On interactions of stationary particles and rising bubbles

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Abstract

The attachment of the bubbles onto a collecting surface or sphere (solid, air bubble and liquid droplet) is a ubiquitous natural phenomenon, playing a critical role in numerous practical processes. The goal of this project is to broaden our knowledge about hydrodynamic interactions between bubbles and solids of comparable size that occur in the flotation process. We studied the collision process between a single rising bubble and stationary spherical particle. The experimental research was focused on the description of a bubble trajectory, collision efficiency and the determination of the limit collision angle. A high-speed digital camera was used for the experimental documentation. According to the obtained results, the trajectory deviation caused by the presence of a stationary solid spherical particle is very small. No collision occurs for initial angles greater than 75°. A good starting point for theoretical description of interaction between a small bubble and a large particle is the study of a bubble collision with an inclined plane, representing a particle of infinite size. The model theory was created for this type of interaction. The bubble trajectory before and during the interaction with the inclined plane was calculated and compared with experimental data. The agreement between the theoretical and measured data is good.

Keywords: bubble - particle interaction, collision efficiency, bubble trajectory, plastics flotation

1. Introduction

The fundamental process in flotation is the formation of bubble – particle aggregate, which is formed due to the interaction between bubbles and particles. Originally developed in the mining industry to recover valuable minerals from mined ore deposits, flotation employs air bubbles as carriers to recover hydrophobic entities from the complex slurry. Due to its high separation efficiency, cost effectiveness and simplicity in operation and maintenance, flotation has been extended to other industries dealing with solid-solid and solid-liquid separations. This includes bitumen recovery from oil sands, de-inking in paper-recycle pulp, de-oiling in heavy-oil exploration, plastics separation and recycling, toxic effluent treatment in chemical and mining industry, and fine solid removal in industrial and domestic water treatment, just to name a few. The exploitation of flotation in plastics separation is growing in line with the need of their recycling. In contrast with the previous efforts stemmed from the needs of mineral flotation aimed at particles much smaller than bubbles, our research activities are aimed at the investigation of behaviour of particles greater or comparable in size with bubbles, which has not been satisfactorily covered yet. To simplify the problem, we started our research with interactions between stationary spherical particle and single rising bubble.

The bubble – particle interaction can be described by three independent steps (Nguyen and Schulze, 2004):

- collision - approach of the bubble and the particle to the contact distance and the subsequent collision

- attachment - adhesion of the bubble on the particle surface

- stability - detachment of the bubble from the particle surface, when exists an instability of the bubble – particle aggregate

Theoretical models assume that particle-bubble collision occurs over the section of the particle surface between angles $\varphi=0$ and $\varphi=\varphi_{c,max}$, where the angle φ is measured from the front stagnation point of the bubble, and $\varphi_{c,max}$ is the maximum possible collision angle. The initial angle φ_0 is formed by the vertical axis of the stationary particle and the connecting line between the particle centre and the bubble centre if no trajectory change is observed. This angle characterizes the bubble position in sufficient distance from the solid particle. The collision angle φ_c is formed by the vertical axis of the stationary particle and the line connecting the particle centre and the bubble centre at the collision point. Finally, the adhesion angle φ_a is formed by the vertical axis of the stationary particle and the line connecting the particle centre and the bubble centre when the bubble is adhered on the particle. In this setup the three-phase contact area has a fixed and stable perimeter; this angle φ_a characterizes the stable bubble position on the solid surface. The bubble trajectory around the stationary spherical solid particle is shown in Figure 1.

For the simulation and analysis of the bubble-particle interaction processes, a measure of the effectiveness of the processes is required. The effectiveness of the whole process is often represented by the efficiency of each step,

$$E = E_c E_a E_s \qquad [1]$$

Here E, E_c, E_a, and E_s, are the capture, collision, attachment, and stability efficiencies, respectively. The effectiveness of each step is defined as the ratio of the actual to the ideal rates (numbers) of the particles participating in the respective interaction process (Dai et al., 1998; Dai et al., 1999; Wang.et al., 2003) and can be mathematically described as follows:

$$E_j = \frac{N_r}{N_i} \quad [2]$$

Here, r and i stands for actual (real) and ideal number of particles. Efficiency of encounter (collision) E_c, attachment E_a and detachment E_d yields:

$$E_c = \frac{N_{cr}}{N_{ci}} \quad [3]$$

$$E_a = \frac{N_{ar}}{N_{cr}} \quad [4]$$

$$E_d = 1 - \frac{N_{col}}{N_{ar}} \quad [5]$$

N_{col} is the number of particles collected by the bubble. The relations $N_{cr} = N_{ai}$ and $N_{ar} = N_{di}$ are used here. All these relations were originally developed for situations where bubble diameter is greater then particle diameter. For the inverse configuration, the intent remains identical.

