Optimizing the Petroleum Supply Chain at PETROBRAS

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Abstract

The main objective of this study is to describe how mathematical programming is being used to solve the Petroleum Allocation Problem at Petrobras. We propose a Mixed Integer Linear Programming formulation of the problem which relies on a time/space discretization network. The formulation involves some inequalities which are redundant to the mixed integer model but no necessarily so to the Linear Programming relaxation of it. We also use some inequalities which are associated with polytopes that have been extensively studied in the literature. Furthermore, separation routines for strong valid inequalities associated with these polytope are readily available in some commercial solvers. Use of this feature allowed a substantial reinforcement of the underlying Linear Programming relaxation to be attained. Our formulation was tested on an industrial-size instance of the problem involving 11 crude oils, 6 tanker types, 5 maritime terminals involving 8 docks, 6 refineries, and 8 distillation units over a time horizon of 60 discretized intervals. The instance has 28,000 binary variables, 19,000 continuous variables, and 14,000 constraints and has been effectively solved under the proposed formulation. Feasible mixed integer solutions, guaranteed to be at no more than 5% of optimality, were obtained in less than 4000 CPU seconds under the mixed integer solver XPRESS-MP.

Keywords: Petroleum allocation, Planning applications, Optimization

1. Introduction

Petrobras is a vertically integrated oil company dealing with a large range of activities extending from petroleum exploration to refining and distribution. The logistic activity at Petrobras is split into three steps, with information being exchanged among the different levels. The first one, i.e., the general supply planning strategy is performed through PLANAB (a multi period linear programming model) which is run every two months over a planning horizon of six months. The following step deals with petroleum allocation, which is the main focus of this work. Petroleum allocation is performed on a monthly basis taking into account a time horizon of two months. Finally, at a third step, operational planning is conducted on a daily basis for a time horizon of one week. Petroleum allocation plays a central role in the petroleum supply chain at Petrobras. It

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overlaps strategic planning with operational demands. The problem involves decisions related with lot sizing of crude oils from offshore platforms in Brazil and production sites abroad, allocation of crude oils to refineries, inventory control at maritime transhipment terminals and refineries, and planning/scheduling operations at crude distillation units. Up to now, no efficient tool is available to perform petroleum allocation at Petrobras. Apart from the obvious financial losses incurred, this fact results in a lack of integration of the logistics activities described above. Additionally, no references can be found in the literature of algorithms to tackle the problem in its entire range. Indeed, if a level of simplification and abstraction is not exercised, any real world instance would remain out of reach in practical terms. Typically, in the literature, the problem is divided into two sub problems: ship scheduling (Zabal, 1984 and Miller, 1987) and planning operations at refineries (Lee et al, 1996 and Pinto et al, 2000). In this study we have chosen to consider a simplified version of the overall problem.
This paper is organised as follows: Section 2 describes the problem, followed by our proposed model, in Section 3. We then present a case study in Section 4 and close the paper with some concluding remarks in Section 5.

2. Problem Definition

Petroleum allocation must be programmed so that sufficient supplies of required crude oil reach refineries along the planning horizon. This must be done by taking into account strategic planning (PLANAB) and operational constraints along the petroleum supply chain.

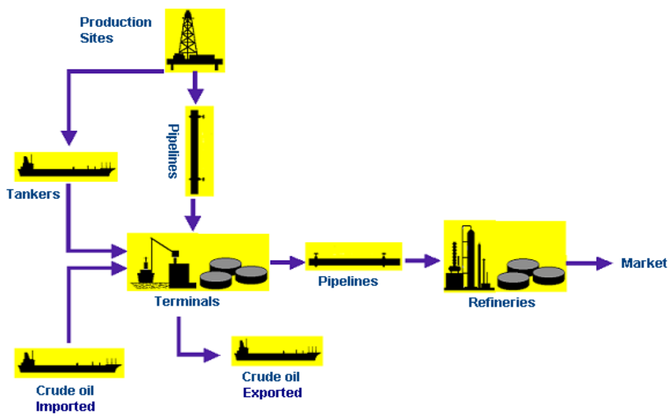

Figure 1: Infrastructure of PETROBRAS' Supply Chain

Figure 1 above, depicts the infrastructure used to allow crude oil supplies to reach refineries. As shown, crude oil could either be locally produced or imported from abroad. Local crude oil comes from production sites, mostly offshore, and are transported either by tankers or pipelines. Imported oil is only transported by tankers. After reaching maritime terminals, crude oils are either exported or shipped to Petrobras refineries. At the refineries, petroleum is processed in crude distillation units (CDUs) on daily scheduled production campaigns. These campaigns are defined by: consumption rates of different petroleum categories, duration, release date, and deadlines to completing them. For the sake of simplification, crude oils are aggregated into categories involving some basic characteristics such as, among others, sulphur contents. For the problem being tackled, the key decisions are related to when crude oil should be loaded at production sites, which type of tanker should be used, which volume should be dispatched, which terminal should be used for discharging and the refineries which will eventually receive these supplies. Another important decision is the scheduling of production campaigns at CDUs.

