

Optimization Under Fuzzy If-Then Rules Using Stochastic Algorithms

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Abstract

A new approach to optimization of processes described by fuzzy rules, in which the functional relationship between the decision variables and the objective function is not completely known, is introduced in this paper. It is based on stochastic algorithms and allows to determine optimal values of state variables and to optimize fuzzy rules (parameters of membership functions). Stochastic algorithms have many advantages like their robustness and, in most cases, global convergence properties. Here, a new algorithm (FICRS) was developed, being applied to the solution of a case study taken from the recent literature.

Keywords: optimization, fuzzy logic, stochastic algorithms

1. Introduction

Methods and techniques of optimization have been successfully used in various fields and related to technical systems of relatively well-defined structure and behavior (Reklaitis et al., 1983). Unfortunately, many technical processes, like bioprocesses, work with uncertainties and non-quantified factors (such as smell, cloudiness, etc.). In practice, for good performance of these systems subjective estimations of the experienced technologist are very important. For these reasons, this kind of processes can be described by linguistic variables and fuzzy rules, being necessary the design of efficient optimization algorithms to obtain an optimal solution.

The aim of this paper is to introduce a new approach to optimization of processes described by fuzzy rules. Suppose we are given a mathematical programming problem in which the functional relationship between the decision variables and the objective function is not completely known. The knowledge-base part consists of a block of fuzzy if-then rules, where the antecedent part of the rules contains some linguistic values of the decision variables, and the consequence part consist of a linguistic value of the objective function. There exist a number of generalizations of the optimization problem when fuzzy elements exist (Luhandjula, 1988; Qian et al., 1992). However, most of them are practically non-applicable to real problems, because of computational efforts. In last years alternative strategies like genetic algorithms (Herrera et al., 1994; Angelov and Guthke, 1997) were proposed to solve this kind of optimization problems. More

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recently, Carlsson and Fullér (2001) suggested the use of Tsukamoto's fuzzy reasoning method to determine the crisp functional relationship between the objective function and the decision variables, and solved the resulting non-linear programming problem (NLP) to find an optimal solution.

Stochastic algorithms are robust alternatives for the global optimization of processes, especially when the process models have discontinuities or multimodal performance indexes (Banga and Seider, 1995; Carrasco and Banga, 1997). In this paper we propose a new adaptive stochastic algorithm, FICRS (Fuzzy Integrated Controlled Random Search), as a reliable and efficient alternative for the global optimization of processes described by fuzzy if-then rules. To best achieve this goal the paper is structured as follows: Section 2 describes the fuzzy optimization problems. Section 3 introduces the foundations of stochastic methods and their application to the solution of fuzzy optimization problems. Section 4 deals with the use of stochastic methods to solve one concrete case taken from the recent literature.

2. The Fuzzy Optimization Problem

Fuzzy optimization problems can be stated and solved in many different ways. Usually, the authors consider optimization problems of the form (Carlsson and Fullér, 2001):

$$\min f(\underline{x}) \text{ subject to } \underline{x} \in X \quad (1)$$

where f or/and X are defined by fuzzy terms. Then they are searching for a crisp $\underline{x}^*$ which (in certain) sense minimizes f under the fuzzy constraints X . For example, fuzzy linear programming (FLP) problems are stated as (Rommelfanger, 1996):

$$\min f(\underline{x}) = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + \dots + \tilde{c}_n x_n \quad (2)$$

subject to:

$$\tilde{a}_{i1} x_1 + \tilde{a}_{i2} x_2 + \dots + \tilde{a}_{in} x_n \leq \tilde{b}_i, \quad i = 1, \dots, m \quad (3)$$

where $\underline{x} \in \mathbb{R}^n$ is the vector of crisp decision variables, and $\tilde{a}_{ij}$, $\tilde{b}_i$ and $\tilde{c}_j$ are fuzzy quantities. The operations addition and multiplication by a real number of fuzzy quantities are defined by Zadeh's extension principle (Zadeh, 1975). The inequality relation ($\leq$) is given by a certain fuzzy relation. Function f is to be minimized in the sense of a given crisp inequality relation between fuzzy quantities, and the (implicit) X is a fuzzy set describing the concept " $\underline{x}$ satisfies all the constraints".

