Coordinated Feedback and Switching for Robust Control of Hybrid Nonlinear Processes

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Abstract

A robust hybrid control strategy, that coordinates feedback and switching, is proposed for a broad class of hybrid nonlinear processes with constrained inputs and uncertain dynamics. The proposed strategy involves the coordinated synthesis, via multiple Lyapunov functions, of a family of bounded robust nonlinear feedback controllers that stabilize the constituent modes of the hybrid system, together with robust switching laws that orchestrate safe transitions between these modes and their respective controllers, in a way that respects input constraints and guarantees stability of the overall switched uncertain closed-loop system. The proposed strategy is illustrated using a chemical process example with switched dynamics, input constraints, and model uncertainty.

1. Introduction

The study of hybrid systems in process control is motivated by the fundamentally hybrid nature of process transients, which depend on an intricate interaction between discrete and continuous variables. The continuous behavior typically arises from the underlying physical laws, such as momentum, mass, and energy conservation, and is described by continuous-time differential equations. Discrete events, on the other hand, may arise from inherent discontinuities in the continuous dynamics (e.g., phase changes), instrumentation with discrete actuators and sensors, or from the use of logic-based switching for supervisory and safety control.

The abundance of hybrid phenomena in many engineering systems in general, and in the chemical process industries in particular, together with the need to design control and supervisory schemes for these systems has fostered a large and growing body of research work in this area (see, e.g., (Grossman et al., 2001; DeCarlo et al., 2000; Bempporad and Morari, 1999; Hu et al., 1999). Recently, we have developed in (El-Farra and Christofides, 2001a) a nonlinear hybrid control methodology for a broad class of switched nonlinear systems with input constraints. These are hybrid systems that comprise of a finite family of continuous nonlinear dynamical modes, subject to hard constraints on their manipulated inputs, together with higher-level supervisors that govern the transitions between the constituent modes. The key feature of the proposed methodology was the integrated synthesis, via multiple Lyapunov functions (MLFs), of: 1) lower-level feedback controllers that stabilize the constituent constrained modes and provide, simultaneously, an explicit characterization of the stability region for each mode, and 2) upper-level switching laws that orchestrate the transitions between the continuous modes and their respective controllers, in a way that ensures stability of the overall switched closed-loop system despite its constrained and changing dynamics.

In this paper, we extend the scope of our previous work to deal with switched nonlinear systems whose dynamics are both constrained and uncertain. Typical sources of uncertainty include plant-model mismatch (e.g., modeling errors, unknown process parameters) as well as time-varying exogenous disturbance inputs. To address the problem, we propose a robust hybrid control strategy that uses multiple Lyapunov functions to: 1) synthesize a family of robust bounded nonlinear feedback controllers that enforce robust stability in the constituent constrained uncertain modes, 2) explicitly characterize the stability region for each mode under uncertainty and constraints, and 3) design robust switching laws that coordinate safe transitions between the modes in a way that guarantees closed-loop stability of the overall switched closed-loop system. The proposed approach is successfully applied to a switched exothermic chemical reactor with input constraints, and model uncertainty.

2. Switched nonlinear processes with uncertain dynamics

2.1 State space description

We consider the class of switched uncertain nonlinear processes described by the following state-space representation:

