

Opportunities and challenges of using sequential quadratic programming (SQP) for optimization of petroleum production networks

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Abstract

There is increasing interest in using model-based optimization in petroleum production operations. In particular, attention is being paid to sequential quadratic programming (SQP). This paper analyzes a type of optimization problems encountered in petroleum production operations and evaluates the performance of SQP.

Keywords: optimization, sequential quadratic programming, petroleum production

1. Introduction

Optimizing the operation of oil and gas production is naturally an area of great interest for both petroleum companies and governmental authorities. Small changes in production rates may have large impact on total recovery and on revenues. Selecting a good production strategy is a difficult challenge with many considerations, and model-based optimization may be an efficient tool supporting such a selection.

Mature hydrocarbon reservoirs will typically experience a decline in reservoir pressure, and the wells will be producing three fluid phases: oil, gas, and water. The reduced energy available in the reservoir, and the restricted capacity of the well network to handle large water and gas rates, will then make the oil rate decline.

Related to the production strategy, many types of optimization problems may be defined, for instance depending on the time horizon involved. Striving to maximize the total recovery or the net-present value of the total revenue involves long-term goals, requiring special optimization techniques. Production engineers are also faced with the challenge of optimizing the *daily* production rate, leading to time-independent (or steady-state) optimization problems. Only the latter type of problem will be studied here.

Several commercial software packages for modelling fluid transport in a well network are available, and these models may offer a good basis for the optimization process. Since the objective function and the system constraints are, in general, nonlinear functions of the available optimization variables, nonlinear optimization techniques are

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recommended (Wang, 2003). Among them, sequential quadratic programming (SQP) is being increasingly popular; see e.g. Barua et al. (1997), Dutta-Roy & Kattapuram (1997), Dutta-Roy et al. (1997), and Wang (2003).

SQP has been largely studied and used in other areas of engineering. Extensive research has been dedicated to develop and improve SQP algorithms, e.g. to handle large-scale problems. A comprehensive description of SQP theory and algorithms can be found in Bazaraa et al. (1994), Boggs & Tolle (1995), Mangasarian (1969), and Nocedal & Wright (1999). The current status and future trends in SQP are described in a recent review by Biegler & Grossmann (2004).

The aim of this paper is to investigate how the SQP algorithm behaves for the problem of optimizing the instantaneous behaviour of a petroleum production network. The paper is organized as follows. Section 2 briefly describes the main elements of a petroleum production network and how they are modelled. Section 3 gives a brief introduction to SQP. Section 4 discusses the application of SQP to petroleum production networks and a simple example is discussed. Finally, section 5 summarizes the main conclusions of the paper.

2. Petroleum production networks

A petroleum production system is comprised of the following subsystems:

- A reservoir, which is a subsurface body of rock having sufficient porosity and permeability to store and transmit fluids.
- A system of wells, i.e. perforations to access the reservoir at diverse locations.
- A gathering system, i.e. the pipeline network that transports and controls the flow of fluids from the wells to the storage or processing plant.
- Surface/Topside facilities for separation and processing.

Figure 1 illustrates these elements for an offshore network in the North Sea.

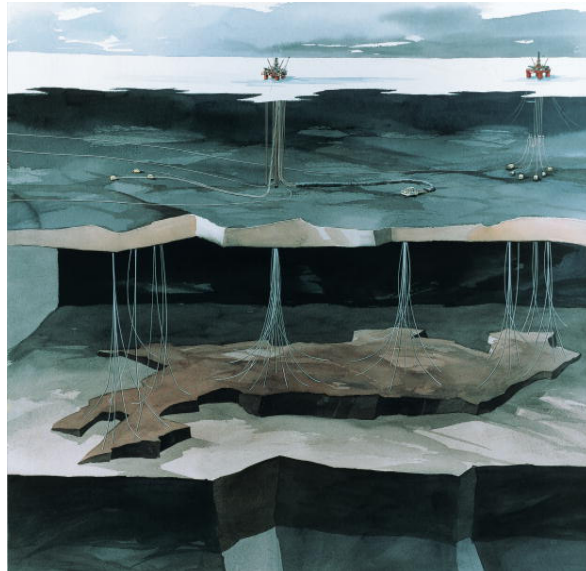


Figure 1: Schematic diagram of the Vigdis and Snorre B fields (source: Norsk Hydro)

