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A Robust Hybrid Search Technique for Solving Distributed Wastewater Treatment Systems

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Abstract

This work addresses the design of distributed wastewater networks for processing a set of streams and reducing the concentration of several contaminants below given concentration limits at a minimum cost. A simplified problem formulation is presented and a hybrid search procedure is introduced for solving the proposed model. Solution feasibility is always ensured by starting the search from a feasible point and allowing only feasible moves, thus reducing the search space as well. This is achieved by considering those changes consistent with local and global balances. Finally, different case studies are addressed and results obtained are discussed. Search performance is analysed regarding objective function evolution as well as the progress of significant bounded variables.

Keywords: Wastewater, Networks, Optimisation, Stochastic Search.

1. Introduction

The need for water conservation and the growing pressure of water quality regulations have demanded addressing the design and use of optimised distributed wastewater treatment systems for integrated water management. The design technique proposed by Wang and Smith (1994) has been followed by several works presenting different NLP and MINLP models for the design superstructure. The nature of the problem includes decisions on recycling and models require bilinear terms. A great effort has been dedicated to reduce the convergence difficulties that standard local optimisation techniques have for solving such nonconvex non-linear models including multiple local minima.

Galan and Grossman (1998) introduced a search procedure based on the successive solution of a relaxed linear model and the original MINLP problem that proved to reach global or near global optimum solutions in many cases. The recent work by Hernández-Suárez et al. (2004) uses a superstructure decomposition and parametric optimisation approach for dividing the search space before addressing the set of sub-problems produced. Solution feasibility is a key point in these math-programming approaches. Much less attention has been paid to stochastic or meta-heuristic optimisation techniques. Tsai and Chang (2001) reported interesting results on the implementation of

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a genetic algorithm for solving the NLP model corresponding to network superstructure they proposed.

This work presents a simplified NLP formulation of the problem. This formulation shows the same convergence problems reported when addressed using standard solvers such GAMS/CPLEX, although they may be mitigated to some unsatisfactory extent by introducing feasible starting points. However, given a complete set of decision variable values the model may be iteratively solved, which allows a simulation-based approach to address the optimisation problem.

2. Problem formulation

A simplified problem formulation is derived from the problem superstructure, which is illustrated in Figure 1 along with problem variables and parameters. A set of streams *s* and allied input splitters (*in*) distribute the entering flows (F_s^o) to the different treatment lines (*k*) composed by mixers, treatment units and the following splitters (*out*), which may redistribute the flows for further processing. Each input stream delivers a set of contaminant flows (f_{js}^o) and each contaminant j is removed at each treatment unit by B_{jk}. Thus, the split fractions x_{sk}^{in} and x_{ki}^{out} are the decision variables determining the final contaminant concentrations.

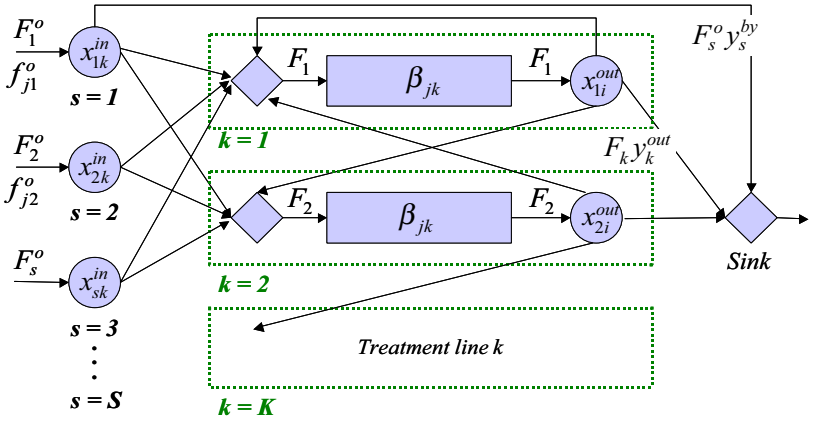

Figure 1. Problem superstructure.

