints is expanded to consider pricing of the product, the effect of advertisement and competition. We use a modification of the Cobb-Douglas model for consumer utility and include the effect of advertising.

Keywords: Capacity Planning Problem, Planning, Pricing.

1. Introduction

The investment capacity planning problem has been so far formulated for enterprises where the demand and price were considered as parameters, sometimes uncertain (Sahinidis et al., 1989, Liu and Sahinidis, 1996, Iyer and Grossmann, 1998, Ahmed and Sahinidis, 1998, etc.). Recently, Guillen et al., (2003) showed that elementary pricing decisions can be added to scheduling problems. Extensions of Guillen et al. (2003)'s model to capacity planning are straightforward. This approach, however, does not take into account the effect of advertisement, or the influence of competition.

This paper addresses the simultaneous addition of pricing, advertisement cost and competition to investment and capacity planning problems. A microbrewery capacity planning problem (Spencer et al., 2004) is used as basis for illustration

2. Background

Spencer et al. (2004) considered the typical capacity planning problem with expansions, with different potential locations and transportation costs for the plants and applied it to the location of beer microbreweries. The model, which maximizes NPV, was crafted so that reinvestment is allowed by using portions of proceeds and no new capital. The model output provides the expansions through the time horizon, the anticipated sales, and the advertising strategies and expenses. The novel aspect of the model was that sales in each potential market were connected to advertisement expenditures and the effect of competition was considered. Competition was modeled in the following way:

$$\text{Sales} < \text{Market Demand} * (\text{Market share without competition} - \text{Effect of competition})$$

Thus, the effect of competition is to reduce the market share in proportion to the number of breweries in the market. Advertisement was considered as follows:

$$\text{Market Share} = \text{Market share without competition} + b * \text{Advertisement}$$

where b is the constant relating market share increase per dollar spent in advertisement. The assumption is that the sales to advertisement efforts behave linearly. In advertisement theory (Rao, 1970), sales grow linearly with advertisement until they reach saturation (they level off) after which there is a small decline (over-saturation phase). When advertisement is not present, Spencer et al (2004) showed that location changes to places where a larger market share can be obtained with less advertisement efforts and/or a location closer to more markets. Spencer et al (2004) considered prices fixed. This paper relaxes this assumption and also considers competition and advertisement in a different way.

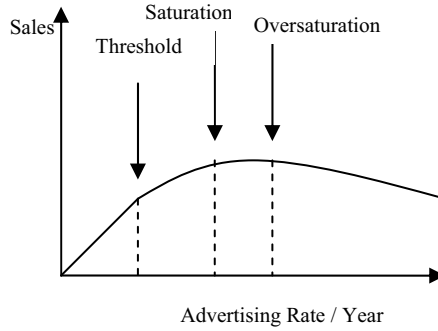


Figure 1. Sales vs. advertising effort.

3. Pricing and Competition

Assume first that there is an established market for the new product (in this case, beer) and that the total demand is assumed constant. The question is what price will be the right one and what level of advertisement efforts is needed to attract the optimal number of customers and the new demand associated to it. Let p_1 be the targeted new product selling price, d_1 its demand, p_2 the average competitor's product price, and d_2 is the corresponding competitor demand. We start assuming the following relation as valid:

$$p_1 d_1 = c p_2 d_2 \quad (1)$$

In this relation c is a constant. This relation can be derived using the concept of indifference curves (Hirshleifer and Hirshleifer, 1998), which express how much equal utility (happiness) a combination of product 1 and 2 will provide to a customer. Such an expression, compatible with (1) is $d_1^{r_1} d_2^{r_2} = U$, where U is the utility and $c = r_1 / r_2$. This form of the utility function is known as the Cobb-Douglas utility, which requires $r_1 + r_2 = 1$. Thus, (1) is obtained by maximizing U subject to a budget constraint $p_1 d_1 + p_2 d_2 \leq Y$, where Y is the total consumer budget. In the case where the prices

of the competition (p_2) and d_2 are fixed, the expression reduces to $p_1 d_1 = \text{const}$, which corresponds to a market structure of monopolistic competition, (Dixit and Stiglitz, 1977) which is in turn based on two assumptions: a) the firm maximize its own profit by choosing an optimal price for given prices by other firms, and b) each demand is small relative to entire market. The use of the above formula for pricing is sometimes conflictive. For instance, if $d_2 \gg d_1$ if an equality is used, p_1 will be very small, and likely incompatible with the economics. If one uses an inequality (e.g. $p_1 d_1 \leq c p_2 d_2$) the restriction becomes meaningless and the price chosen will be too high. To overcome this, it is proposed to use the plant capacity instead of d_2 . This is equivalent to assuming that the market size is of the capacity of the plant. Indeed, if such a market is shared 50/50, then prices will be equal (for $c=1$).