$$\begin{array}{rcl} \dot{x}(t) & = & f_{\sigma(t)}(x(t)) + G_{\sigma(t)}(x(t))u_{\sigma(t)} + W_{\sigma(t)}(x(t))\theta_{\sigma(t)}(t) \\ \sigma(t) & \in & \mathcal{I} = \{1, \cdots, N\} \end{array} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ denotes the vector of continuous-time process state variables, $u(t) = [u_1(t) \cdots u_m(t)]^T \in \mathcal{U} \subset \mathbb{R}^m$ denotes the vector of control inputs taking values in a nonempty compact convex subset of $\mathbb{R}^m$, $\theta(t) = [\theta_1(t) \cdots \theta_q(t)]^T \in \Theta \subset \mathbb{R}^q$ denotes the vector of uncertain (possibly time-varying) variables taking values in a nonempty compact convex subset of $\mathbb{R}^q$. The uncertain variables $\theta(t)$ may describe time-varying parametric uncertainty and/or exogenous disturbances. $\sigma : [0, \infty) \rightarrow \mathcal{I}$ is the switching signal which is assumed to be a piecewise continuous (from the right) function of time, implying that only a finite number of switches is allowed on any finite interval of time. For each value that the discrete state σ assumes in $\mathcal{I}$, the temporal evolution of the continuous state, x, is governed by a different set of differential equations. Processes of the form of Eq.1 are therefore of variable structure; they consist of a finite family of N continuous-time uncertain nonlinear subsystems (or modes) and some rules for switching between them. This class of systems arises naturally in the context of coordinated supervisory and feedback control of chemical process systems (see section 4 for an example). Note that, by indexing the uncertain terms in Eq.1 by σ, the constituent modes are, in general, not assumed to share the same uncertain variables nor be equally impacted by them. The uncertainty is therefore allowed to influence the dynamics of different modes, differently.

Throughout the paper, the notation t_{i_k} and t_{i_k+1} is used to denote, the k-th times that the i-th subsystem is switched in and out, respectively, i.e. $\sigma(t_{i_k}^+) = \sigma(t_{i_k+1}^-) = i$. With this notation, it is understood that the continuous state evolves according to $\dot{x} = f_i(x) + G_i(x)u_i + W_i(x)\theta_i$ for $t_{i_k} \leq t < t_{i_k+1}$. It is assumed that all entries of the vector functions $f_i(x)$, the $n \times m$ matrices $G_i(x)$, the $n \times q$ matrices $W_i(x)$ are sufficiently smooth on $\mathbb{R}^n$ and that $W_i(0) = 0$ for all $i \in \mathcal{I}$. We also assume that the state x does not jump at the switching instants, i.e. the solution $x(\cdot)$ is everywhere continuous. Finally, we recall the definition of a robust control Lyapunov function which will be used in the development of the main result of this paper.

Definition 1 (Freeman and Kokotovic, 1996): *A robust control Lyapunov function for a system of the form $\dot{x} = f(x) + G(x)u + W(x)\theta$ is a smooth, proper, and positive definite function $V : \mathbb{R}^n \rightarrow \mathbb{R}$ with the property that for every fixed $x \neq 0$:*

$$L_G V = 0 \implies \sup_{\theta \in \Theta} \{L_f V + L_W V \theta\} < 0 \tag{2}$$

where $L_f V = \frac{\partial V}{\partial x} f(x)$, $L_G V$ and $L_W V$ are row vectors of the form $[L_{g_1} V \cdots L_{g_m} V]$ and $[L_{w_1} V \cdots L_{w_q} V]$, respectively, with g_k and w_k referring to the k-th columns of the matrices G and W, respectively.

2.2 Problem formulation

Consider the switched nonlinear process of Eq.1, where, for each $i \in \mathcal{I}$, a robust control Lyapunov function, V_i, is available, the vector of control inputs, u_i, is constrained by $|u_i| \leq u_i^{max}$, and the vector of uncertain variables is bounded by $|\theta_i| \leq \theta_{bi}$. Given that switching is controlled by a higher-level supervisor, the problem is how to coordinate switching between the constituent modes, and their respective controllers, in a way that respects the constraints and guarantees closed-loop stability in the presence of uncertainty. To address the problem, we formulate the following objectives. The first is to synthesize, using a family of Lyapunov functions, a family of N bounded robust nonlinear feedback control laws of the general form

$$u_i \quad = \quad -k_i(V_i, u_i^{max}, \theta_{bi})(L_{G_i}V_i)^T, i = 1, \cdots, N \tag{3}$$

that: 1) enforce robust asymptotic stability, for their respective closed-loop subsystems, and 2) provide, for each mode, an explicit characterization of the set of admissible initial conditions starting from where this mode is guaranteed to be stable in the presence of model uncertainty and input constraints. The gain, $k_i(\cdot)$, of the L_GV controller in Eq.3 is to be designed so that $|u_i| \leq u_i^{max}$ and the energy of each mode, as captured by V_i, is monotonically decreasing whenever that mode is active. The second objective is to construct a set of robust switching laws that supply the supervisor with the set of switching times that guarantee stability of the constrained uncertain switched closed-loop system.