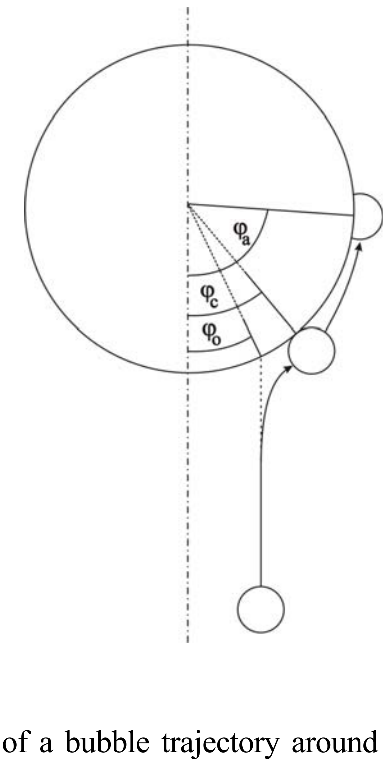

Fig. 1: Schematic drawing of a bubble trajectory around the stationary spherical solid particle; φ_o, φ_c and φ_a are the initial, the collision and the adhesion angles respectively

In this project we primarily focused on the collision process between stationary spherical particle and single rising bubble (Basařová et al., 2006; Hubička et al., 2007). The collision efficiency is defined by eq. 3 and is determined as the ratio of the actual number of bubbles to the ideal number of bubbles encountering the particle. In the ideal encounter, the bubble motion towards the solid surface is not considered to be influenced by the presence of the particle. In this case, only the leading surface of the particle contributes to the ideal bubble-particle encounter since the bubble flux to the rear of the particle does not occur. For the ideal collision number N_{ci} we obtain (Nguyen and Schulze, 2004):

$$N_{ci} = \pi\left(R_p + R_b\right)^2 CV_{rel} \qquad [6]$$

Here, C is the concentration of the bubbles in the liquid phase and V_{rel} is the bubble velocity relative to the particle. The actual encounter is affected by the bubble motion towards the particle surface, which is diverted by the distortion of the liquid flow due to the presence of the particle. Thus, not all the bubbles in the path swept by the encounter the particle. The real encounter number is therefore smaller than the ideal one and the collision efficiency is not larger than one. The encounter efficiency is determined through the location of the so-called grazing trajectory (radius R_c), which is defined to be the trajectory that separates the trajectories that encounter the particle from those that do not. Then, we obtain another expression for the bubble-particle encounter efficiency:

$$E_c = \left\{\frac{R_c}{R_p + R_b}\right\}^2 = \sin^2\left(\varphi_{o,\lim}\right) \qquad [7]$$

Here, $\varphi_{0,\lim}$ is the initial angle which corresponds with the grazing trajectory. This expression has been derived based on a simple geometrical interpretation of the particle-grazing trajectory. It is convenient for the simulation of the bubble-particle encounter efficiency and it provides a possibility for the experimental determination of the encounter efficiency by probing the location of particle grazing trajectories (Nguyen and Schulze, 2004).

A good starting point for theoretical study of interaction between a small bubble and a large particle is the study of a bubble collision with an inclined plane, which represents a particle of infinite size. The bubble adhesion on the plane is controlled by the bubble ability to push out the liquid film, which separates them. The model theory is based on the motion of a single solid sphere. The bubble rises up along the vertical axis. During this motion, the liquid is compressed and expelled upwards and sidelong. The stream function ψ is given as (Nguyen et al., 1997):

$$\psi(r,\theta) = \frac{-3\pi\upsilon_S a}{2k}\left(2 + \frac{3ka}{4}\right)(1+\cos\theta)\left[1 - e^{-kr(1-\cos\theta)}\right] \qquad [8]$$