The interaction between the reservoir and the well is typically given by two relationships: the vertical lift performance (VLP) and the inflow performance relationship (IPR). The former accounts for the hydrodynamic and friction effects in the well, while the latter reflects the ability of the reservoir to deliver fluid to the well. The IPR/VLP intersection thus gives the feasible pressure-flowrate pairs in the well. Several alternative approaches for describing the IPR/VLP relationship are used by commercial software, ranging from use of tabular data or empirical correlations to use of detailed spatial mechanistic flow models. In any case, we get some relationships between the reservoir pressure $P_{reservoir}$, the bottomhole pressure $P_{w,down}$, and the well flowrate

q_{well} . Note that:

- For a given constant pressure at the top of the well, the IPR and VLP curves may intersect at one or several points (normally they intersect at two points).
- For multi-phase production where the gas or water rate varies strongly as function of the total production rate, additional relationships are required.
- If gas lift is used to enhance flow, i.e. gas is injected into the well to reduce the hydrostatic pressure of the fluid column, then the VLP has to be modified to include such an effect. The amount of gas used for gas lift is typically one of the variables we can use for production optimization.

Normally, at the top of the well there is a choke valve device used to control fluid flow rate and/or bottomhole pressure. Choke opening is thus an optimization variable, and the relationship between the flowrate q_{well} and the pressure drop in the choke ΔP_{choke} should be included in the model.

The model of the gathering system (or pipeline network) is comprised of:

- A mathematical description of pressure drop as a function of flowrate for each pipeline.
- A conservation mass law for each node of the network.
- A requirement that Kirchoff's law is satisfied for each node of the network, i.e. the node pressure calculated from the pressure drops in the pipelines upstream of the node has to be equal to the node pressure calculated from the pressure drops in the pipelines downstream of the node.

Finally, the topside equipment is typically included as a terminal node of the network, where the pressure is fixed and where the maximum rates of water and gas that can be treated are also fixed.

The complexity of the model varies greatly with the size of the network and with the type of mathematical descriptions used for the pressure-flowrate relationships. In general, the total production rate and other network variables vary nonlinearly with the available optimization variables: gas lift rates and choke openings.

3. Sequential Quadratic Programming

SQP belongs to the class of optimization techniques called nonlinear programming (NLP). A NLP problem is typically formulated as follows:

$$\begin{aligned}
& \min_x f(x) \\
& s.t. \quad g_i(x) \leq 0 \quad \text{for } i = 1, 2, \dots, n_{ineq} \\
& \quad h_j(x) = 0 \quad \text{for } j = 1, 2, \dots, n_{eq}
\end{aligned}$$

where x represents the optimization variables, $f(x)$ represents the scalar objective function, the vector $g(x) = \{g_i(x)\}$ represents the inequality constraints, and the vector $h(x) = \{h_j(x)\}$ represents the equality constraints. The Lagrangian function

$$L(x, \lambda, v) = f(x) + \lambda^T h(x) + v^T g(x)$$

is required to write the first-order (or necessary) optimality conditions, also called Karush-Kuhn-Tucker (KKT) conditions:

$$\nabla_x L|_{(x, \lambda, v)_{\min}} = \nabla_x f|_{x_{\min}} + \lambda_{\min}^T \nabla_x h|_{x_{\min}} + v_{\min}^T \nabla_x g|_{x_{\min}} = 0$$

$$h(x)|_{x_{\min}} = 0$$

$$g(x)|_{x_{\min}} \leq 0$$

$$v_{\min} \geq 0$$

$$v_{\min}^T g(x)|_{x_{\min}} = 0$$

SQP is an iterative and approximate method to solve the KKT conditions (Newton's method). At each iteration, the NLP becomes a quadratic programming (QP) problem:

$$\min_d \left\{ \nabla f(x_k)^T d + \frac{1}{2} d^T H_L(x_k, \lambda_k, v_k) d \right\}$$

$$\begin{aligned}
s.t. \quad & g_i(x_k) + \nabla g_i(x_k)^T d \leq 0 \\
& h_j(x_k) + \nabla h_j(x_k)^T d = 0
\end{aligned}$$

Here, the objective function is a quadratic approximation of the NLP's Lagrangian function, and the constraints are linear approximations of the NLP's constraints. Convergence to the solution depends, among other factors, on the way the Hessian of the Lagrangian H_L is calculated (analytical, finite differences, etc) .

SQP shows the best performance when dealing with smooth (i.e. twice differentiable) and convex (only one minimum) optimization problems, and when a good starting value of the optimization variables is used.