Minimising the cost of the treatment is an objective that is formulated in a simplified way by the minimisation of the total of the flows to be processed by each of the units:

$$\min Z = \sum_{k=1}^{K} F_k \tag{1}$$

subject to the balance in each of the splitters:

$$\sum_{k=1}^{K} x_{sk}^{in} + y_s^{by} = 1 \quad \forall s \; ; \qquad \sum_{i=1}^{K} x_{ki}^{out} + y_k^{out} = 1 \quad \forall k \tag{2}$$

$$0 \le x_{sk}^{in} \le 1 \quad \forall s,k \qquad 0 \le x_{ki}^{out} \le 1 \quad \forall k,i \tag{3}$$

$$0 \le y_s^{by} \le 1 \quad \forall s \qquad 0 \le y_k^{out} \le 1 \quad \forall k \tag{4}$$

being y_s^{by} the fractions bypassing the treatment lines and y_k^{out} the fractions leaving them. The flows (total and for each contaminant) in each treatment line are given by the fresh contribution plus the flows (R_k and r_{jk}) recycled from other treatment lines.

$$F_k = \sum_{s=1}^{S} F_s^o x_{sk}^{in} + R_k = \sum_{s=1}^{S} F_s^o x_{sk}^{in} + \sum_{i=1}^{K} F_i x_{ik}^{out} \qquad \forall k \tag{5}$$

$$f_{jk} = \sum_{s=1}^{S} f_{js}^o x_{sk}^{in} + r_{jk} = \sum_{s=1}^{S} f_{js}^o x_{sk}^{in} + \sum_{i=1}^{K} f_{ji}\left(1-\beta_{jk}\right) x_{ik}^{out} \qquad \forall k, j \tag{6}$$

Total mass balance is also to be satisfied:

$$\sum_{k=1}^{K} F_k y_k^{out} + \sum_{s=1}^{S} F_s^o y_s^{by} = \sum_{s=1}^{S} F_s^o = F^T \tag{7}$$

Finally, the treatment target may be formulated in a simplified way by forcing the final contaminant concentrations to result below the limits established:

$$\sum_{k=1}^{K} f_{jk}\left(1-\beta_{jk}\right) y_k^{out} \le C_j^{\max} \cdot F^T \qquad \forall j \qquad \left(q_j \le Q_j^{\max}\right) \tag{8}$$

2.1 Feasible solutions and bounds

This simplified NLP formulation presents the same problems reported by Galan and Grossmann (1998) referred to convergence and the existence of local optima. Instead of addressing these problems by means of a series of LP relaxations, this work proposes the finding of some feasible starting points to be subsequently improved focussing in a practical way to obtain realistic solutions. Certainly, the most obvious and expensive solution consists of mixing and processing all the input streams through all treatment lines (Figure 2). This simple rule provides a set of feasible solutions as well as valuable information such as bounds for the objective function and the minimum concentrations attainable, which also allows checking the existence of at least one feasible solution.

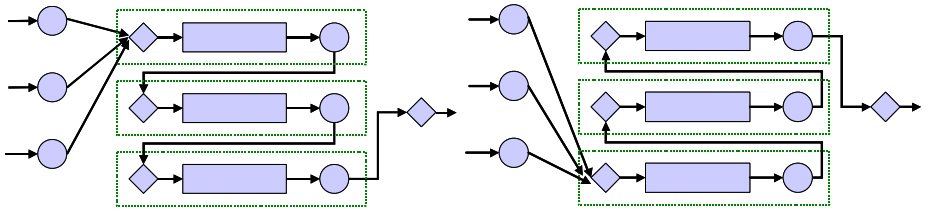

Figure 2. Feasible starting points.

The use of these starting points helped the solution of the NLP when using GAMS/CPLEX for solving simple examples (two contaminants, two streams, two lines) that previously did not converge. However, this proved to be insufficient when addressing more complex examples.

3. Simulation and search procedure

The solution scheme proposed is based on system simulation, for which recycle solving is achieved iteratively once the set of decision variables is fixed:

$$\left.\begin{array}{ll} F_k = \sum_{s=1}^{S} F_s^o x_{sk}^{in} + R_k^{(n)} & \forall k \\ R_k^{(n+1)} = \sum_{i=1}^{K} F_i x_{ik}^{out} & \forall k \\ \left| R_k^{(n+1)} - R_k^{(n)} \right| \le \varepsilon & \forall k \end{array}\right\} \qquad \left.\begin{array}{ll} f_{jk} = \sum_{s=1}^{S} f_{js}^o x_{sk}^{in} + r_{jk}^{(n)} & \forall k, j \\ r_{jk}^{(n+1)} = \sum_{i=1}^{K} f_{ji} \left(1-\beta_{jk}\right) x_{ik}^{out} & \forall k, j \\ \left| r_{jk}^{(n+1)} - r_{jk}^{(n)} \right| \le \varepsilon & \forall k, j \end{array}\right\} \tag{9}$$

