

Advances in Robust Optimization Approaches for Scheduling under Uncertainty

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Abstract

The problem of scheduling under uncertainty is addressed. We propose a novel robust optimization methodology, which when applied to Mixed-Integer Linear Programming (MILP) problems produces “robust” solutions that are, in a sense, immune against uncertainty. The robust optimization approach is applied to the scheduling under uncertainty problem. Based on a novel and effective continuous-time short-term scheduling model proposed by Floudas and coworkers (Ierapetritou and Floudas 1998a, 1998b; Ierapetritou et al. 1999; Janak et al. 2004; Lin and Floudas 2001; Lin et al. 2002, 2003), three of the most common sources of uncertainty in scheduling problems can be addressed, namely processing times of tasks, market demands for products, and prices of products and raw materials. Computational results on a small example with uncertainty in the processing times of tasks are presented to demonstrate the effectiveness of the proposed approach.

Keywords: Process scheduling, uncertainty, robust optimization, MILP

1. Introduction

The issue of robustness in scheduling under uncertainty has received relatively little attention, in spite of its importance and the fact that there has been a substantial amount of work to address the problem of design and operation of batch plants under uncertainty. Most of the existing work has followed the scenario-based framework, in which the uncertainty is modeled through the use of a number of scenarios, using either discrete probability distributions or the discretization of continuous probability distribution functions, and the expectation of a certain performance criterion, such as the expected profit, which is optimized with respect to the scheduling decision variables. Scenario-based approaches provide a straightforward way to implicitly incorporate uncertainty. However, they inevitably enlarge the size of the problem significantly as the number of scenarios increases exponentially with the number of uncertain parameters. This main drawback limits the application of these approaches to solve practical problems with a large number of uncertain parameters. A recent review of scheduling approaches, including uncertainty, can be found in Floudas and Lin (2004).

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In this work, we propose a novel robust optimization approach to address the problem of scheduling under uncertainty. The underlying framework is based on a robust optimization methodology first introduced for Linear Programming (LP) problems by Ben-Tal and Nemirovski (2000) and extended in this work for Mixed-Integer Linear Programming (MILP) problems.

2. Problem Statement

The scheduling problem of chemical processes is defined as follows. Given (i) production recipes (i.e., the processing times for each task at the suitable units, and the amount of the materials required for the production of each product), (ii) available equipment and the ranges of their capacities, (iii) material storage policy, (iv) production requirement, and (v) time horizon under consideration, determine (i) the optimal sequence of tasks taking place in each unit, (ii) the amount of material being processed at each time in each unit, (iii) the processing time of each task in each unit, so as to optimize a performance criterion, for example, to minimize the makespan or to maximize the overall profit.

The most common sources of uncertainty in the aforementioned scheduling problem are (i) the processing times of tasks, (ii) the market demands for products, and (iii) the prices of products and/or raw materials. An uncertain parameter can be described using discrete or continuous distributions. In some cases, only limited knowledge about the distribution is available, for example, the uncertainty is bounded, or the uncertainty is symmetrically distributed in a certain range. In the best situation, the distribution function for the uncertain parameter is given, for instance, as a normal distribution with known mean and standard deviation. In this paper, we will discuss bounded uncertainty as well as uncertainty with a known distribution.

3. Robust Optimization for MILP Problems

Consider the following generic mixed-integer linear programming (MILP) problem:

$$\begin{aligned}
 & \underset{x,y}{Min / Max} \quad c^T x + d^T y \\
 & s.t. \quad \quad \quad Ex + Fy = e \\
 & \quad \quad \quad Ax + By \leq p \\
 & \quad \quad \quad x^L \leq x \leq x^U \\
 & \quad \quad \quad y = \{0,1\}
 \end{aligned} \tag{1}$$

Assume that the uncertainty arises from both the coefficients and the right-hand-side parameters of the inequality constraints, namely, a_{lm} , b_{lk} and p_l . We are concerned about the feasibility of the following inequality.

$$\sum_m a_{lm} x_m + \sum_k b_{lk} y_k \leq p_l \tag{2}$$

Our objective here is to develop a robust optimization methodology to generate “reliable” solutions to the MILP program, which are immune against uncertainty. Two types of uncertainty are addressed, (i) bounded uncertainty and (ii) uncertainty with a known distribution.

3.1. Bounded Uncertainty

Suppose that the uncertain data range in the following intervals:

$$|\tilde{a}_{lm} - a_{lm}| \leq \varepsilon |a_{lm}|, \quad |\tilde{b}_{lk} - b_{lk}| \leq \varepsilon |b_{lk}|, \quad |\tilde{p}_l - p_l| \leq \varepsilon |p_l| \quad (3)$$

$\tilde{a}_{lm}$, $\tilde{b}_{lk}$ and $\tilde{p}_l$ are the “true” values, a_{lm} , b_{lk} and p_l are the nominal values, and $\varepsilon > 0$ is a given (relative) uncertainty level. We call a solution (x, y) robust if: (i) (x, y) is feasible for the nominal problem, and (ii) whatever are the true values of the coefficients and parameters within the corresponding intervals, (x, y) must satisfy the l -th inequality constraint with an error of at most $\delta \max[1, |p_l|]$, where δ is a given infeasibility tolerance.