4. Advertising

Equation (1) does not express the dynamics of the competition process, but rather the state of equilibrium among competitors at some point in time. When $c=1$, equation (1) reveals that equal prices of the new product and that of the competitors' product, results in an evenly shared market. Such outcome would be realistic if the following conditions are met: a) The new product and the existing ones have been in the market for a long time. b) The quality of each product is the same. c) Advertising campaigns are equally effective, and d) Production capacities of all competitors can satisfy the demands. Since these conditions are hardly met for a new product, the above model needs to be improved. First, the competitors will have a clear advantage over the new product because they will have been established in the market for a number of years. They will have earned loyal customers and will have successful advertising campaigns in place. On the other hand, the new product will have the advantage of being a superior product, which will either increase convenience of use or save money and time to the customer. To account for influences from advertising, the model is modified as follows:

$$\beta(t, a) \cdot p_1 d_1 = c p_2 d_2 \cdot \alpha(t, a) \quad (2)$$

This still corresponds to an indifference curve of the form $d_1^{r_1} d_2^{r_2} = U$, where now r_1 and r_2 are functions of $\alpha(t, a)$ and $\beta(t, a)$. The function $\alpha(t, a)$ is a function of time and the advertisement campaign efforts for the new product. This function ranges between zero and one. At the beginning ($t=0$), $\alpha(0, a) = \alpha_0$ (a small number) indicating that the demand of the new product d_1 is small, no matter what the price is. As time goes by, the function approaches one, reflecting equal opportunities for all competitors in terms of advertisement. Thus, before the value of $\alpha(t, a)$ reaches one, the competitors have a competitive advantage by virtue of their longer standing in the market, with an established customer base. Thus, we call $\alpha(t, a)$ the Inferiority Function for the new product. In turn, $\beta(t, a)$ is also a function of time and the advertisement campaign of the new product (a). At the beginning ($t = 0$), $\beta(0, a) = 1$

indicating that there is no initial advantage for the new product. As time increases, β approaches zero asymptotically, becoming zero only if the competition disappears. It represents the superiority of the new product and ultimately its competitive edge. Therefore, we call it the Superiority function. We envision these two functions to have concave and convex forms as shown in figure 2.

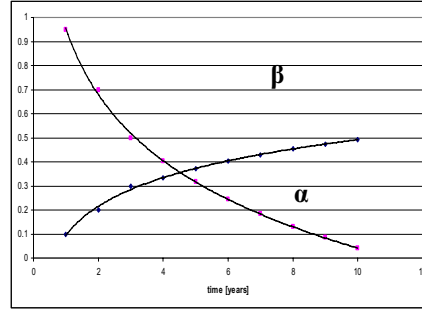


Figure 2. Inferiority and superiority functions.

One gets some estimated values for $\alpha(t, a)$ and $\beta(t, a)$ using data from the performance of similar novel products in the past. We note that this is a simplified model ignores the advertisement efforts of the competition. The assumption that the inferiority and superiority functions are linear with advertisement efforts is first made. For simplicity, we also assume $\beta(t, a) = 1$, that is the existing product is as good as the new one in the consumer minds for a long time. The following form of the inferiority function can be assumed: $\alpha(t, a) = [\alpha_1 - (\alpha_1 - \alpha_0)e^{-(\gamma_t + \gamma_{at}a)t}]$, where α_0 and α_1 are the inferiority function values at time zero and infinity, respectively. It is also assumed that within the time horizon, $[\gamma_t + \gamma_{at}a]t \ll 1$. Under these conditions, (2) renders a linear relationship between d_1 and advertisement efforts consistent with figure 1. This inferiority function does not penalize higher prices and therefore, it leads to answers where the price chosen is always the highest. This can be easily verified by substituting (2) in the following simplified profit function $Profit = p_1 d_1 - (g_1 + g_2 d_1) - a$ where $(g_1 + g_2 d_1)$ is the operating cost and realizing that reducing d_1 and increasing p_1 leads to higher profits. One needs to realize that as much as advertisements changes consumer attitude towards buying the product, price has the adverse effect, so we propose to use $\alpha = \alpha(t, a, p_1) = e^{\gamma_p(p_1 - p_2)} [\alpha_1 - (\alpha_1 - \alpha_0)e^{-(\gamma_t + \gamma_{at}a)t}]$. Thus, if the price is higher than the market, then revenues are higher, but the inferiority function is also lower.