3. Stochastic Algorithms and Fuzzy Optimization

The new FICRS algorithm proposed here is an adaptation of the original ICRS algorithm (Banga and Casares, 1987). It has three heuristic parameters, which default values are $k_1 = 1/3$, $k_2 = 1/2$ and $n_e = 4$. The k_1 parameter controls the initial step size of the search, which will be subsequently reduced using the parameter k_2 . The n_e integer parameter controls the rate of convergence. For problems without local optima, these default values are adequate. For highly multimodal problems, k_1 must be increased to values from 1.0 to 2.0 as shown by Banga and Seider (1995).

The proposed algorithm uses the more convenient and, in some cases, only possible description of processes by fuzzy rules. Stochastic algorithms have many advantages like their robustness and, in most cases, global convergence properties (Carrasco and Banga, 1997). On the other hand, the fuzzy models allow a more detailed description that can be closer to the nature of the processes described and can include linguistic and qualitative information. The flow-chart of the proposed strategy can be sketched as shown in Figure 1.

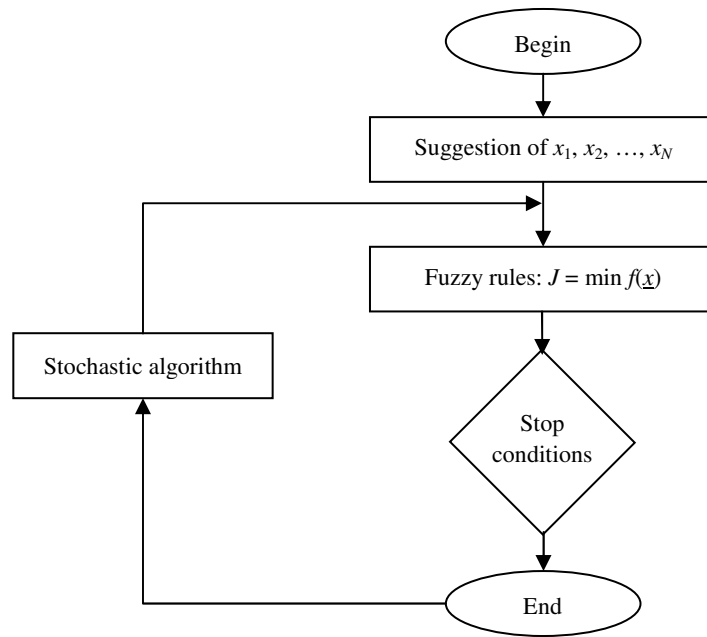


Figure 1. Flow-chart diagram of the proposed strategy

As you can see, the only modifications respect to the original algorithm is that the algebraic expressions for the calculation of the value of the objective function, and/or the constraints, have been substituted by fuzzy expressions that use the corresponding membership functions and inference method.

The proposed strategy suggested in this paper can be easily implemented. The algorithm is initialized from a suggested starting decision vector $\underline{x}_0 = (x_1, x_2, \dots, x_N)$. Then, objective function's value is evaluated by fuzzy rules. If stop conditions are not satisfied the stochastic algorithm generates new decision vectors, recalculating again a new value of the objective function en each step, until those conditions are achieved. In the case study considered here, for every decision variables vector, the algorithm computes the objective function, via product t-norm inference method, using triangular norms membership functions.

4. Case Study: A Fuzzy Mathematical Problem

Let us take a particular example, recently studied by Carlsson and Fullér (2001), which consider the following optimization problem:

$$\min f(\underline{x}) \quad (4)$$

where $f(\underline{x})$ is given linguistically as:

$$\mathbf{R}_1(\underline{x}): \text{ if } x_1 \text{ is } \textit{small} \text{ and } x_2 \text{ is } \textit{small} \text{ then } f(\underline{x}) \text{ is } \textit{small} \quad (5)$$

$$\mathbf{R}_2(\underline{x}): \text{ if } x_1 \text{ is } \textit{small} \text{ and } x_2 \text{ is } \textit{big} \text{ then } f(\underline{x}) \text{ is } \textit{big} \quad (6)$$

the universe of discourse for the linguistic value of f being the interval $[0,1]$. The membership functions, which are the same for both variables, in the rule-base $\mathbf{R}$ are defined by:

$$\textit{small}(t) = 1 - t, \quad \textit{big}(t) = t, \quad t \in [0,1] \quad (7)$$

