European Symposium on Computer Aided Process Engineering – 15
L. Puigjaner and A. Espuña (Editors)

The Escape Time of Small Heavy Particles from a 3D Vortex

U. Sánchez[a] and M. J. Moreno-López[b]

[a]Fluid Mechanics Division, University of Huelva
Campus La Rábida, Ctra. Huelva-Palos s/n, 21819-Palos de la Frontera (Huelva),
urbano.sanchez@dcaf.uhu.es Tel +34959217565 Fax +34959217560

[b]Chemical Engineering Department, University of Huelva
Campus El Carmen, Avda. Fuerzas Armadas s/n, 21071-Huelva,
moreno@uhu.es Tel +34959218208 Fax +34959219983

Abstract

The time that small heavy particles remain inside a 3D vortex until they escape is an important feature in many practical situations. Previous works focused in the case of a 2D vortex. In this paper we analyse the dynamics of small heavy particles initially situated inside a three-dimensional vortex and, as a result, we settle down a formulation of the time that they need to escape from the vortex. Afterwards, by means of numerical simulation we verify the validity of the obtained formulation; a very good agreement is found.

Keywords: modelling and simulation, CFD, environment**.**

1. Introduction

Some authors, as Yule (1984), Fung (1997), Dávila and Hunt (2001), have demonstrated that the most important flow structures in relation with suspended particles are vortices. The knowledge of the time that heavy particles need to escape from a 3D vortex is important in many fields of Science and Technology; the escape time is a suitable parameter to determine the validity of a PIV system to visualize vortical flows. It is also very important to estimate the residence times of dust particles from wind erosion (Nielsen, 1984) or manmade pollutants (Tooby et al., 1977). In the atmosphere, the permanence times of various aerosols determine how long the aerosols remain suspended in air. In the last years, the trapping and settling of particles by vortical flows has been an important field of study. Stommel (1949) showed that particles without inertia in 2D periodic flows may remain suspended and describe closed orbits. Maxey & Corrsin (1986) proved that aerosol particles in randomly oriented cellular flows can be trapped only if the effect of the particle inertia is neglected. However, the work of McLaughlin (1988) demonstrates the possibility of suspension of heavy inertial particles in two-dimensional periodic flows. Dávila and Hunt (2001) established the dimensionless parameters that govern the movement of the particles and they studied the dynamics of heavy particles near a horizontal vortex and

in turbulence under gravity. A common characteristic of some of the studies above mentioned is the fact of the particle ejection from the vortex centre.
One of the most important papers relative to the trapping of particles in a line vortex is that of Fung (2000). It can be considered as the precedent of this work. Fung studied the residence time of inertial particles in the restricted case of a two-dimensional vortex. He found that the residence time is a function of the particle inertia, its fall velocity and the vortex's Reynolds number. Nevertheless, he does not develop a formulation for the residence time, and as it has been already mentioned, he analysed only the two-dimensional problem.
Here we generalize the results mentioned above to the case of a three-dimensional vortex. We develop a formulation for the escape time of heavy particles from the 3D vortex and we verify its validity by means of numerical simulation.

2. Flow Field

In order to show the main features of the dynamics of small heavy particles in vortical flows, we have chosen the Rankine vortex model because it simulates with very suitable approximation the flow field of a vortex. As it is well known, there are some models of vortices (Burgers-Rott, Sullivan, Lamb-Oseen, Kaufmann...), but the Rankine vortex has a relatively easy formulation and it is suitable to our objectives. It would be an unnecessary complication to choose another vortex model, with radial or longitudinal velocities.
The normalized 3-D velocity field in cartesian co-ordinates relatives to the axis of the vortex is given by

$$u_x^* = \frac{-2y^*\Gamma^*}{(R_v^{*2} + x^{*2} + y^{*2})} \quad , \quad u_y^* = \frac{2x^*\Gamma^*}{(R_v^{*2} + x^{*2} + y^{*2})} \quad , \quad u_z^* = 0 \qquad (1)$$

where Γ^* is the circulation of the vortex and R_v^* the radius of the vortex so the maximum value of the velocity U^* is given on $r^*=R_v^*=\Gamma^*/U^*$. The inclination of the vortex is defined by the azimuthal angle θ, i.e., the angle between the axis of the vortex and the fixed Z axis, which is anti-parallel to the direction of gravity.

