

Optimal Scheduling of Supply Chains: A New Continuous-Time Formulation

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Abstract

In this paper a new continuous-time mathematical formulation is proposed for the optimal schedule of industrial supply chains. This introduces novel concepts to represent the supply chain *instances* (sites, equipment units, transportation facilities, etc) *suitable events* (transforming operations, transportation tasks, storage) and *material states*. Furthermore, it establishes a better commitment between the events allocation and the time scale associated.

A single level formulation is obtained that allows the computation of the optimal supply chain schedule for a defined economical or operational performance criterion while accounting simultaneously for the explicit integration of different topological and operational characteristics of the supply chain dynamics.

The formulation is a Mixed Integer Linear Programming (MILP) that is solved using a standard Branch and Bound (B&B) procedure.

The applicability of the proposed formulation is illustrated through the solution of a practical example involving an industrial pharmaceutical chain.

Keywords: supply chain, schedule, optimization, continuous-time.

1. Introduction

For many years companies managed their logistic processes, procurement, production and distribution in a non integrated way. However, due to increasing competition, this attitude has been changing and companies do now consider the integration of their supply chain as a key business issue. Consequently, the accessibility to tools that will allow an integration of such structures both at the design and operational levels is crucial (Goetschalckx et al, 2002).

Some important contributions within the supply chain modeling and optimization have been proposed recently. Two important research strategies were followed, a stochastic and a deterministic approach. The former, results frequently into complex non linear optimization problems requiring strong assumptions to achieve an optimal or near-optimal solution. On the other hand, the deterministic counterpart results typically into large mixed integer linear problems based essentially in a discrete time approach to represent the time domain. The main drawback of these formulations is that the resulting problems can become hard to solve within a reasonable margin of optimality.

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Furthermore, in the works published few authors considered the different supply chain activities integration and a lack of models generality exists (Shah, 2004).

This work tries to address some of these limitations and presents a continuous time model for the integrated supply chain operation. Novel concepts are introduced that allow the complete representation of supply chain structure and its operability while keeping the mathematical formulation linearity. The resulting MILP problems are solved using a standard Branch and Bound (B&B) procedure.

The model is characterized below and some considerations are made on the assumptions made. A real case-study is then solved showing the model applicability.

2. Model Characterization

A supply chain is an operational structure that produces and distributes a set of materials using its internal resources (production, storage and transportation facilities) and some external resources (raw-materials, utilities). The resources are distributed amongst *sites* that guarantee a supply chain specific task (production, storage, distribution, etc.). These sites can be grouped into different *clusters* based on their operational functionality: (a) supplying, (b) transformation, (c) packing and (d) distribution. The latter supplying a set of aggregated customer regions (Amaro and Barbosa-Póvoa, 2004).

The supply chain sites and the associated connectivity structures form the supply chain topology. Each site and connectivity structure is then characterized by the associated resources (e.g. facility, warehouses and trucks) availability, suitability and capacity amongst other requirements.

In terms of operation two major set of events describe the supply chain operability. These are respectively *transforming* and *transportation* events. The former represents general processing operations involving a material “transformation” (e.g. reaction step, filling operation, storage) and are referred as *tasks*, $i=1, \dots, NT$. The latter describes the materials transportation and are named as *transportation flows* or simply *flows*, $l=1, \dots, NL$.

The linkage between the chain topology and the events occurrence is derived automatically based on the suitability criteria. For the task events a single suitable linkage is assumed. Does for a given transforming operation that can be performed at NE independent resource instances, a set of NE distinct event tasks is generate. Nevertheless, the same resource can be suitable to perform more than a task (at different time domains) and a compatibility set is generated to represent the set of task events that can be performed at each resource. For the transportations events, each transportation flow is defined by a material state, a path and a connectivity structure (Amaro and Barbosa-Póvoa, 1999). Each connectivity structure may have a set of transportation resources to perform every suitable event. The suitability criterion establishes the linkage between the transportation flow and the connectivity structure. Any transport event (e.g. transport of material A from S1 to S3 site) is represented by a single transportation flow with NV suitable resource allocations belonging to the same connectivity structure, or by a set of Nl transportation flows defined over an equal number of available connectivity structures. To account for the multipurpose nature of the transportation resources (a resource may be shared by more than a material state during a specific time domain) a compatibility criteria is also defined and evaluated for each pair of transportation flows, l and l' , defined over the same connectivity structure. The sets of compatible flows defined over each connectivity structure are grouped into a