This problem has the following equality and inequality constraints:

$$x_1 + x_2 = 1/2 \quad (8)$$

$$0 \leq x_1 \leq 1/2 \quad (9)$$

$$0 \leq x_2 \leq 1/2 \quad (10)$$

We will compute the firing levels of the rules by the product t-norm. The individual rule outputs are those showed in Figure 2. As you can see, the membership functions are all triangular norms. This kind of membership function was introduced by Schweizer and Sklar (1963) to model the distances in probabilistic metric spaces. The rules given by Eqs. (5) and (6) represent the knowledge for the fuzzy optimization problem. In order to compare with the result reported by Carlsson and Fullér (2001) the same Tsukamoto's reasoning scheme has been chosen. This scheme has been used because the functional relationship between the input vector $\underline{x}$ and the system output $f(\underline{x})$ can be relatively easily defined performing only inversion operations.

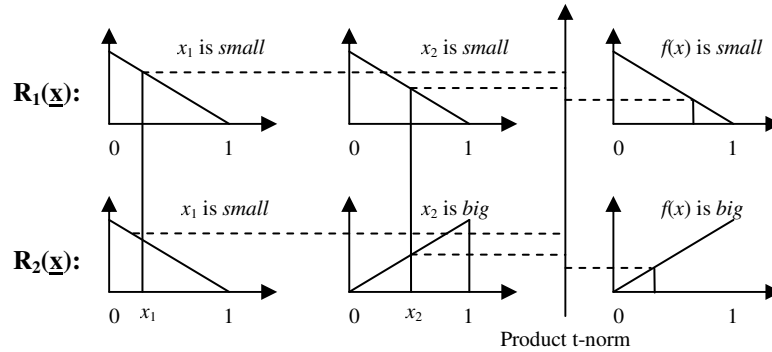


Figure 2. Individual rule outputs for case study

5. Results and Discussion

Carlsson and Fullér (2001) obtained an “analytical” solution for this case study. They found the global optimum at $x_1 = x_2 = 1/4$ with an optimum value for the objective function $f(\underline{x}) = 0.375$. This case study was successfully solved using Fortran 77 (double-precision) implementations of the FICRS algorithm. One of our objectives was to demonstrate that this stochastic method is efficient enough to solve optimization

problems described by fuzzy if-then rules using low-cost computing platforms. Therefore, the program was compiled (using GNU Fortran 3.4.1 compiler) and run on a PC Pentium IV/2.0 GHz (with Linux Mandrake 10.1). In order to make meaningful comparisons between CPU times obtained in this work and other platforms a performance database server based on the Linpack benchmark is available in the Internet at: <http://performance.netlib.org>.

The algorithm was executed using always the same starting decision vector $\underline{x} = (0.5, 0)$. Several executions, using different seed values and all with a value of 10^{-3} for the stop criterion, were done (Table 1). FICRS converged to the global optimum in all cases. The best result obtained was $f(\underline{x}) = 0.37500000$ and $\underline{x} = (0.24999, 0.25001)$ which is very close to the analytical solution. As you can see, all results were achieved in very short CPU times.

Table 1. Results of the FICRS algorithm.

Run	Obj. Func.	State variables	Func. Eval.	t_{CPU} (s)
1	0.37500000	(0.24997, 0.25002)	17271	2.5
2	0.37500001	(0.25006, 0.24994)	3328	0.6
3	0.37500000	(0.24999, 0.25001)	199	0.1
4	0.37500025	(0.25035, 0.24965)	603	0.4
5	0.37500000	(0.25002, 0.24998)	6715	1.6
6	0.37500001	(0.25006, 0.24994)	104	0.1
7	0.37500002	(0.24999, 0.25001)	2869	0.5
8	0.37500000	(0.24999, 0.25001)	2869	0.5
9	0.37500000	(0.24998, 0.25002)	2910	0.5
10	0.37500000	(0.25001, 0.24999)	1019	0.3

6. Conclusions

It has been shown that the FICRS algorithm is a simple alternative for the optimization of problems described by fuzzy if-then rules. Besides, it also has other important advantages, including very easy implementation and rapid convergence. With respect to the case study considered here, which has been taken from the recent literature, the algorithm is efficient enough to be used in low-cost computing platforms with very reasonable CPU times.

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