3. Mathematical Formulation

After considering various alternatives, a Mixed Integer Linear Programming (MILP) model was adopted. This model is based on a fixed charge network flow structure over a discretized time representation. Time intervals of equal duration are considered and activities allocated to a given interval must be capable of being performed within it. An Initial Model is presented first. Next this model is refined with the introduction of a "cut inequality" which tightens the associated Linear Programming (LP) relaxation. Finally, the model is substantially enforced through a reformulation that allows facet defining inequalities of a well studied polytope to be incorporated into the model.

3.1 Initial Model

Constraints:

Volume balance at production sites:

$$ep_{o,t} - ep_{o,t-1} - P_{ot} + \sum_{b,cl} VN_{cl} \cdot spb_{o,b,cl,t} = 0 \quad \forall o,t \tag{1}$$

At most one tanker should visit a production site at each time period:

$$\sum_{b,cl} spb_{o,b,cl,t} \leq 1 \quad \forall o,t \tag{2}$$

At most one tanker should arrive at a berth at each time period:

$$\sum_{o,cl} spb_{o,b,cl,t} \leq 1 \quad \forall b,t \tag{3}$$

Volume balance at berths:

$$\sum_{cl} spb_{o,b,cl,t} - \sum_{r,c} ct_{o,c,z,r,t+TV_{o,z}} = 0 \quad \forall o,z,t \tag{4}$$

Notice that these inequalities implicitly define the refineries to be supplied.

Volume balance at terminals:

$$vct_{c,z,r,t} - vct_{c,z,r,t-1} + st_{c,z,r,t} - \sum_{o} ct_{o,c,z,r,t} = 0 \quad \forall c,z,r,t \tag{5}$$

Volume balance at refineries:

$$vcrn_{c,r,t} - vcrl_{c,r,t} - vcrll_{c,r,t} - vcri_{c,r,t} - vcrn_{c,r,t-1} + vcrl_{c,r,t-1} + vcrll_{c,r,t-1} + vcri_{c,r,t-1} + st_{c,z,r,t} - \sum_{cp,u} CAMP_{cp(u),c} \cdot cpb_{u,cp(u),t} = 0 \quad \forall c,z,r,t \tag{6}$$

Solution deviation from PLANAB:

$$dplan_{o,r} + \sum_{c,z,t} ct_{o,c,z,r,t} \geq PLAN_{o,r} \quad \forall o,r \tag{7a}$$

$$dplan_{o,r} - \sum_{c,z,t} ct_{o,c,z,r,t} \geq PLAN_{o,r} \quad \forall o,r \tag{7b}$$

Assignment of production campaign to time slots within valid time windows:

$$\sum_{cp(u)} cpb_{u,cp(u),t} = 1 \quad \forall u, TS_{cp(u)} \leq t \leq TD_{cp(u)} \tag{8a}$$

$$\sum_{t=TS_{cp(u)}}^{TD_{cp(u)}} cpb_{u,cp(u),t} = DC_{cp(u)} \quad \forall u, cp(u) \tag{8b}$$

If a change of campaign takes place, then a crude distillation set-up is necessary:

$$su_{u,t} + cpb_{u,cp(u),t+1} - cpb_{u,cp(u),t} \geq 0 \quad \forall u,t \tag{9}$$

Maximum number of tankers used during the horizon study for each class:

$$df_{cl} \geq \lceil FU_{cl} \rceil NCL_{cl} - \sum_{o,b} spb_{o,b,cl,t} \quad \forall c,t \tag{10}$$

The objective function consists of minimizing voyage costs, costs for freighting extra tankers, set-up costs due to change of campaigns, penalty for oil shortage at refineries and penalty for PLANAB deviation.

$$\min\Big\{ \sum_{o,b,cl,t} CV_{o,z,cl} \cdot spb_{o,b,cl,t} + \sum_{c,r,t} CRMC_{c,r} \cdot vcrl_{c,r,t} + \sum_{c,r,t} CRPC_{c,r} \cdot vcrll_{c,r,t} + \sum_{o,r} CPL_{o,r} \cdot dplan_{o,r} + \sum_{u,t} CSP \cdot su_{u,t} + \sum_{cl} CF_{cl} \cdot df_{cl} \Big\} \tag{11}$$