novel modeling instance called a *family*. The supply chain transport operations are then represented by the whole sets of compatible and incompatible family flows. Finally, the material existence along the supply chain is modeled by a set of material states. These represent the linkage between each material and the associated chain location.

3. Mathematical Formulation

In the present model a continuous time formulation is developed where a common time grid for all resources is assumed (Castro et al, 2001). The time horizon is divided into several intervals (slots), with events taking place only at the interval boundaries. Each slot has an *a priori* unknown duration that will be determined by the time limiting event. This limiting time can result from different occurrences: (1) the allocation of a single processing task or transportation flow to a given resource; (2) the allocation of a set of batches of the same processing tasks (replication of the same batch) to the same resource; (3) the occurrence of a linear combination of different task events allocated to the same processing resource instance (assemble of batches) and (4) a linear combination of transportation events allocated to suitable transportation resources. Note that, cases (2), (3) and (4) can only occur if the required material to process or transport, during the slot time, is available at the beginning of the slot. The same availability is applied to the resources used. Furthermore, the products obtained through the processing tasks or transportation flows will only be available at the end of the slot. Therefore occurrences of any of these situations imply a non requirement, during the slot time, of the material and resources involved (see case-study below). Also, associated storage capacity must exist during the slot so as to account for eventual releases of material. Furthermore, as in Castro et al (2001), and for points (1), (2) and (3) the concept of non-limiting task is used. Tasks are allowed to last longer than their processing times if no demand exists on the associated material and/or resource. Additionally, point (4) may represent the occurrence of different transport flows performed by the same family or the allocation of different transport families to a given suitable transport resource defined within a connectivity structure. A sharing mode of operation is allowed for the allocation of each transportation family to a given transport resource (each transport equipment may be shared by the whole set of flows defined within a family, during a specific time domain). Overlapped and the non-overlapped operation modes are considered. The former represents a sharing mode of operation where the limiting time describes the time usage of a transport resource by the simultaneous occurrence of a set of transport flows belonging to the same family. In the latter, the limiting time accounts for the linear combination of transportation times characterizing the allocated set of compatible (single family) or incompatible (different families) transportation flows.

These new concepts allow a reduction on the number of event points required to describe the global time horizon and does a reduction in the global model may be obtained.

In terms of variables the model considers both binary and continuous variables. The binary variables represent decisions within the supply chain (e.g. allocation of transportation flows and transforming tasks to suitable resources) while the continuous variables describe operational requirements (e.g. task dimension, amounts transported and material amounts).

In terms of constraints these are of different forms and account for various situations such as: time slot bounds and linkages with events allocation, capacity requirements and limitations, equipment instances suitability and allocation, transportation policies, events pre-conditions and compatibilities, material supplies, demand levels and due dates, amongst others. Due to the lack of space the mathematical model will not be presented in this paper.

As objective function the maximization of the supply chain profit is adopted. This involves processing, storage, transport and raw-materials costs as well as product values.

The final model solution optimizes simultaneously the slot dimension and the scheduling objective while accounting for the explicit integration of different topological and operational characteristics of the supply chain.