Given an infeasibility tolerance δ , to generate robust solutions, the following so-called (ε, δ) -Interval Robust Counterpart (IRC $[\varepsilon, \delta]$) of the original uncertain MILP problem can be derived.

$$\begin{aligned} & \underset{x, y, u}{\text{Min / Max}} \quad c^T x + d^T y \\ & \text{s.t.} \quad \begin{aligned} & Ex + Fy = e \\ & Ax + By \leq p \\ & \sum_m a_{lm} x_m + \varepsilon \sum_{m \in M_l} |a_{lm}| u_m + \sum_k b_{lk} y_k + \varepsilon \sum_{k \in K_l} |b_{lk}| y_k \\ & \quad \leq p_l - \varepsilon |p_l| + \delta \max[1, |p_l|], \forall l \\ & -u_m \leq x_m \leq u_m, \forall m \\ & x^L \leq x \leq x^U \\ & y_k \in \{0, 1\}, \forall k \end{aligned} \end{aligned} \quad (4)$$

where M_l and K_l are the set of indices of the x and y variables, respectively, with uncertain coefficients in the l -th inequality constraint. The derivation of this formulation can be found in the full-length manuscript of Lin et al. (2004). Note the mathematical model given in (4) remains an MILP model and compared to the original deterministic MILP problem, the robust counterpart has a set of auxiliary variables u_m and a set of additional constraints relating the variables x_m and u_m .

3.2. Uncertainty with a Known Distribution

Assume that in inequality constraint l , the true values of the uncertain parameters are obtained from their nominal values by random perturbations:

$$\tilde{a}_{lm} = (1 + \varepsilon \xi_{lm}) a_{lm}, \quad \tilde{b}_{lk} = (1 + \varepsilon \xi_{lk}) b_{lk}, \quad \tilde{p}_l = (1 + \varepsilon \xi_l) p_l \quad (5)$$

where ξ_{lm} , ξ_{lk} and ξ_l are independent random variables and $\varepsilon > 0$ is a given (relative) uncertainty level. In this situation, we call a solution (x, y) robust if: (i) (x, y) is feasible for the nominal problem, and (ii) for every l , the probability of the event

$$\sum_m \tilde{a}_{lm} x_m + \sum_k \tilde{b}_{lk} y_k > \tilde{p}_l + \delta \max[1, |p_l|] \text{ is at most } \kappa, \text{ where } \delta > 0 \text{ is a given}$$

feasibility tolerance and $\kappa > 0$ is a given reliability level. If the distributions of the random variables ξ_{lm} , ξ_{lk} and ξ_l in the uncertain parameters are known, it is possible

to obtain a more accurate estimation of the probability measures involved. Denote a new random variable ξ as the following:

$$\xi = \sum_{m \in M_l} \xi_{lm} | a_{lm} | x_m + \sum_{k \in K_l} \xi_{lk} | b_{lk} | y_k - \xi_l | p_l | \quad (6)$$

Assume that the distribution function of ξ is:

$$F_{\xi}(\lambda) = \Pr\{\xi \leq \lambda\} = 1 - \Pr\{\xi > \lambda\} = 1 - \kappa \quad (7)$$

where κ is a given reliability level and the inverse function (quantile) can be represented as follows:

$$F_{\xi}^{-1}(1 - \kappa) = f(\lambda, | a_{lm} | x_m, | b_{lk} | y_k, | p_l |) \quad (8)$$

Then, given an infeasibility tolerance, δ , and a reliability level, κ , to generate robust solutions, the following so-called $(\varepsilon, \delta, \kappa)$ -Robust Counterpart (RC $[\varepsilon, \delta, \kappa]$) of the original uncertain MILP problem can be derived. The additional constraints in the RC problem:

$$\begin{aligned} \sum_m a_{lm} x_m + \sum_k b_{lk} y_k + \varepsilon f(\lambda, | a_{lm} | x_m, | b_{lk} | y_k, | p_l |) \\ \leq p_l + \delta \max[1, | p_l |], \forall l \end{aligned} \quad (9)$$

Several different distribution functions can be modeled this way including the uniform distribution, normal distribution, difference of normal distributions, and several discrete distributions such as Poisson or binomial (Janak et al., 2005).

This robust optimization methodology can be applied to address the problem of scheduling under uncertainty, including three classes of problems: (i) uncertainty in processing times/rates of tasks, (ii) uncertainty in market demands for products, and (iii) uncertainty in market prices of products and raw materials. In this work, we will only consider uncertainty in the processing times/rates of tasks.

4. Example Problem

Consider the following example process that was first presented by Kondili et al. (1993) and has been widely studied in the literature. Two products can be produced from three feeds according to the state-task network as shown in Figure 1. The objective is to maximize the profit from sales of products manufactured in a time horizon of 12 h.

