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Entire Supply Chain Optimization in Terms of Hybrid in Approach

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Abstract
As an essential part of decision-making in recent business processes, this study has called attention on entire network associated with supply-chain management. To optimize such large-scale and complex systems, we proposed a hybrid method compose of meta-heuristic method and mathematical programming after decomposing the problem into two sub-problems in a hierarchical manner. Then we provide a unique method to adjust and unify each result of sub-problems solved individually. Through numerical experiments, we verified the performance of the proposed method.

Keywords: supply chain optimization, hybrid method, multi-level model, capacitated hub problem, meta-heuristics

1. Introduction

Recently, industries have been paying keen interests on supply chain management (SCM) that might be viewed as a re-engineering method managing life cycle activities of business process. It tries to deliver value-added products and services to customers in just-in-time and agile manners. As an essential part of SCM, many researchers have engaged in logistic optimisation known as *p*-hub center problem, *p*-median problem, etc. from a variety of approaches (for examples, Campbell, 1994; O'Kelly and Miller, 1994; Drezner & Hamacher, 2002; Ebery et al., 2000; Lee et al., 2001) as well as classical location problems (Shimizu, 1999). Taking a *p*-median problem of distribution center (DC) problems in SCM, we also developed previously a method to optimize the total transport cost with respect to location of DCs and route from plants to customers via DCs (Shimizu & Wada, 2004).

However, since none of those have called attention on the entire system composed both of the DC problem and collection center (CC) problem, we can concern with decision-making on economical partnership in SCM more relevantly by optimising the entire system. To deal with such large-scale and complex problems practically, in this study, we decompose the problem into two sub-problems in a hierarchical manner, and apply a hybrid method employing meta-heuristic method and mathematical programming. Through numerical experiments, we will examine validity of the proposed method.

2. Problem Statement

Taking a supply chain network (SCN) composed of suppliers, CCs, plants, DCs, and customers as shown in Figure 1, we have formulated the problem under the conditions

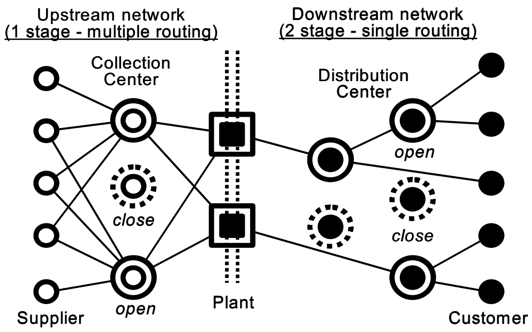

Figure 1. Supply chain network concerned here.

that capacity of facility is constrained, and demand, supply and per unit transport cost are given a priori. Moreover, multiple and single-stage routing is allowed for CC problem while single and two-stage routing for DC problem. After all, it refers to a non-linear integer programming deciding simultaneously the location of hub centers and routes to meet the demands of all SCN members while minimizing total cost.

$$Min \quad \sum_{i\in I}\sum_{j\in J} D_i C1_{ij} r_{ij} + \sum_{j\in J}\sum_{j'\in J}\left(\sum_{i\in I} D_i r_{ij}\right) C2_{jj'} s_{jj'} + \sum_{j'\in J}\sum_{k\in K}\left(\sum_{j\in J}\left(\sum_{i\in I} D_i r_{ij}\right) s_{jj'}\right) C3_{j'k} t_{j'k}$$
$$+ \sum_{k\in K}\sum_{l\in L} C4_{kl} u_{kl} + \sum_{l\in L}\sum_{m\in M} C5_{lm} v_{lm} + \sum_{j\in J} F1_j x_j + \sum_{l\in L} F2_l y_l \qquad subject \quad to$$

$$\sum_{j\in J} r_{ij} = 1 \qquad \forall i \in I \tag{1}$$

$$\sum_{i\in I} D_i r_{ij} \le P_j x_j \qquad \forall j \in J \tag{2}$$

$$\sum_{j'\in J} s_{jj'} = x_j \qquad \forall j \in J \tag{3}$$

$$\sum_{j\in J}\left(\sum_{i\in I} D_i r_{ij}\right) s_{jj'} \le P_{j'} s_{j'j'} x_{j'} \quad \forall j' \in J \tag{4}$$