3. Equation for Particle Motion

Assuming that the particles have a spherical shape, that the particle-particle interactions are negligible, this is, dilute regime assumption and that the presence of particles in the fluid does not affect the structure of the flow, the only forces considered are the forces from the fluid-particle interactions and the gravity effect (or buoyancy force). Therefore, the dynamics of the particle is governed by the Maxey-Riley Equation (1983), the general form of which is presented in this section:
PARTICLE ACCELERATION = BUOYANCY FORCE + PRESSURE GRADIENT FORCE +VIRTUAL MASS + STOKES DRAG + BASSET HISTORY TERM
The equation reads:

$$m_p^* \frac{d\vec{v}^*}{dt^*} = (m_p^* - m_F^*)\vec{g}^* + m_F^* \frac{D\vec{u}^*}{Dt^*} - \frac{1}{2} m_F^* \frac{d}{dt^*}\left[\vec{v}^* - \vec{u}^* - \frac{1}{10} R_p^{*2} \nabla^2 \vec{u}^*\right]$$
$$- 6\pi R_p^* \mu^* \vec{Y}^* - 6\pi R_p^{*2} \mu^* \int_0^{t^*} d\tau^* \frac{d\vec{Y}^* / d\tau^*}{\sqrt{\pi \nu^* (t^* - \tau^*)}} \tag{2}$$

with $\vec{Y}^* = \vec{v}^* - \vec{u}^* - 1/6 R_p^{*2} \nabla^2 \vec{u}^*$ and $\vec{v}^*$ particle velocity, $\vec{u}^*$ local fluid velocity, t^* time, m_p^* particle mass, m_F^* fluid mass, $\vec{g}^*$ gravitational acceleration, R_p^* particle radius, μ^* fluid viscosity.
Assuming the particle diameter is small compared to the flow scale, the Faxen correction terms can be neglected (indeed, a small particle diameter implies that the velocity curvature has negligible effect on the calculation of the drag force on the sphere at low particle Reynolds number, which is what Faxen correction terms stand for).
Equation (2) is valid for low particle Reynolds number and it is rather complicate. Given that this paper focuses on the case of heavy particles, the Basset history term will be neglected. Also, for heavy particles with $m_p^* >> m_F^*$ and $\rho_p^* >> \rho_F^*$ (with ρ_p^* particle density and ρ_F^* fluid density) the equation (2) reads:

$$m_p^* \frac{d\vec{v}^*}{dt^*} = 6\pi R_p^* \mu^* (\vec{u}^* - \vec{v}^*) + m_p^* \vec{g}^* \tag{3}$$

The determination of the particle motion from this equation is a complex problem due to the nonlinearity introduced by the velocity field and we have to obtain computational solutions. To integrate equation (3) it is necessary to write it in dimensionless form introducing the suitable dimensionless parameters. If particles are sufficiently small (i.e. if the Reynolds number of the particles, based on their diameter and their terminal velocity, is small), only two dimensionless parameters are enough to describe the motion of small spherical particles: a gravitational and an inertial parameter. The gravitational parameter is the ratio of the terminal velocity of the particles in still flow V_t^* to the characteristic velocity of the fluid U^*: $V_t = V_t^*/U^* = m_p^* g^* / 6\pi R_p^* \mu^*$. For the inertial parameter, Dávila and Hunt (2001) showed that the average properties of small particles trapped in regions of concentrated vorticity (in which case the characteristic velocity of the particles is that of the fluid) can be characterized by the Stokes number $St = \tau_p^*/\tau_r^*$, where $\tau_p^* = m_p^*/6\pi R_p^* \mu^* = V_t^*/g^* = (\beta - 1) d_p^{*2}/(18\nu^*)$ is the viscous response time of the particles and $\tau_r^* = R_v^*/U^*$ is the characteristic residence time of the fluid flow. Here β is the ratio between the particle density and the fluid density, d_p^* is the particle diameter and ν^* is the kinematic viscosity of the fluid. This is the case of turbulent flows, if the time that the particles need to escape from regions of intense vorticity is much greater than the life time of those vortex structures. The Stokes number determines the importance of the particle inertia such that when St is very small, inertial effects are small. However, if the particles are falling from above a vortex with circulation Γ^*, their inertia is characterised by the particle Froude number

$F_p=\tau_p{*}^3 g{*}^2/\Gamma{*}$. Considering $\vec{x}=\vec{x}^*/R_v^*$, $\vec{v}=\vec{v}^*/U^*$, $t=t^*U^*/R_v^*$ the scaled nondimensional equations of motion are:

$$\frac{d\vec{x}(t)}{dt}=\vec{v}(t) \tag{4a}$$

$$\frac{d\vec{v}(t)}{dt}=(\frac{1}{St})\left[\vec{u}(\vec{x},t)-\vec{v}(t)+\vec{V}_t\right] \tag{4b}$$

The trajectories of particle are obtained integrating equations (4) using a fourth order variable step size Runge-Kutta method. The initial condition at the position $\vec{x}_o$ is that the particles have the velocity of the fluid at $\vec{x}_o$.