4. Case Study

A supply chain is considered where different pharmaceutical solutions are produced for hospitals as well as ambulatory use. The supplying cluster is characterized by a main production structure, plant P1. This has four operational sections, two of them performing water treatment operations (WT1 and WT2) followed by a mixing step (MH) where chemicals additives are homogenised with treated water. A filling section follows where a blow, fill and sell process is carried out (BFS). The water treatment produces two water types, W_1 and W_2, and some water wasted solutions, WW. W_1 is sterile water and is used for general nursing purposes and to produce the water solution (W_2), suitable for injection purposes at the second treatment section. Chemical sterile additives are then mixed and homogenised with W_2 and the resulting solution goes to the filling section. Four different solutions are obtained: IS1, IS2, IS3 and IS4 at different capacity containers: 100, 250, 500 and 1000 ml. The P1 raw-materials, water treatment chemicals and additives as well as the polymer, are fulfilled by external suppliers based on pre-defined supplying capacities and time scales. The water required is an inner resource (locally explored).

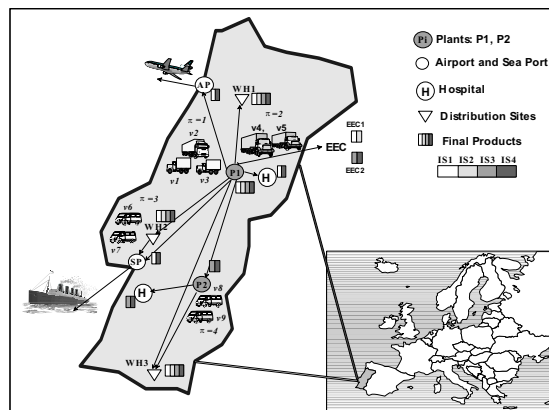


Table 1 - Family flows description

Sites	P1	P2	WH2
H1	<i>f1</i>		
WH1	<i>f2</i>		
WH2	<i>f3</i>		
WH3	<i>f6</i>	<i>f7</i>	
P2	<i>f5</i>		
SP	<i>f4</i>		<i>f9</i>
AP	<i>f12</i>		
H2		<i>f8</i>	
EEC1	<i>f10</i>		
EEC2	<i>f11</i>		

Figure 1 - Supply chain structure.

In terms of country distribution P1 supplies different points in the supply chain: P2 a generic drug plant with IS3 and IS4; a hospital site with IS1 and IS4; and three

distribution sites (warehouses WH1 to WH3) located at different country regions, figure 1 and table 1.

Furthermore, IS1 and IS4 are also delivered to an African distribution partner using a free on board strategy for the material fulfilments carried at the sea port. Finally, delivers to EEC partners are performed. Two of them involve a road distribution and a latter one a free on fly strategy carried out at the airport site, figure 1. The former EEC partners (EEC1 and EEC2) demand respectively, IS1 and IS2, and IS3 and IS4 while the later one (EEC3) requires IS1 and IS4. The above distribution network is guaranteed essentially by P1 dedicated transport resources $\pi=1$ and $\pi=2$ (see Figure 1) that involve respectively resources $v1, v2, v3$ (in-house) and $v4, v5$ (contracted). Furthermore, WH2 and P2 also have their own transport resources ($\pi=3$ and $\pi=4$). The former may transport materials to the sea port while the later may send the materials to the WH3 distribution site and to the south hospital site, H2. This is allowed based on a cross-docking strategy.

Concerning the storage policies, it is assumed that at the beginning of the time scheduling all supply chain sites have a storage level of 25% of capacity dedicated to each suitable material state. Also, no storage level less than 5% is allowed.

Based on the above characteristics the supply chain schedule is optimized for five days of operation so as to maximize its profit while guaranteeing pre-defined demands and due dates. Some of the results obtained are shown in figures 2 and 3. In terms of operation it can be seen from figure 2 that only IS4 and IS2 are produced. This is explained by the fact that the available storage levels of IS1 and IS3 are enough to guarantee the amounts required for distribution. On the other hand, the high level of production of IS4 is due to its high market requirements that are not supported by the available storage levels. Within the plant schedule (figure 2) different time limiting occurrences are observed such as: the replication of the same task (task 7 in slot 1, from 0 to 4) and a combination of different tasks (tasks 7 and 5 in slot 5). Also, the occurrence of non-limiting tasks is observed (represented by task horizontal lines, task 1 in slot 1 and task 3 in slot 5, etc.)

