## A MILP Decomposition Approach for the Risk Management within A Flexible Recipe Framework

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## 1. Summary

The aim of the present work is to provide a tool to support the scheduling under uncertainty of batch chemical processes taking full advantage of a flexible recipe framework. To address this problem, a large scale multi-objective stochastic Mixed-Integer-Linear-Programming (MILP) model is presented. This model is based on the general precedence model and explicitly includes the trade-off between risk and profit. The outcome of such formulation consists of a set of Pareto-optimal solutions from which the decision-maker should choose the best one according to his/her preferences. A decomposition strategy based on the sampling average approximation (SAA) has been studied as a way of overcoming the numerical difficulties associated with the application of the proposed strategy to large scale industrial scenarios.

Keywords: demand uncertainty, risk management, flexible recipe, sample average algorithm

## 2. Extended Abstract

Batch processes have received great attention over the last years because of their higher flexibility compared to continuous processes and the increasing demand for specialty, high added-value chemical and pharmaceutical products. Within this context, the short-term scheduling deals with the optimal allocation of a set of scarce plant resources over time to manufacture one or more products following a batch recipe. Most of the scheduling approaches assume that batch processes are operated at nominal conditions following predefined fixed production recipes. However, in many cases a flexible recipe operation may be a suitable way of incorporating systematic recipe adaptations depending on the actual process conditions (Romero et al., 2003). Furthermore, the complexity of the scheduling problem is increased by the high degree of uncertainty brought about by external factors, such as continuously changing market conditions and customer expectations, and internal parameters, such as product yields, qualities and processing times. Although it has been widely

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recognized the importance of incorporating uncertainties in the scheduling formulations, most of these models are deterministic, i.e. they assume that all the problem data is known in advance. Thus, the accuracy of the solutions generated using deterministic models may depend on the degree of uncertainty. Moreover, most of the stochastic models devised to date to address scheduling under uncertainty optimize the total expected performance measure, and do not provide any control on its variability over the different scenarios. That is to say, they assume that the decision-maker is risk neutral. However, different attitudes towards risk may be encountered (Bavaro and Bagajewicz, 2004).

The aim of the present work is to provide a tool to support decision making during the development of a scheduling policy in an uncertain market environment while incorporating the trade-off between risk and profit at the decision level. To achieve our goal, this work propose an efficient MILP-based framework that manages the risk in the decision-making strategies by incorporating as an additional feature the flexibility of the batch processes recipes is presented. The problem is mathematically posed as a multi-scenario multi-objective two-stage stochastic model, which accounts for the maximization of the expected profit and minimization of a risk measure. The former metric is indeed appended to the objective function to allow controlling its variability over the entire range of scenarios. In our stochastic formulation, the decisions associated with the scheduling tasks are represented by first-stage variables whose value must be determined before the uncertainty is unveiled. On the other hand, the sales are computed once the uncertain events take place at the end of the time horizon. The resulting model suffers from the "curse of dimensionality" since it is indeed very sensitive to the number of scenarios considered. To overcome the numerical difficulties associated with such mathematical formulation, decomposition strategy based on the Sample Average Approximation (SAA) is introduced. This decomposition technique provides near optimal solutions and incurs in much less CPU time than the monolithic formulation. The main advantages of our approach are highlighted through a case study, in which a set of solutions appealing to decision makers with different attitudes toward risk are obtained. Moreover, the convenience of exploiting the capabilities of the flexible recipe framework as a way of hedging the financial risk associated with the batch process operation is also discussed through comparison with the traditional approach which operates at nominal conditions.

Financial support received from the European Community projects (MRTN-CT-2004-512233; INCO-CT-2005-013359), the Departament d'Educació i Universitats de la Generalitat de Catalunya and the European Social Fund is fully appreciated. Gonzalo Guillén-Gosálbez expresses also his gratitude for the financial support received from the Fulbright/Spanish Ministry of Education and Science visiting scholar program.

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