4. Application of SQP to petroleum production networks

The optimization problem described in Section 2 is a nonlinear, constrained problem, and a nonlinear programming technique such as SQP may be well suited for the problem. However, the performance of SQP depends greatly on the availability and quality of the gradient and Hessian information. If either the objective function or the

constraints have discontinuities with respect to the optimization variables, then gradient information can not be calculated, and the performance of SQP solvers can be poor in regions close to the discontinuities. A simple example will be used to illustrate the behaviour of the SQP algorithm. Consider a simple network of three wells in parallel connected to a platform with a single oil-water-gas separator. One of the wells (well 1) produces oil naturally, while the other two (wells 2 and 3) require gas lift in order to be able to produce. Wells 2 and 3 produce water together with the oil. The separator can only handle a limited amount of water. The objective function to maximize is the total oil production, the optimization variables are the amounts of gas lift used in wells 2 and 3, and the constraints are the network equations and the water capacity at the separator. The network has been modelled and optimized using a commercial simulator (GAP, 2004). Figure 2 shows the objective function and the water constraint as a function of the optimization variables. Figure 2 was obtained by running simulations of the network model in an automatic way from an external interface (MATLAB, 2004).

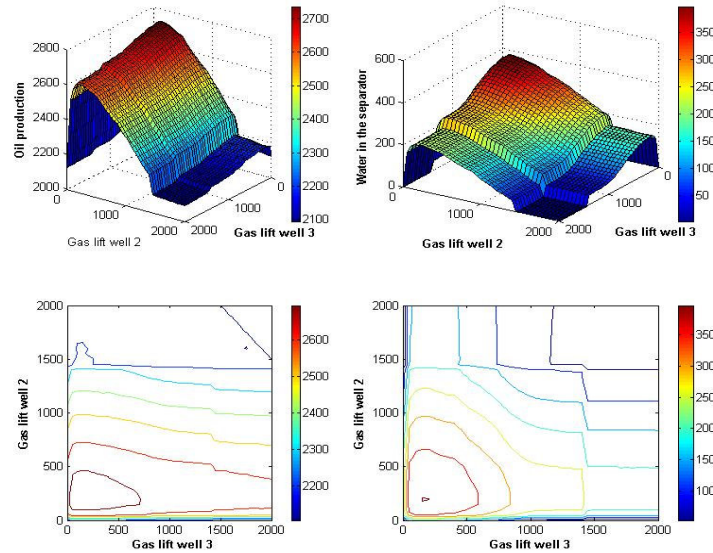


Figure 2: Objective and constraint as a function of optimization variables

Figure 2 illustrates quite well the standard behaviour of gas lift. For small gas rates, an increase in gas rate enhances the oil production. However, for large gas rates, friction effects become so large that increasing the gas rate becomes detrimental, i.e. less oil is produced. The well may even stop producing oil. In the network model, the death of a well causes sudden drops in the total oil and water rates, as observed in Figure 2. For this specific example, the discontinuities do not pose any serious problem to SQP, because they are located far away from the optimum. Hence, when SQP is applied for different values of the water constraint the obtained results make perfect sense: the more restrictive the constraint, the smaller the total oil production. Note that for more complex networks, the optimum may be closer to discontinuities, and then the performance of the SQP algorithm will be poor. The SQP algorithm may be combined

with derivative-free methods in such situations, or heuristic rules can be added to the SQP algorithm to handle discontinuities.

Optimization of production networks does not generally lead to convex problems, and typically, several local minima exist. In general, the more branches the network has, the more local optima exist in the network. SQP has limited globalization abilities (typically a merit function is used for such a purpose). Hence, standard SQP algorithms will tend to get trapped in local optima. This is certainly disadvantageous when the local optima have considerable worse objective function values than the global, or whenever the local optima are not good operating points in practice. Again, heuristic rules or the combination of SQP with a global optimization method may be needed.

SQP is not a feasible method (Boggs et al., 1995). This means that the constraints may be violated in certain cases. The parameters of the optimization algorithm should be tuned such that constraints are fulfilled to a small tolerance, and the values of the constraints in the optimal solution should always be checked to ensure that the values are acceptable. It is also important to take into account the model uncertainties when judging the results provided by the optimizer.

5. Conclusions

The problem of optimizing the instantaneous production rate from a petroleum production network has been analyzed. Since this typically leads to a constrained nonlinear problem, a method such as SQP is suitable. However, SQP may perform poorly in regions where the model contains discontinuities. The existence of several optima, together with possible violation of constraints, must be handled properly. A careful inspection of the results should always be carried out, and a combination of SQP with global solvers or heuristic rules may be necessary to improve the optimization process.

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