Having the system response, next step is a mechanism for proposing test solutions to be evaluated. Having a feasible solution as well, moves will be defined so that feasibility is kept. At each splitter (*in* or *out*) any generic variable x_q (x_{sk}^{in}, x_{ki}^{out}, y_s^{by} or y_k^{out}) is locally constrained by eq.2. Thus, a feasible move is defined by:

$$\sum_{p=1}^{P} x_p^{(n)} = 1 \;\rightarrow\; \left\{\begin{array}{ll} x_q^{(n+1)} = x_q^{(n)} \pm \delta x & \\ x_p^{(n+1)} = x_p^{(n)} \mp \dfrac{\delta x}{P} & \forall p \neq q \end{array}\right\} \;\Rightarrow\; \sum_{p=1}^{P} x_p^{(n+1)} = 1 \tag{10}$$

$$\text{where:} \quad \delta x = \begin{cases} \min\left\{ dx, \left(1 - x_p\right) \right\} & \textit{if } dx \ge 0 \\ \max\left\{ dx, \left(-x_p\right) \right\} & \textit{otherwise} \end{cases} \tag{11}$$

being dx the basic step size applied to variable x_q and equally balanced by the rest of the variables in the set.

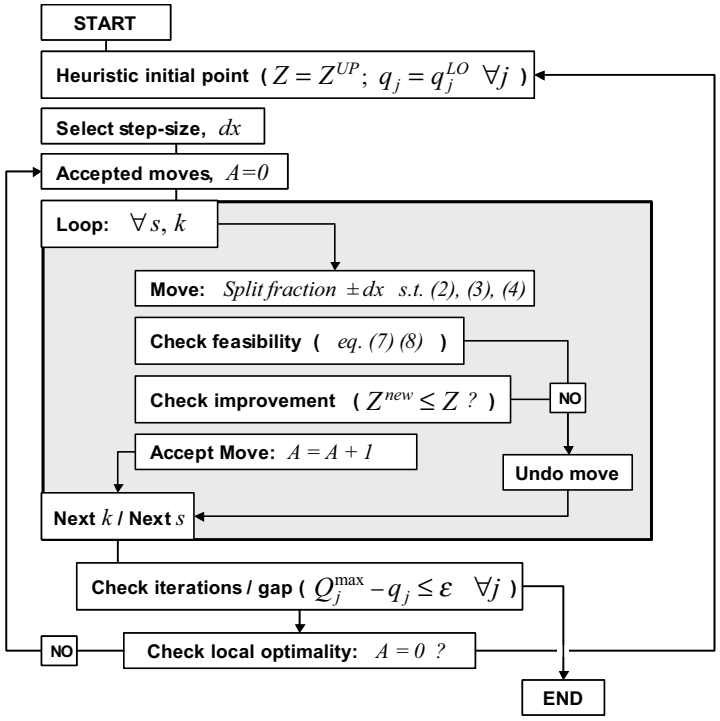

Figure 3. Search procedure.

The solution obtained using the move defined by eq. 10 and 11 is partially feasible in terms of the set of balances at each splitter (eq. 2). Confirming solution feasibility is immediate by checking total balance (eq. 7) and final concentrations (eq. 8). Hence, starting from a feasible point, this kind of moving allows exploring a reduced subset of the solution space especially dense in terms of feasibility.

The existence of local optima is a consequence of the limitations of the optimisation techniques and depends on the search procedure employed and the step-size adopted. A local optimum is the best solution among all possible surrounding solutions, attainable from a certain starting point once a search technique is given and a step-size defined. In this case, local optimum identification is clearly affordable since requires moving twice ($\pm dx$) the $\Omega = S{\cdot}(K+1)+K{\cdot}(K+1)$ decision variables.

Following the Mixed Stochastic Enumerative Search (MSES) scheme (Graells et al. 2001), the strategy presented is based on a greedy/stochastic search for fast down hill moving, coupled with the exhaustive search (Ω) of the neighbourhood at each step in order to determine if progress is no longer possible with the current search procedure (local optimum). Only when such a situation is detected, up hill moves are justified.

Figure 3 describes this search procedure. The loop for all s and k required may be simply sequential or following a random sequence each time.