4. Results

The study of the motion of particles around vortices with axial symmetry is clearly simplified projecting the trajectories over a plane perpendicular to the axis of the vortex and considering the component of gravity which is not parallel to the axis of the vortex, $V_t^{*'}=V_t^* sin\theta$. In the following, we will refer to that projection.
In the limit of non-inertial particles, the mentioned projection of the trajectories of some particles are closed curves, as it can be seen in figure 1. In the case of inertial particles, figure 2 shows that they are never trapped inside the vortex (Maxey and Corrsin, 1986), they gradually drift outwards due to the inertial centrifugal force (Fung, 2000).
The trajectories of the particles are characterised by the position of the equilibrium points of the mentioned projection, where $d\vec{x}^*/dt^*=d\vec{v}^*/dt^*=0$. There are an unstable focus (X^*_{E1}) near the centre of the vortex and a saddle point (X^*_{E2}):

$$\vec{x}^*_{E1,2}=\frac{\Omega^*\pm\sqrt{\Omega^{*2}-\dfrac{4V_t^{*2}sen^2\theta}{R_v^{*2}}}}{\dfrac{2V_t^* sen\theta}{R_v^{*2}}} \tag{5}$$

both separated by a distance

$$\left|\vec{x}^*_{E2}-\vec{x}^*_{E1}\right|=\frac{2\Gamma^*\sqrt{1-\left(\dfrac{V_t^{*'}}{U^*}\right)^2}}{V_t^{*'}} \tag{6}$$

When $V_t^*\to 0$, $\vec{x}^*_{E1}\to(0,0)$, $\vec{x}^*_{E2}\to\left(R_v^{*2}\Omega^*/V_t^* sen\theta,0\right)$. The limit trajectories starting at the saddle point enclose a region that can not be reached by particles coming from above the vortex.
For the objective to determine the escape time of particles from the vortex, it is necessary to give a criterion to consider that a particle has escaped. In this way, we

consider that the particle has escaped when its distance to the focus point is 1.5 times the distance between the equilibrium points. Figure 2 shows that this criterion is reasonable.

From (6) it follows that the characteristic length scale of this problem is $\Gamma^*/V_t'^*$, the characteristic time scale for the particles is $\Gamma^*/V_t'^{*2}$ and $|u|\sim V_t'^*$. This scaling, used by Dávila and Hunt (2001), means that the inertia of the particle is characterised by the particle Froude number F_p. From (3):

$$\vec{v}^* \approx \vec{u}^* + \vec{V}_t^* + \tau_p^* \frac{d\vec{u}^*}{dt^*} \tag{7}$$

If the particles has not inertia $\tau_p^*=0$, following closed trajectories, but due to the inertia, they escape with a velocity $\tau_p^* \, d\vec{u}^* / dt^*$. This velocity is a space divide by a escape time and considering the scales mentioned above it follows

$$\frac{space}{T_e^*} \approx \tau_p^* \frac{d\vec{u}^*}{dt^*} \Rightarrow \frac{\Gamma^* / V_t'^*}{T_e^*} \approx \tau_p^* \frac{V_t'^*}{\Gamma^* / V_t'^{*2}} \tag{8}$$

From (8),

$$T_e^* = K_e \frac{\Gamma^{*2}}{\tau_p^* V_t'^{*4}} \tag{9}$$

where K_e is a constant independent of the parameters of the problem. K_e is a dimensionless escape time.

