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A Multi-level Programming Optimization Approach to Enterprise-wide Supply Chain Planning

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Abstract

Current economy-dominant enterprises generally consist of a large number of entities which are heterogeneous in terms of characteristics and geographically distributed in their locations. Recently highlighted Supply Chain Management (SCM) is initiated after recognising overall management of such multi-faceted principles of individual organizations. Explicitly addressing varieties and perhaps conflicts between heterogeneous entities within decision-making frameworks is an important issue. This paper contributes by providing a multi-level programming framework in capturing complex supply chain decision making processes and proposing a novel solution methodology to compute the resulting multi-level programming problems. The proposed solution methodology transforms multi-level programming SCM problems into a series of bilevel programming problems

Keywords: supply chain, multi-level programming

1. Introduction

Companies always seek to increase their profits, which can be done by meeting customers' requirement. When a company failed to listen to the voices of customers, they are likely to lose their place to other competitors. The key issue is that customers are always changing and updated along technological advances and social trends. Companies therefore have been continuously seeking to ways of innovating faster than any other organizations. As one way of innovations, companies nowadays collaborate with others to increase their competitiveness. Some of such companies are developed

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into currently economy-dominant enterprises consisting of geographically distributed entities, for instance, raw material suppliers in middle Asia, manufacturing and processing facilities in Eastern Europe and Asia, distribution centres and markets in North America, etc. Each entity has its own decision making structure, or distinctive decision-making features.

The success of enterprises depends on the overall performance of entire enterprise-wide process networks. In order to increase overall performance, which is, in most cases, profits, multi-faceted principles of those heterogeneous organizations should be properly addressed in their overall management. Recently highlighted supply chain management (SCM) is initiated after recognising such industrial needs.

However, there still remain challenges of explicitly addressing such varieties and perhaps conflicts within decision-making frameworks. Because the problem involves a large number of entities, one of the key issues in supply chain planning is how to rigorously consider their multifaceted natures.

Real entities can not be fully controlled on a single perspective: each entity has its own business standards which can be described as its objective function as an academic term. It is probable that such objective functions between entities may be different, sometimes conflicting each other. The performance of entire supply chain depends on how to persuade each participating elements with reasonable information. In this regards, mathematical modelling of supply chains can be an important decision-making supporting tool for supply chain planning. This paper therefore contributes by formulating the above issues as a multi-level programming problem.

In the literature of developing such supply chain planning issues into mathematical models, Ryu et al. (2004) proposed a bilevel programming model in order to capture the dynamics of supply chain activities. The key point of their approach is that operations of production and distribution entities are formulated into a bilevel programming problem and the solutions of the problem are computed using parametric programming techniques. However, further research is requested in terms of expanding the bi-level approach into multi-level enough to handle multi-level cases. It is also desirable for the model formulation to incorporate discrete decisions in addition to continuous ones. There is little research to my knowledge of explicitly handling them.

Particularly, this paper focuses on planning issues which are concerned with incorporating various activities of enterprises, such as procurement of raw materials, a series of external or internal process operations, and distribution of final products to customers, etc. Each entity wants to focus on its core competencies. In focusing on its own purpose, different opinions may appear, which should be properly analyzed and explained to prevent unnecessary waste of resources. So far, it was seen that multiple level of new types of optimization problems should be considered as a practical decision-support tool. This paper focuses on multi-level programming problems. To author's understanding, there has been little research on this in spite of above explained practical advantages. The rest of their paper is as follows. In the next section, how supply chain planning problems can be transformed into multilevel programming problems will be described. A solution methodology to solve the problems will be afterwards presented with an illustration of a numerical example.

2. Multi-level Programming Modelling

Consider an enterprise consisting of K entities which have own decision-making procedure. The overall objective of the enterprise can be mathematically formulated as follows:

$$\begin{aligned} &\underset{X}{Max}\ \ F(X) \\ &s.t.\quad G(X) \le 0 \end{aligned} \tag{1}$$

where F(X) is often the profit function of the entire enterprise and X denote all participating decision variables. In real industries, it is quite difficult to compute the profit of enterprises by solving one single problem. Some variables can be determined based on the limited information before the profit is computed. Therefore ***(1)*** can be reformulated as follows:

$$\begin{aligned} &\underset{X_H}{Max}\ \ F_H(X_H, X_L) \\ &s.t.\quad G(X_H, X_L) \le 0 \\ &\underset{X_L}{Max}\ \ F_L(X_H, X_L) \\ &s.t.\quad G_2(X_1, X_2) \le 0 \end{aligned} \tag{2}$$

where F_H denote the original profit of ***(1)*** which lies in the higher level and F_L denote the lower level local decision making problem. X_H and X_L respectively denote variables computed at each problem. ***(2)*** is the bilevel programming problem. However, it should be highlighted that some variables previously computed may be determined after other variables were computed. Then the decision variables are determined hierarchically in the form of a multilevel programming problem. The general model can be formulated into the following multi-level programming problem:

$$\begin{aligned} &\underset{X_1, y_1}{Min}\quad F_1(X_1, X_2, X_3, \cdots, X_N, y_{1,} y_{2,} y_{3,} \cdots y_M) \\ &s.t.\quad G_1(X_1, X_2, X_3, \cdots, X_N, y_{1,} y_{2,} y_{3,} \cdots y_M) \le 0 \\ &\underset{X_2, y_2}{Min}\quad F_2(X_1, X_2, X_3, \cdots, X_N, y_{1,} y_{2,} y_{3,} \cdots y_M) \\ &s.t.\quad G_2(X_1, X_2, X_3, \cdots, X_N, y_{1,} y_{2,} y_{3,} \cdots y_M) \le 0 \\ &\qquad\qquad\qquad \vdots \\ &\underset{X_K, y_K}{Min}\quad F_K(X_1, X_2, X_3, \cdots, X_N, y_{1,} y_{2,} y_{3,} \cdots y_M) \\ &s.t.\quad G_K(X_1, X_2, X_3, \cdots, X_N, y_{1,} y_{2,} y_{3,} \cdots y_M) \le 0 \end{aligned} \tag{3}$$

3. A Solution Methodology

This section introduces a solution methodology to compute the solutions of above formulated multilevel programming problem. To the author's knowledge, there has been little research on solution methodology for the problem. Based on the parametric optimization, the following methodology is proposed.