3.2 Changeover cut

As shown by Yee and Shah (1998), the presence of non-productive activities in a MILP scheduling model may lead to a large relaxation gap. To overcome this difficulty, usually some cut constraints are added which enforce that a minimum number of changeover tasks must be performed, i.e., that

$$\sum_{t} chc_{u,t} \geq \left|NCP_u\right| - 1 \quad \forall u \tag{12}$$

3.3 Reformulation of volume balance at production sites

As one may appreciate, it is not obvious what could be done with inequalities (1) in order to reinforce the overall model. However these inequalities may be replaced with advantage by the following inequalities,

$$\sum_{\tau=0}^{t}\sum_{b,cl} VN_{cl} \cdot spb_{o,b,cl,\tau} \le VPI_o + \sum_{\tau=0}^{t} P_{o,\tau} \quad \forall o \in O, \forall t \in T \tag{13a}$$

$$\sum_{\tau=0}^{t}\sum_{b,cl} VN_{cl} \cdot spb_{o,b,cl,\tau} \ge VPI_o + \sum_{\tau=0}^{t} P_{o,\tau} - CMAP_o \quad \forall o \in O, \forall t \in T \tag{13b}$$

These define cascading sets of knapsack type inequalities which could be reinforced through associated Lifted Minimum Cover Inequalities (Wolsey, 1998). Indeed this is automatically done in some commercial MILP solvers.

4. Case study

Experiments were conducted on a Pentium III 833MHz 512Mb RAM PC and the code was compiled with gcc under a LINUX Platform. Table 1 shows, respectively, results for the Initial Model, Initial Model plus Changeover cuts, and Initial Model reformulated with inequalities (13a) and (13b) plus Changeover cuts.

Table 1: Summary of Computational Results

Instance	**Initial Model**	**With Changeover**	**With Changeover Cuts and Reformulation**
Number of constraints	13857	13855	14395
Number of variables	45069	45069	44398
Number of binaries variables	22333	22333	22333
Number of nodes visited	>> 1000000	>> 1000000	55763
CPU(seconds)	>> 1000000	>> 1000000	3524
LP solution (Z_{lp})	22856.5	28610.7	28610.7
Best solution (Z_o)	55180.64	41122.3	41117.1
Relaxation gap $100*(Z_{ob} - Z_{lp}) / Z_{ob}$ where $Z_{ob} = \min\{Z_o\}$	44.4	30.4	30.4

5. Conclusions

In this study we depart from what is traditionally done in the literature by considering together Ship Scheduling and Refinery Planning. Although this is done under a simplified model of the overall problem, advantages may be attained by integrating the two sub-problems. Furthermore our computational experiments indicate that the suggested approach is attractive in computational terms.

We plan to further refine the proposed model by reformulating, in the style of inequalities (13a) and (13b), similar structures found within it. Namely volume balance at terminals and refineries.

Nomenclature

Variables:

$spb_{o,b,cl,,t}$: 1 if crude type *o* is sent to berth b by tanker *cl* over interval *t*; 0 otherwise.
$ep_{o,t}$: Amount of crude type *o* stored over interval *t* .
$ct_{o,c,z,r,t}$: Amount of crude type *o* of category *c* that arrives at terminal *z* to supply refinery *r* over interval *t*.
$st_{c,z,rt}$: Amount of category *c* pumped from terminal *z* to refinery *r* over interval *t*.
$vcrn_{c,r,t}$, $vcrl_{c,r,t}$, $vcrll_{c,r,t}$, $vcri_{c,r,t}$: Stock of category *c* at refinery *r* over interval *t*, normal, low, very low and infeasible, repectively.
$dplan_{o,r}$: Deviation from PLANAB of crude type *o* supplied to refinery *r* .
$cpb_{u,cp(u),t}$: 1 if campaign *cp(u)* takes place at CDU *u* over interval *t*; 0 otherwise.
$su_{u,t}$: 1 if a set-up is necessary in CDU *u* over interval *t*.
df_{cl} : Number of tankers having to be either freighted or rented during the horizon study.

Parameters:

$P_{o,t}$: Amount of production of crude type o over interval *t*.
VN_{cl} : Average transportation capacity of tanker *cl*.
$CAMP_{cp(u),c}$: Consumption rate of category *c* of campaing *cp(u)*.
$PLAN_{o,r}$: Amount of crude oil type *o* planned for refinery *r* during the horizon study.
TV_{oz} : Voyage time between production site *o* and terminal *z*.
$TS_{cp(u)}$: Release date for campaign *cp(u)*.
$TD_{cp(u)}$: Deadline for completing campaign *cp(u)*.
$DC_{cp(u)}$: Duration of campaign *cp(u)*.
FU_{cl} : Fraction of the number of tanker *cl* that could be used during the horizon study.
NCL_{cl} : Number of tankers in class *cl*.
$CMAP_o$: Maximum storage capacity at production site *o*.
VPI_o : Stock of crude oil *o* in production site at the beginning of the horizon study.

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