In the following figures we verify the validity of equation (9) by means of numerical simulation. For that propose, we have taken the average values for a mesh of 500 uniformly distributed particles situated inside the trapping region. In figure 3 it is represented $K_e = T_e^* \tau_p^* V_t'^{*4} / \Gamma^{*2}$ obtained from (9) versus F_p for various values of the Stokes number. The average escape time is obtained with numerical simulation. From this figure it is clear that $<T_e^*>$ follows equation (9) for sufficiently small values of F_p, in agreement with the hypotheses used to obtain that equation. We have obtained a value of $K_e \to 0.036$ when $F_p \to 0$. For F_p of order unity equation (9) is not longer valid because the size of regions where the effects of inertia can not be considered small (close to equilibrium points) is of the order of F_p times the size of the trapping region (Dávila and Hunt, 2001). It is important to notice that in the case of heavy particles, the terminal velocity V_t^* is always much smaller that the characteristic velocity of the flow; then, the trapping region is much larger that the vortex core and the dependence on the Stokes number disappears. For that reason K_e is only a function of F_p and, as it can be seen in figure 3, all data points collapse in a single curve.

The interest of the obtained scaling of the escape time can be also shown in terms of physical variables. In figure 4 we plot the average escape time versus the diameter of the particles for some values of the circulation of the vortex that could be found in a typical turbulent flow. The solid lines represent the values obtained with equation (9) and K_e=0.036. A very good agreement for sufficiently small particles can be observed.

5. Figures

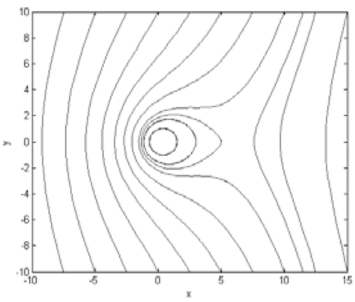

Figure 1. Trajectories of non- inertial particles

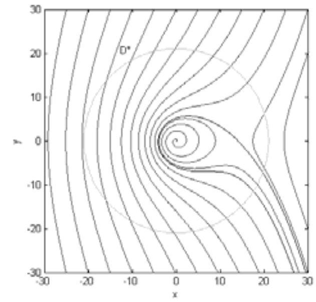

Figure 2. Trajectories of inertial particles and escape distance

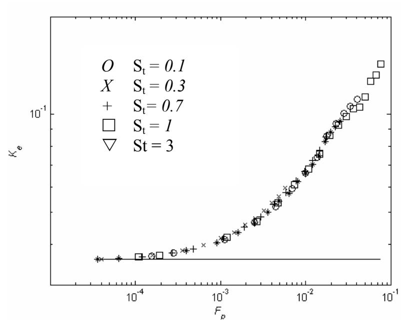

Figure 3. Dimesionless escape time versus the particle Froude number

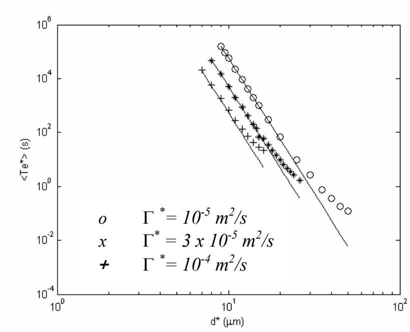

Figure 4. Average escape time versus particle diameter

References

Dávila, J. and J. C. R. Hunt, 2001, Settling of small particles near vortices and in turbulence, J. Fluid. Mech., 440, 117-145.

Fung, J. C. H., 1997, Gravitational settling of small spherical particles in unsteady cellular flow fields, J. Aerosol. Sci., 28(5), 753-787.

Fung, J. C. H., 2000, The residence time of inertial particles in a vortex, J. Geophysical Res., 105 (65), 14261-14272.

Maxey, M. R. and S. Corrsin, 1986, Gravitational settling of aerosol particles in randomly oriented cellular flow field, J. Atmos. Sci., 43, 1112-1134.

Maxey, M. R. and J. J. Riley, 1983, Equation of motion for a small rigid sphere in a non-uniform flow, Phys. Fluid, 26, 883-889.

McLaughlin, J. B., 1988, Particle size effects on Lagrangian turbulence , Phys. Fluids, 31, 2544.

Nielsen, P., 1984, On the motion of suspended sand particles, J. Geophys. Res., 89, 616-626.

Stommel, H., 1949, Trajectories of small bodies sinking slowly through convection cells, J. Marine Res., 8, 24-29.

Tooby, P. F., G. L. Wick, and J. D. Isaacs, 1977, The motion of small sphere in a rotating velocity field: a possible mechanism for suspending particles in turbulence, J. Geophysical Res., 82, nº 15, 2096-2100.

Yule, A. J., 1980, Investigations of eddy coherence injet flows. In *The Role of Coherent Structures in Modelling Turbulence and Mixin,*. (ed. J. Jiménez),188-207, Springer.