$$\sum_{k\in K} t_{j'k'} = s_{j'j'} \qquad \forall j' \in J \tag{5}$$

$$\sum_{j'\in J}\left(\sum_{j\in J}\left(\sum_{i\in I} D_i r_{ij}\right) s_{jj'}\right) t_{j'k} \le Q_k \qquad \forall k \in K \tag{6}$$

$$\sum_{l\in L} u_{kl} = \sum_{j'\in J}\left(\sum_{j\in J}\left(\sum_{i\in I} D_i r_{ij}\right) s_{jj'}\right) t_{j'k} \qquad \forall k \in K \tag{7}$$

$$\sum_{k\in K} u_{kl} \le S_l y_l \quad \forall l \in L \tag{8}$$

$$\sum_{k\in K} u_{kl} = \sum_{m\in M} v_{lm} \quad \forall l \in L \tag{9}$$

$$\sum_{l\in L} v_{lm} \le T_m \quad \forall m \in M \tag{10}$$

$$r, s, t \in \{0,1\},\ x, y \in \{0,1\}, u, v \in \text{Real number}$$

where binary variables x_i and y_i take 1 if each center i is open, and r_{ij}, s_{ij}, t_{ij} become 1 if there exist routes between customer i and DC j, DC i and DC j, and DC i and plant j, respectively, and 0 otherwise in all cases. On the other hand, u_{ij} and v_{ij} denote shipping amounts from CC j to plant i and from supplier j to CC i respectively. Moreover, D_i is demand of customer i, and P_i , Q_i, S_i and T_i represent capacities of DC, plant, CC, and supplier, respectively.

On the other hand, terms from 1_{st} through 5_{th} of the objective function denote transport costs, and the 6_{th} and 7_{th} terms fixed charge costs of DC and CC respectively. Moreover, each constraint means such that : Eqs.(1), (3), and (5) require each customer, DC and plant must select only one location respectively in the downstream network; Eqs.(2), (4), (6), (8), and (10) stand for the capacity constraints on the 1_{st} and the 2_{nd} stage DCs, plant, CC, and supplier respectively; Eqs.(7) and (9) represent balances between input and output of plant and CC respectively. Owing to the product terms among the binary variables in objective function and Eqs.(4), (6) and (7), this refers to a mixed integer non-linear programming problem (MINLP).

3. Hierarchical Procedure for Solution

3.1 Decomposition into sub-models

Since solution of MINLP belongs to a NP hard class, developing a practical solution method is more desirable than aiming at rigid optimality. With this point of view, we decompose the original SCN into two sub-networks originating from the plants, i.e., upstream chain, and downstream chain. The former solves a problem how to supply raw materials from suppliers to plants via CCs while the later how to distribute the products from plants to customers via DCs.

Then, to obtain a consequent result of the entire supply chain from each solved individually, we have given an effective way to combine them consistently by adjusting a coupling constraint. That is, instead of using Eq.(7) directly as the coupling constraint, we transform it into a suitable condition for operating auction mechanism based on an imaginary cost that makes trade off gains between the sub-networks. For this purpose, let us assume first that the optimal cost associated with the procurement in the upstream chain is C_{proc}*, i.e.

$$\sum_{k\in K}\sum_{l\in L} C4_{kl}\, u_{kl} + \sum_{l\in L}\sum_{m\in M} C5_{lm}\, v_{lm} + \sum_{l\in L} F2_l\, y_l = C^*_{proc} \tag{11}$$

Then, dividing C_{proc}* into each plant according to the amount of production, i.e., C_{proc}* $=\sum_k V_k$, we can view V_k as a certain shipping cost from each plant. Now, writing such unit procurement cost by ρ_k, we obtain the equation like

$$\rho_k \sum_{j'\in J}\left(\sum_{j\in J}\left(\sum_{i\in I} D_i r_{ij}\right) s_{jj'}\right) t_{j'k} = V_k \qquad \forall k \in K \tag{12}$$

Using this as a coupling condition instead of Eq.(7), we can decompose the entire model into each sub-model as follows.