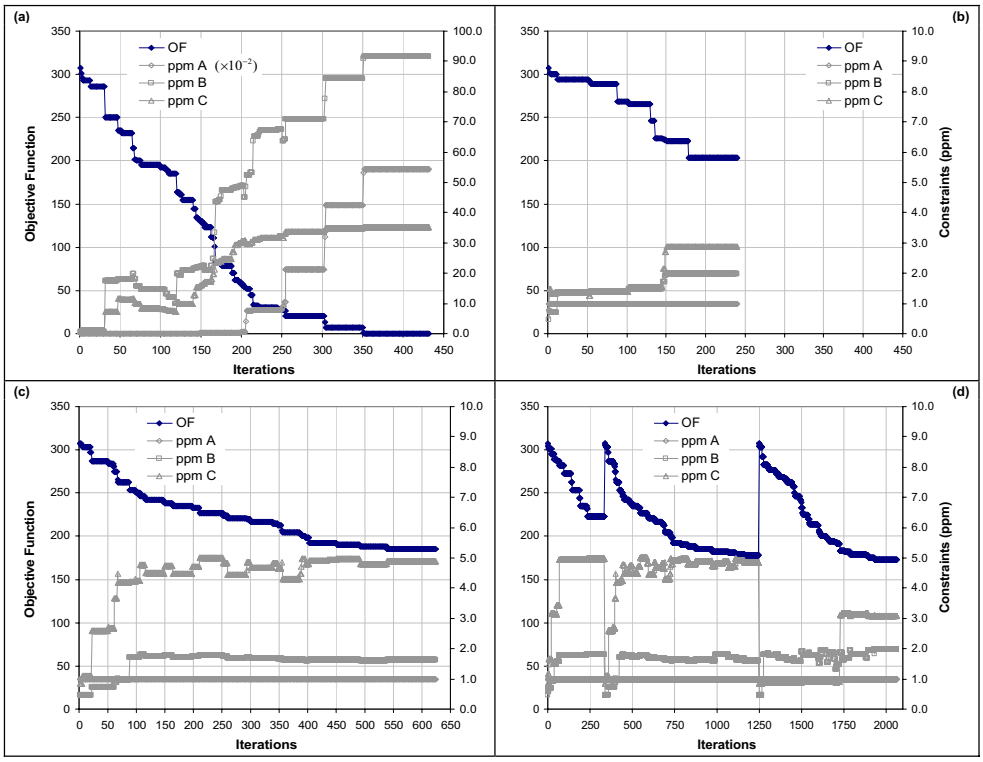

Figure 4. Different optimisation runs for the case study.

3.1 Case Study

A well-known example, revisited by Galan and Grossmann as Example 4, consists of three feeds, three contaminants (A,B,C) and three treatment lines.

Figure 4 shows the performance of the search procedure when addressing this problem. Figure 4a corresponds to the relaxed problem for which concentration limits are not considered: the search readily evolves to the obvious zero-cost solution consisting of all feed streams bypassing the treatments. The same dx=0.2 applied to the constrained case ($C_{A,max}$ 2 ppm; $C_{B,max}$ 2 ppm; $C_{C,max}$ 5 ppm) results in an early local optimum finding, while dx=0.1 produces a more extended search and a much better result (Fig. 4c). Figure 4d shows some loops for which, according to the algorithm (Fig. 3), each time a local optimum is detected step-size is changed ($dx/2$) or a different starting point is selected.
These runs confirm the difficulties multiple local optima and degeneracy pose to the solution of this kind of optimisation problems. Moreover, these local optima define new search subspaces and starting points that could be used by NLP solvers. The granularity and precision given by the step-size is a key point in the trade-off between the paired advantages and shortcomings of both algorithms. However, when dx is significantly reduced the neighbourhood search performed is equivalent to a gradient search and results are the same. Thus, the best local optimum reported could be this way attained.

4. Conclusions and further work

The design of distributed wastewater networks has been addressed at three levels. First, feasible starting points are provided by means of a simple rule; next a search procedure allows these solutions rapidly evolve to a feasible solution that is locally optimum in terms of the step adopted for the algorithm. Finally, at each decision step, simulation allows recycle solving and objective function evaluation, upon which a move is accepted or discarded. Different examples have illustrated the performance of this algorithm.
The main advantages of this approach are basically due to the robust procedure providing feasible complete solutions at any search level. Moreover, the simulation-based approach allows supporting any kind of objective function or removal rate function for further consideration of more realistic situations and processes. This could even contemplate black box models for the modelling of the treatment units. Further work may also include bounding the treatment flows (F_k) for modelling and optimising the operation of the network.

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