[Step 1] Solve the most inner problem by transforming into a multi-parametric programming problem where variables of the remaining outer problems are treated as uncertain parameters.

[Step 2] Using the parametric solutions of the inner problem, formulate the next inner problem into a family of the paramtric programming problems. The solutions can be expressed as a family of explicit functions of the variables of the remaining outer problems and the uncertain parameters with the corresponding valid regions.

[Step 3] Continue to solve the next inner problem to the final outer problem. As a result, the final outer problem is formulated as a family of parametric optimization problems. By solving these parametric programming problems, solutions of the original problem are obtained.

4. An Illustrating Example

This section is devoted to illustrate the proposed solution methodology to solve numerical example. Consider the following three-level programming problem :

$$\begin{aligned} &\underset{x}{Min}\ 3x + y + 4z \\ &\underset{y}{Min}\ -x + 2y - 3z \\ s.t.\quad & x + y + z \geq 1 \\ &\underset{z}{Min}\ 2x - 4y + z \\ s.t.\quad & 2x + y + z \geq 5 \\ & -x - y + 3z \leq 3 \\ \\ & 0 \leq x, y, z \leq 10 \end{aligned} \tag{3}$$

Here, the variables *x,y,z* are bounded between 0 and 10., which is valid in the entire three levels. According to the proposed methodology, the most inner problem which is the third-level problem is separated by treating the two variables as parameters:

$$\begin{aligned} &\underset{z}{Min} \quad z+2x-4y \\ &s.t. \quad -z \le -5+2x+y \\ &\qquad\; 3z \le 3+x+y \end{aligned} \tag{4}$$

This is equivalent to the following formulation:

$$\begin{aligned} &\min_{z} z+[2 \; -4]\begin{bmatrix} x \\ y \end{bmatrix} \\ &s.t. \quad \begin{bmatrix} -1 \\ 3 \end{bmatrix} z \le \begin{bmatrix} -5 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \end{aligned} \tag{5}$$

The following indicate the solution of above parametric programming problem

$$z = \begin{cases} 0 & if \quad -2x-y \le -5 \\ -2x-y+5 & if \begin{cases} 2x+y \le 5 \\ -1.75x-y \le -3 \end{cases} \end{cases} \tag{6}$$

Using the above result in ***(6)***, the second-level problem corresponds to the following two LP problems, ***(7)*** and ***(8)*** respectively

$$\begin{aligned} &\underset{y}{Min} \quad -x+2y \\ &s.t. -y \le -1+x \\ &\quad -y \le -5+2x \end{aligned} \tag{7}$$

$$\begin{aligned} &\underset{y}{Min} \quad 5x+5y-15 \\ &s.t. -y \le -3+1.75x \\ &\qquad 0 \le x \le 4 \end{aligned} \tag{8}$$

The parametric solution of problem ***(7)*** and ***(8)*** are as follows:

$$y = \begin{cases} 0 & \text{if } 2.5 \le x \le 10 \\ -2x+5 & \text{if } 0 \le x \le 2.5 \end{cases} \tag{9}$$

$$y = \begin{cases} x & \text{if } 1.7429 \le x \le 4 \\ -1.75x+3 & \text{if } 0 \le x \le 1.7429 \end{cases} \tag{10}$$

$$\begin{aligned} &\underset{x}{Min} \quad 3x \\ &s.t. 2.5 \le x \le 10 \end{aligned} \tag{11}$$

$$\begin{aligned} &\underset{x}{Min} \quad x+5 \\ &s.t. 0 \le x \le 2.5 \end{aligned} \tag{12}$$

$$\begin{aligned} &\underset{x}{Min} \quad -8x+20 \\ &s.t. 1.71429 \le x \le 4 \end{aligned} \tag{13}$$

$$\begin{aligned} &\underset{x}{Min} \quad 0.25x+11 \\ &s.t. 0 \le x \le 1.71429 \end{aligned} \tag{14}$$

The solutions of above four problems are summarized in the following Table and the optimal solution corresponds to 5 when (*x,y,z*) is (0,5,0).

Table 1: Final Optimal Solution of Example

Variable	Subproblems			
	(11)	(***12***)	***(13)***	***(14)***
x	2.5	0	4	0
y	0	5	4	3
z	0	0	-7	2
F	7.5	5	-12	11
Optimality	suboptimal	optimal	infeasible	suboptimal

5. Conclusion

In order to survive in continuously changing economic environments, companies should increase their efficiencies across their entire activities. Supply Chain Management has drawn attention based on such an imperative necessity. However, the actual methodology to incorporate various activities within the rigorous decision supporting framework is still a challenge requiring further research. This paper provided a multilevel programming framework as an alternative strategy. In addition to the proposed modelling framework, more complicated multi-layer supply chain decision making issues can be addressed within the framework. Its solution methodology is unique in handling multilevel programming problems. The proposed solution methodology may be introduced to compute the solutions of various complex industrial applications as well. A numerical example is presented to illustrate the potential of the proposed solution methodology and detailed supply chain planning result can be found at the unabridged paper by the author.

References

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