5. Model

The model is presented succinctly because of space reasons. As stated above, the Spencer et al's (2004) model is used, which considers multiple potential sites for plants. Each plant can send its products to different markets. We also consider several different locations for raw materials. The transportation costs of raw materials as well as products are taken into account. Budgeting constraints are set so that the capital investment is used at the beginning and all the expansions are financed by the proceeds of the project. Thus, the model decides if it is profitable to reinvest. Finally, the following pricing

model is used: $p_1 d_1 = c p_2 Cap [\alpha_1 + (\alpha_1 - \alpha_0)(\gamma_t + \gamma_{at} a) t e^{\gamma_p(p_2 - p_1)}]$, which assumes monopolistic competition. (p_2 and d_2 fixed) and includes a linearization of the inferiority function using the assumption that $[\gamma_t + \gamma_{at} a] t \ll 1$ over the time horizon. Although time is already discretized into periods in this model, the expression $p_1 d_1$ is non-linear. To linearize the expression and use MILP solvers prices were discretized. Thus, assuming that a set of n_m values of prices per market m are chosen ($p_{m,i}$, $i=1..n_m$), the following equations are used:

$$\sum_{i \in P_m} z_{m,i} p_{m,i} sales_m \leq c p_{c,m} Cap \cdot \left[1 + (1 - \alpha_{0,m})(\gamma_{m,t} + \gamma_{m,at} a_m) t e^{\gamma_p(p_{c,m} - p_{m,i1})} \right] \quad (3)$$

$$\sum_{i \in P_m} z_{m,i} = 1 \quad (4)$$

where Cap is the final capacity, $p_{c,m}$ is the average price of the competition, and $z_{m,i}$ a binary variable that forces the model to choose only one price. This price stays constant through the life of the project (although this condition is also easy to relax). This is the same strategy as the one used by Guillen et al. (2003). Linearization of (3) is straightforward. Notice that sales instead of demand is used in eq (3) together with an inequality, because sales (which are lower than demand) may be limited by capacity.

4. Example

Microbreweries are defined by the industry as small breweries that produce less than 15,000 barrels of beer per year and distribute the product for consumption off-premise. A pale ale has been chosen for the recipe of the beer to be produced because it is lighter in taste than other microbrews, but it has more taste than the watered-down national brands. The main raw materials used in the production of micro brewed beer are hops, malted barley (yellow dots in figure 3), and yeast (blue dots). Several locations in the US were considered (black dots). The Markets are each of the 48 US contiguous states. Using a maximum of 1.5 million of initial investment, fixed prices and the aforementioned advertisement and competition model, Spencer et al. (2004) found a solution that chooses to build only one brewery (Phoenix, AZ) for only one market (AZ), with expansions in year two, three and four. For a \$3 million initial investment, two breweries are built (Phoenix, AZ and Las Vegas, NV) and expanded afterwards.



Figure 3. Raw materials and potential plant locations

When the effect of competition was eliminated and the advertisement was eliminated and the market share considered fixed (slightly higher than the one without advertisement) the model used by Spencer et al (2004) chose to build only one brewery in Milwaukee, Wisconsin. Since the model was not allowed to increase its market percentage through advertising, it was forced to sell to more markets than before. The net present worth came out to be about half of that with advertising included. The new model was run using $\alpha_1 = 0.95$, $\alpha_0 = 1/7$, $\gamma_t = 0.029$, $\gamma_p = \ln\{p_2 1.5 / p_{1max}\} / (p_{1max} - p_2)$, which corresponds to 66% of the maximum potential achieved if $p_1 = p_2$, and γ_{at} such that the negative effect of the highest price ($p_1 = p_{1max}$) is equivalent to $a = \$50,000$ at year 10. Only 3 choices of price were given ($0.975p_2$, p_2 and $1.025p_2$). With these parameters, when considering Phoenix, the model puts one plant with a small capacity at first, building up to full capacity (15,000) in year 7, using only about 1.25 million of the 1.5 million available. It sells and advertises in Arizona with the lowest price possible, and decides to sell in New Mexico at a highest price *without any advertising*, for only two years, after which it abandons that market. One interesting result is that it advertises in Arizona until year 9, time at which decides to stop, because of the natural growth of the inferiority function. When γ_t was halved, Nevada was added as a market for a few years to later abandon it: the investment is slightly lower (1.175 million).

5. Conclusions

A multi-site capacity planning problem recently developed, which contained location dependent transportation costs and budgeting constraints as well as sales, prices and advertisement costs as variables, was developed. A combination of low prices and advertisement can be a good strategy coupled with capacity planning expansions. Future work will include larger initial investment, uncertainty and risk.

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