The continuous-time formulation proposed by Floudas and coworkers (Ierapetritou and Floudas 1998a, 1998b; Ierapetritou et al. 1999; Janak et al. 2004; Lin and Floudas 2001; Lin et al. 2002, 2003) is used to solve this simple scheduling problem. The example is implemented with GAMS (Brooke et al., 1988) and solved using CPLEX 8.1 on a Linux 3.0 GHz workstation. The “nominal” solution is shown in Figure 2, which features intensive utilization of units U2 and U3 and an objective value (profit) of 3639. However, this solution can become completely infeasible when there is uncertainty in the processing times of the tasks. Consider the case where the uncertainty of the processing times is bounded and the (relative) uncertainty level, ε , is 15% and the infeasibility tolerance level, δ , is 10%. Then, by solving the IRC $[\varepsilon, \delta]$ problem, a “robust” schedule is obtained, as shown in Figure 3, which takes into account

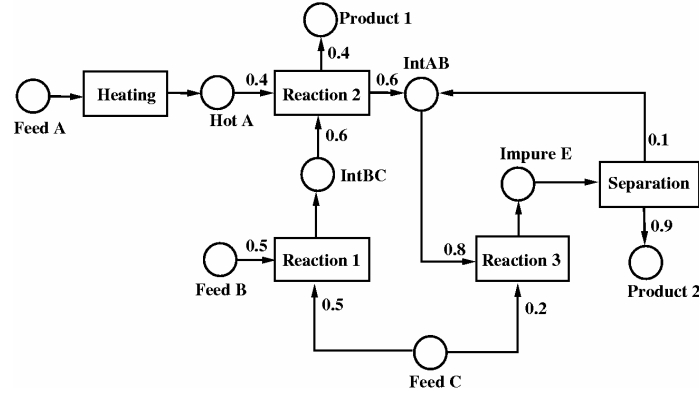


Figure 1. State-task network for the example problem.

uncertainty in the processing times. Compared to the nominal solution, the robust solution exhibits very different scheduling strategies, such as task-unit assignments and task timings. The robust solution ensures that the robust schedule obtained is feasible with the specified uncertainty level and infeasibility tolerance. However, the resulting profit is reduced, from 3639 to 2887, which reflects the effect of uncertainty on overall production. A comparison of the model and solution statistics for the nominal and robust solutions can be found in Table 1.

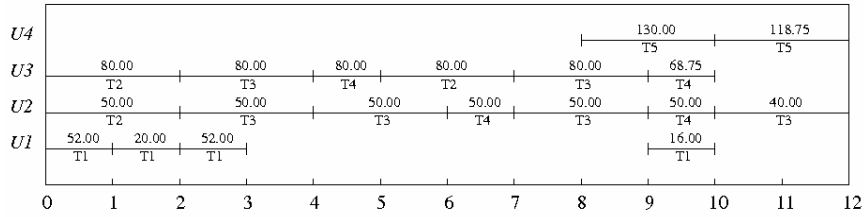


Figure 2. Optimal solution with nominal processing times.

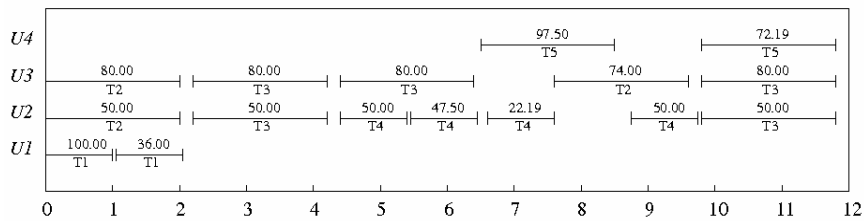


Figure 3. Robust solution with uncertainty level (ϵ) of 15%, infeasibility tolerance (δ) of 10%.

5. Conclusions

In this work, we propose a new approach to address the scheduling under uncertainty problem based on a robust optimization methodology, which when applied to Mixed-Integer (MILP) problems produces “robust” solutions which are in a sense immune

Table 1. Model and solution statistics for the example problem.

	Nominal Solution	Robust Solution
Profit	3638.75	2887.19
CPU Time (s)	0.40	10.10
Binary Variables	64	64
Continuous Variables	346	346
Constraints	489	713

against uncertainties in both the coefficients in the objective function, the left-hand-side parameters and the right-hand-side parameters of the inequality constraints. A unique feature of the proposed approach is that it can address many uncertain parameters. The approach can be applied to address the problem of production scheduling with uncertain processing times, market demands, and/or prices of products and raw materials. Our computational results show that this approach provides an effective way to address scheduling problems under uncertainty, producing reliable schedules and generating helpful insights on the tradeoffs between conflicting objectives. Furthermore, due to its efficient transformation, the approach is capable of solving real-world problems with a large number of uncertain parameters (Lin et al., 2004).

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