3. A robust hybrid control strategy

This section contains the main result of this paper. Theorem 1 below provides the formulae for the family of bounded robust feedback controllers together with the appropriate switching rules that guarantee the desired properties in the constrained switched closed-loop system. The proof of this theorem is omitted due to space limitations.

Theorem 1: *Consider the switched uncertain nonlinear process of Eq.1 under the following family of bounded nonlinear feedback controllers:*

$$u_i \quad = \quad -k_i(V_i, u_i^{max}, \theta_{bi}, \chi_i)(L_{G_i}V_i)^T, \; i = 1, \cdots, N \tag{4}$$

where

$$k_i(\cdot) \quad = \quad \left\{ \begin{array}{cc} \dfrac{L_{f_i}^*V_i + \sqrt{(L_{f_i}^*V_i)^2 + (u_i^{max}|(L_{G_i}V_i)^T|)^4}}{(|(L_{G_i}V_i)^T|)^2[1 + \sqrt{1 + (u_i^{max}|(L_{G_i}V_i)^T|)^2}]} & , |(L_{G_i}V_i)^T| \neq 0 \\ 0 & , |(L_{G_i}V_i)^T| = 0 \end{array} \right\} \tag{5}$$

$$L_{f_i}^*V_i \quad = \quad L_{f_i}V_i + \chi_i|(L_{W_i}V_i)^T|\theta_{bi} \tag{6}$$

V_i is a robust control Lyapunov function for the i-th subsystem and χ_i is a tunable parameter that satisfies $\chi_i > 1$. Let $\Omega_i^*(u_i^{max}, \theta_{bi})$ be the largest invariant set embedded within the region described by the inequality

$$L_{f_i}V_i + \chi_i|(L_{W_i}V_i)^T|\theta_{bi} \quad \leq \quad u_i^{max}|(L_{G_i}V_i)^T| \tag{7}$$

and assume, without loss of generality, that $x(0) \in \Omega_i^*(u_i^{max}, \theta_{bi})$ for some $i \in \mathcal{I}$. If, at any given time T, the following conditions hold:

$$x(T) \quad \in \quad \Omega_j^*(u_j^{max}, \theta_{bj}) \tag{8}$$

$$V_j(x(T)) \quad < \quad V_j(x(t_{j_*+1})) \tag{9}$$

for some $j \in \mathcal{I}$, $j \neq i$, where t_{j_*+1} is the time when the j-th subsystem was last switched out, i.e. $\sigma(t^+_{j_*+1}) \neq \sigma(t^-_{j_*+1}) = j$, then setting $\sigma(T^+) = j$ guarantees that the switched closed-loop system is asymptotically stable.

Remark 1: The robust feedback controllers given in Eq.4-5 are synthesized by reshaping the gains of the L_GV controllers proposed in (El-Farra and Christofides, 2001a) (see also (Lin and Sontag, 1991)), in order to account for the uncertain variables. This procedure allows us also to obtain an explicit expression, via Eq.7, that can be used to characterize the limitations imposed by uncertainty and constraints on the stability regions of the constituent modes (see (El-Farra and Christofides, 2001b) for details on how to construct the stability regions). In the absence of uncertainty, with $\theta_{bi} = 0$, the controllers of Eq.4 together with the expressions for the stability regions, Ω_i^*, reduce to those developed in (El-Farra and Christofides, 2001a).

Remark 2: The switching rules of Eq.8-9 determine, implicitly, the times when switching from mode i to mode j is allowed. The first rule tracks the temporal evolution of the continuous state, x, and requires that, at the switching time, the continuous state, x, be inside the stability region, $\Omega_j^*(u_j^{max}, \theta_{bj})$, associated with the target mode. This ensures that, once this mode is activated, its Lyapunov function continues to decay for as long as the mode remains engaged. Note that this condition applies at every time that the supervisor considers switching from one mode to another. In contrast, the second switching rule of Eq.9 applies only when the target mode j has been previously activated. In this case, Eq.9 requires that the gain in energy, V_j, from the last "switch out" to the current "switch in" be less than unity. This guarantees that by the time a given mode is re-activated, its energy has not been adversely impacted by the other modes.