<u>Downstream network model</u>

$$Min\ \sum_{i\in I}\sum_{j\in J} D_i C1_{ij} r_{ij} + \sum_{j\in J}\sum_{j'\in J}\left(\sum_{i\in I} D_i r_{ij}\right) C2_{jj'} s_{jj'} + \sum_{j'\in J}\sum_{k\in K}\left(\sum_{j\in J}\left(\sum_{i\in I} D_i r_{ij}\right) s_{jj'}\right) C3_{j'k} t_{j'k} + \sum_{j\in J} F1 x_j$$

subject to Eqs.(1)－(6) and Eq.(12)

Upstream network model

$$Min \quad \sum_{k\in K}\sum_{l\in L} C4_{kl}u_{kl} + \sum_{l\in L}\sum_{m\in M} C5_{lm}v_{lm} + \sum_{l\in L} F2_l y_l$$ subject to Eqs.(8) —(10) and Eq.(13)

$$R_k = \sum_{l\in L} u_{kl} = \sum_{j'\in J}\left(\sum_{j\in J}\left(\sum_{i\in I} D_i r_{ij}^*\right)s_{jj'}^*\right)t_{j'k}^* \qquad \forall k \in K \tag{13}$$

where asterisks means the optimal value for the downstream problem.

3.2 Coordination between sub-models

It is apparent that as long as the optimal values of the coupling quantities, i. e., V_k and R_k are known a priori, we can derive the consistent solution straightforwardly by solving each sub-problem individually. However, since this is not true presently, we need to take an adjusting process as follows.

(1) For the tentative V_k (initially not set forth), solve the downstream problem.
(2) After calculating R_k based on the above result, solve the upstream problem.
(3) Re-evaluate V_k based on the above upstream optimization.
(4) Repeat until no more change of V_k has been observed.

Increasing quality of information on V_k and/or R_k along with iteration may guarantee qualitatively the convergence of such coordination. In addition, we can rewrite the objective function of the downstream problem by relaxing the coupling constraint in terms of Lagrange multiplier as follows.

$$\sum_{i\in I}\sum_{j\in J} D_i C1_{ij} r_{ij} + \sum_{j\in J}\sum_{j'\in J}\left(\sum_{i\in I} D_i r_{ij}\right) C2_{jj'} s_{jj'} + \sum_{j\in J} F1_j x_j - \sum_{k\in K}\lambda_k V_k + \sum_{j'\in J}\sum_{k\in K}\left(\sum_{j\in J}\left(\sum_{i\in I} D_i r_{ij}\right)s_{jj'}\right)(C3_{j'k} + \lambda_k \rho_k)t_{j'k} \tag{14}$$

Then we notice the last term implies that re-costing the transport cost $C3_{jk}$ will conveniently play this role of coordination. In practice, we can perform it as $C3_{jk}$:=$C3_{jk}$ +Constant× ρ_k. After all, so far statements claim the coordination will be viewed as an auction on the transportation cost so that the procurement becomes most suitable for the entire chain.

3.3 Procedure for coordinated solution

To reduce load of computation, we break down each sub-problem further into two levels, i.e., the upper level problem for locating and the lower for routing. Taking such hierarchical approach, we have the following advantages.

1. In the upper level problem, we can shrink searching space dramatically by confining the search just to the location.
2. We can transform the lower level problem into what is solvable very effectively by using an appropriate mathematical programming.

As a drawback, we need to solve repeatedly one of the two subject to the foregoing result of the other in turn. However, the previous study revealed computational load of such adjustment was moderate and effective. Consequently, we solve the downstream problem simply following the previous method that applies a meta-heuristic method like complete local search with memory (CLM; Ghosh & Sierksma, 2002) for the upper level and a mathematical programming like Dijkstra method for the lower. In contrast, we have developed newly a similar hybrid method that is peculiar to the upstream problem where we use linear programming (LP) to solve the lower level problem. Eventually, the proposed approach is summarized as follows.

Step 1. Set all parameters at their initial values.

Step 2. Under the prescribed parameters, solve the downstream problem using the hybrid meta-heuristic method.

- Provide the initial location of DCs.
- Decide the routes covering the plants, DCs, and customers by the revised Dijkstra method.
- Revise the DCs' location repeatedly until the convergence of CLM search will be attained.