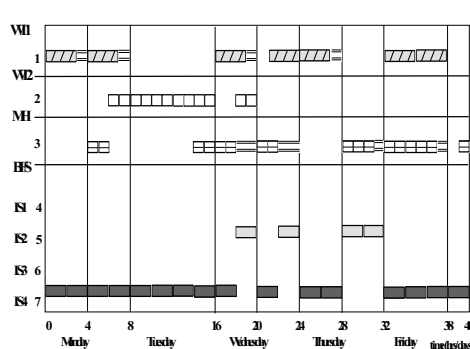


Figure 2 – Plant P1 Schedule

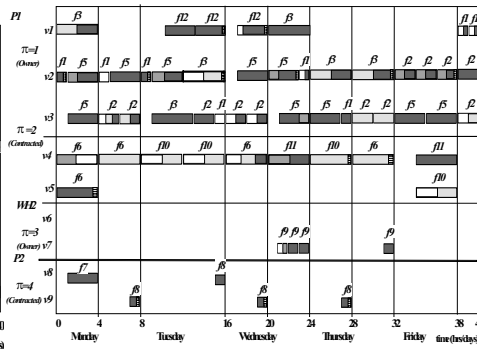


Figure 3–Supply Chain distribution schedule

For the distribution activity (figure 3), structure P1 guarantees most of the distribution requirements since it is the main linking point in the supply chain. P1 supplies directly warehouses WH1, WH2 and WH3, the airport and the associated plant P2. On the other hand, warehouse WH3 is also supplied through P2 and the sea port is supplied by warehouse WH2. Analyzing in more detail the use of the vehicles it can be seen that

due to its higher capacity transportation equipment v2 is fully used during the week. Furthermore, vehicles v1 and v3 that have the same capacity but different variable transportation costs (v3 has a lower cost) are used accordingly and does the later is more used. On the other hand, vehicles v7, v8 and v9 are used occasionally when required to perform some cross-docking operations. These different occurrences combine various transportation policies (see point 3). For instance at the second time slot (from 4 to 8 hours) family flow f2 is allocated twice to vehicle v3 and two non replicated transport operations are done. In the former three different materials are transported (compatible flows of IS1, IS2 and IS4) while at the latter only two of these materials (IS2 and IS4) are transported. Events are overlapped during each family allocation and a sharing mode of operation is observed. In slot 8 to 16, family f3 is allocated to different transportation equipments (v2 and v3) performing in each case different flow tasks. The transport of IS1 and IS2 is performed by v2 using a sharing mode of operation, while IS4 uses fully the capacity of vehicle v3. Finally, at the first Friday slot (32 to 38 hr) a single transport event uses v3 capacity and a non overlapped operation is performed.

The final model was solved using the GAMS/CPLEX solver. It is characterised by a maximum profit of 139982,37 currency units and involves 15261 constraints, 9306 variables, 2719 integer, and took 2605 CPU sec. to solve in a Pentium III.

5. Conclusions

A novel continuous time model was proposed for the optimal schedule of supply chains where an integrated approach of different supply chain activities such as supply, production, storage and distribution is considered. Structural and operational characteristics are model simultaneously where variable tasks and transportation times are allowed. Different storage policies, transportation resources sharing and pre-defined demand requirements are also considered. As final result a detailed operating plan at both the production and transport levels is obtained where processing, storage and transport occurrences are identified.

The continuous nature of the mathematical formulation allows a closer linkage between supply chain major events and the modeling space used to represent it. In this way time dependent events can be easily modeled and novel concepts are considered. This is the case of the non-limiting tasks concept and the occurrence of different combination of processing tasks as well as transportation flows. Furthermore, a transport sharing policy exploring the family concepts is also developed. These novel concepts allow the reduction of the number of events points in the global model resulting in a flexible but quite compact formulation.

A real case study taken from a pharmaceutical industry was studied and the results obtained were promising. Further studies are under development where different supply chain structures and characteristics are being analyzed.

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