Remark 3: For the case when the uncertain variables are non-vanishing, i.e. $W_i(0) \neq 0$ in Eq.1 (see section 4 for an example), it can be shown that, under the conditions of Theorem 1, the trajectories of the switched closed-loop system remain bounded and converge, in finite time, to an arbitrarily small neighborhood around the origin.

4. Application to a switched chemical reactor

Consider a continuous stirred tank reactor where an irreversible first-order exothermic reaction of the form $A \xrightarrow{k} B$ takes place. The reactor has two inlet streams, the first of which continuously feeds pure A at flow rate F, concentration C_{A0} and temperature T_{A0}, while the second has a valve that can be turned on or off, depending on operational requirements. When the valve is on, the second stream feeds pure A at flow rate F^*, concentration C^*_{A0} and temperature T^*_{A0}. Under standard modeling assumptions, the mathematical model for the process takes the form:

$$\begin{aligned} V\frac{dC_A}{dt} &= F(C_{A0} - C_A) + \sigma(t)F^*(C^*_{A0} - C_A) - k_0 e^{\frac{-E}{RT}} C_A V \\ V\frac{dT}{dt} &= F(T_{A0} - T) + \sigma(t)F^*(T^*_{A0} - T) + \frac{(-\Delta H_r)}{\rho c_p} k_0 e^{\frac{-E}{RT}} C_A V + \frac{Q}{\rho c_p} \end{aligned} \quad (10)$$

where C_A denotes the concentration of A, T denotes the reactor temperature, Q denotes the rate of heat input to the reactor, V denotes the reactor volume , k_0, E, ΔH denote the pre-exponential constant, the activation energy, and the enthalpy of the reaction, c_p and ρ, denote the heat capacity and density of the fluid in the reactor. The process parameters and steady state values for this example can be found in (El-Farra and Christofides, 2001a) $\sigma(t)$ is a discrete variable that takes a value of zero when the valve of the second inlet stream is closed and a value of one when the valve is open. Initially, it is assumed that the valve is closed (i.e. $\sigma(0) = 0$). During reactor operation, however, it is desired to open this valve and feed in more reactant material through the second inlet stream (i.e. $\sigma = 1$) in order to enhance the product concentration leaving the reactor.

The above requirement clearly gives rise to two distinct operational modes for the CSTR, between which switching is needed. These modes correspond to the off($\sigma = 0$)/on($\sigma = 1$) conditions of the valve of the second inlet stream. Since the initial operating mode (with $\sigma = 0$) has an open-loop unstable steady-state that corresponds to $T = 395$ K, our control objective will be to stabilize the reactor temperature at this point by manipulating the rate of heat input. However, since switching to the "valve on" mode (with $\sigma = 1$) at some later point in time can potentially disturb the process and cause instability, our switching objective will be to carry out the transition between the two modes at the earliest "safe" time that does not jeopardize process stability. The control and switching objectives are to be accomplished in the presence of: 1) hard constraints on the manipulated input, $|Q| \leq 80$ KJ/min, 2) time-varying external disturbances in the feed temperature of both inlet streams, and 3) time-varying parametric uncertainty in the enthalpy of reaction. For the purpose of simulating the effect of the uncertainty on the process, we consider time-varying functions of the form

$$\theta_k(t) = \theta_{bk} sin(3t) \tag{11}$$

where the upper bounds, θ_{bk}, on the feed temperature disturbances are taken to be 10 K for both streams, and the upper bound on the uncertainty in the enthalpy is taken to be 15% of the nominal value.

To accommodate both the control and operational objectives, we follow the strategy proposed in Theorem 1. With two quadratic Lyapunov function of the form $V_i = \frac{1}{2}c_i(T - T_s)^2$, where $c_i > 0$, we initially use Eq.4 to synthesize two bounded robust controllers, one for each mode, that enforce robust closed-loop stability for their respective modes, and also achieve an arbitrary degree of attenuation of the effect of uncertainty on the reactor temperature. Then, with the aid of Eq.7, we compute the region of guaranteed closed-loop stability associated with each mode, which will be needed to implement the necessary stabilizing switching laws. Details of these computations are omitted due to space limitations and will be presented at the conference.