Step 3. Compute the necessary amount at the plant based on the above result.

Step 4. Solve the upstream problem using another hybrid meta-heuristic method.

- Provide the initial location of CCs.
- Decide the routes covering the suppliers, CCs and plants by LP.
- Revise the CCs location according to CLM search.

Step 5. Check the convergence criterion. If it is satisfied, stop.

Step 6. Re-cost the transport costs between plants and DCs, and go back to Step 2.

4. Numerical Experiments

We evaluated the performance of the proposed method by solving a variety of benchmark problems whose features are summarized in Table 1. They are produced by generating the nodes whose appearance rates become approximately 3:4:1:6:8 among suppliers, CCs plants, DCs, and customers. Then distributing them randomly over the grid point drawn on the plain, we give the value related to Euclid distance between each node as the transport cost per unit demand. On the other hand, demand and capacity are given randomly between certain intervals.

The results summarized in Table 2 reveal that the proposed method can improve the initial feasible solution and expansion of computation load is considerably slow along with the increase in problem size. On the other hand, due to the complicated non-linear structure not reconfigurable by CPLEX, it could not solve the present model. (We showed outstanding advantage of our hybrid approach over CPLEX for the downstream problems (Shimizu & Wada, 2004). However, due to our unsophisticated implementation of LP, we cannot always outperform it for the upstream problems.)

In Figure 2, we present the convergence features including those of downstream and

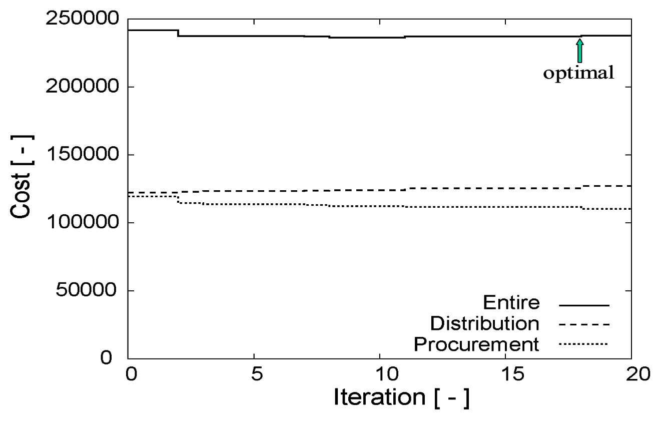

Figure 2.Convergence feature along iteration. (*F1-10*)

upstream problems. There, we observe that the proposed coordination method works adequately to reduce the total cost by making the gain at the procurement chain overwhelm the loss at the distribution chain, and only small number of iterations is required by convergence (See also Table 2). Moreover, relying on the good performance of the hybrid method compared with CPLEX for each decomposed sub-problem, we can claim the converged solution might attain almost at the global optimum by virtue of the generic nature of meta-heuristic algorithm employed presently.

Table 1. Feature of bench-mark problems.

Data set ID	Number of					Number of {(binary, real), constraint}
	Customer	DC	Plant	CC	Supplier	
F1-04	32	24	4	16	12	{(1480, 256), 180}
F1-06	48	36	6	24	18	{(3300, 576), 270}
F1-08	64	48	8	32	24	{(5840, 1024), 360}
F1-10	80	60	10	40	30	{(9100, 1600), 450}

Table 2. Results of benchmark problems.

Data set ID	Objective value		Improved rate [%]	Computation time [sec]	Number of coordination
	Initial	Converged			
F1-04	129031.8	127555.4	1.1	98	3
F1-06	175894.5	167199.7	5.2	859	5
F1-08	192049.0	186287.5	3.1	3680	12
F1-10	241648.4	236349.8	2.2	6842	18

PC={CPU:Athlon 1.0 GHz, Memory:256MB}

5. Conclusion

This study has extended the foregoing distribution problem to the entire supply chain environment while expecting the advantage brought about by expanding the system boundary. To realize a practical resolution method, we have given an idea that tries to decompose first the entire supply chain into the sub-chains, and then combines consistently each result solved individually. Applying two kinds of hybrid methods of meta-heuristic and mathematical programming, we have given an effective algorithm depending on the peculiar feature of each sub-problem and examined its effectiveness through numerical experiments.

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