Several closed-loop simulations were performed to evaluate the proposed control strategy. Figure 1 depicts the controlled output and manipulated input profiles when the reactor is operated in the $\sigma = 0$ mode and no switching is involved. We observe that the controller for this mode successfully stabilizes the reactor temperature at the desired steady-state and simultaneously attenuates the effect of disturbances and model uncertainty on the reactor temperature. In order to increase the product concentration, however, we decide to switch to the $\sigma = 1$ mode at some point. Without using the switching laws of Theorem 1, suppose that we set the switching time to be as early as $t = 21$ min. The result is shown in Figure 2 (dashed lines). It is clear that by switching at this arbitrarily chosen time, the controller for the $\sigma = 1$ mode is unable to robustly stabilize the reactor temperature at the desired steady-state. The reason is that at this time, the state of the system is still outside the stability region for the $\sigma = 1$ mode and therefore, the available control action is insufficient to stabilize the temperature, as can be seen from the input profile. To avoid this instability, we use the switching scheme proposed in Theorem 1. In this case, we start the reactor in the $\sigma = 0$ mode and switch to the $\sigma = 1$ mode only when the condition of Eq.8 is satisfied (note that the condition of Eq.9 is not needed here since the $\sigma = 0$ mode is never re-activated). The controlled output and manipulated input profiles for this case are depicted by the solid lines in Figure 2 which show that the controllers successfully drive the reactor temperature to the desired steady state while attenuating the effect of uncertainty. Switching becomes safe after about 24 minutes of startup.

5. References

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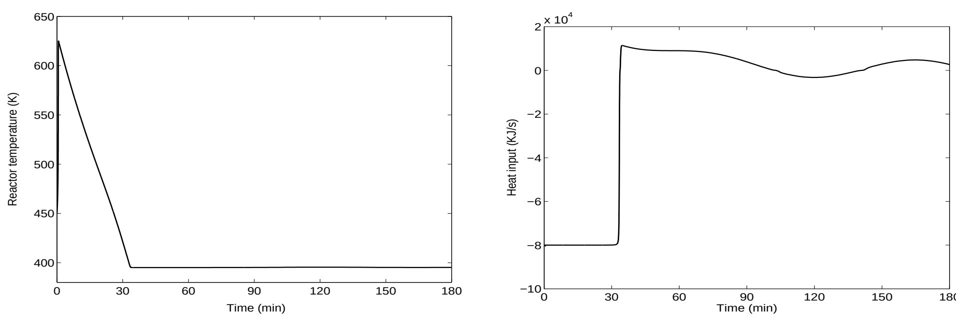

Figure 1: Controlled output and manipulated input profiles with valve closed ($\sigma = 0$).

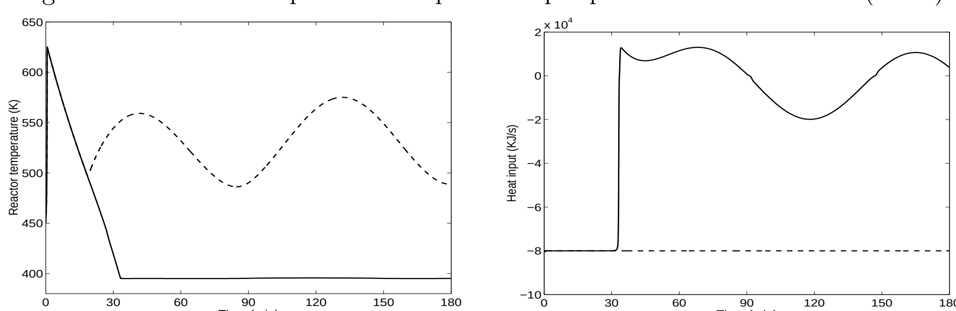

Figure 2: Controlled output and manipulated input profiles when valve opened at $t = 21$ min (dashed) and when valve opened at $t = 24$ min using the switching laws of Theorem 1 (solid).