In this relationship, r,θ are spherical coordinates: r is coordinate and θ is a polar angle. Furthermore, a is a radius of the sphere, ν_{kin} kinematic viscosity and υ_s is velocity of the sphere. Coefficient k is given as

$$k = \nu / 2\nu_{kin} \tag{9}$$

The pressure P around the moving bubble is defined by:

$$P = P_c - \frac{3}{2}\mu\upsilon_S a(1+\frac{3ka}{4})\frac{\cos\theta}{r^3} - \frac{1}{2}\rho\frac{U^2a^3}{r^3}(3\cos^2\theta - 1) \tag{10}$$

Here, P_c is the critical pressure and μ dynamic viscosity.

In a free volume, the liquid is moving only due to the presence of a rising bubble. In the vicinity of a barrier (here, inclined plane with the angle of inclination α) the streamlines are changing. It is possible to imagine the streamlines like some rays. Due to the presence of the barrier thesestreamlines curve. The liquid having velocity $v_{l\infty}$ in the infinity is interacting with the plane. This problem could be solved using a complex potential. For the description of phenomenon occurring during the interaction of bubble with the inclined plane it is more suitable to change the imagination of bubbles. Now the bubble is represented as a cylinder with an equal volume as the sphere. The base of a cylinder is collinear with the inclined plane. Furthermore, the Reynolds number is assumed to be lower than 0,01 and the liquid flow is laminar. The body (cylinder) is moving with the velocity v_0; also the liquid layer between the cylinder base and inclined plane is very thin. A modified Navier – Stokes equation could be obtained:

$$\frac{\partial P}{\partial x} = \mu\frac{\partial^2 v_x}{\partial z^2} \text{ and } \frac{\partial P}{\partial z} = 0 \tag{11}$$

Where P is the pressure which is changed in the direction of horizontal axis x. Then, the pressure on the edge of the cylinder, external force on the rising sphere and the frictional force should be expressed. The detailed description of this procedure is described elsewhere (Horn et al., 2007).

The final equation for the liquid flow Q is

$$Q = \frac{1}{2a}\int_{-a}^{a}\int_{-a}^{a} v_x dydz \approx \frac{v_0}{2a}\left[(h_0 + a)2a - \frac{\pi}{2}a^2{}_0\right] = \gamma v_0\left(h_0 + a - \frac{\pi}{4}a\right) \tag{12}$$

Here, h_0 characterizes the minimal height of the liquid layer.

The aim of this project was to study the interaction between the stationary particle and a single rising bubble. Preferably, the collision process was studied. We focused on the description of initial and collision angles and their mutual dependence. The bubble-particle collision efficiency was evaluated also. The bubble trajectory before and after the interaction with the inclined plane representing a particle of infinite size was measured experimentally and then calculated using the mathematical model.

2. Materials and method

Distilled and de-ionised water at 25°C was used for all measurements. The pH value was 6,13 and conductivity 1,6 μS/cm. For flotation experiments was used a water solution of surface active agent terpineol (supplied by Fluka Company) at concentration 187 mg/l (22 ppm). The surface tensions of distilled water and terpineol solution were 71,6 mN/m and 63 mN/m, respectively. Silanized glass balls with the diameter 6,97 mm, 10,0 mm and 15,0 mm were used for all measurements. These particles were cleaned and degreased in an equimolar mixture of sulphuric acid and hydroperoxide. The silanization was done in (chloromethyl)dimethyl-chlorosilane (supplied by Sigma-Aldrich Company) solution for 25 hours. The value of contact angle was 64^0.

The experimental measurements were done in a special flotation cell (Hubička et al., 2007). The scheme of the experimental apparatus is given in the Figure 2. The air bubbles entered the flotation cell through the bottom using a compressor, air reservoir, reduction valve and precise valve Omega FVL – 404 – 55. Small bubbles were created using the special capillaries with inner diameters 10, 15 and 20 μm. The exposure was realized by cold light source SCHOTT KL 2500 LCD. A high-speed camera Redlake Motion Pro was used for experiment documentation. This camera is able to capture 500 pictures per second in the resolution 1280x1024 pixels, it has variable shutter speed and for our experiments was used 1/1000 sec. The NIS-elements software and programme equipment were employed for image analysis. We measured the bubble size, particle size, bubble velocity and the positions of bubble and particle centres (x, y position).

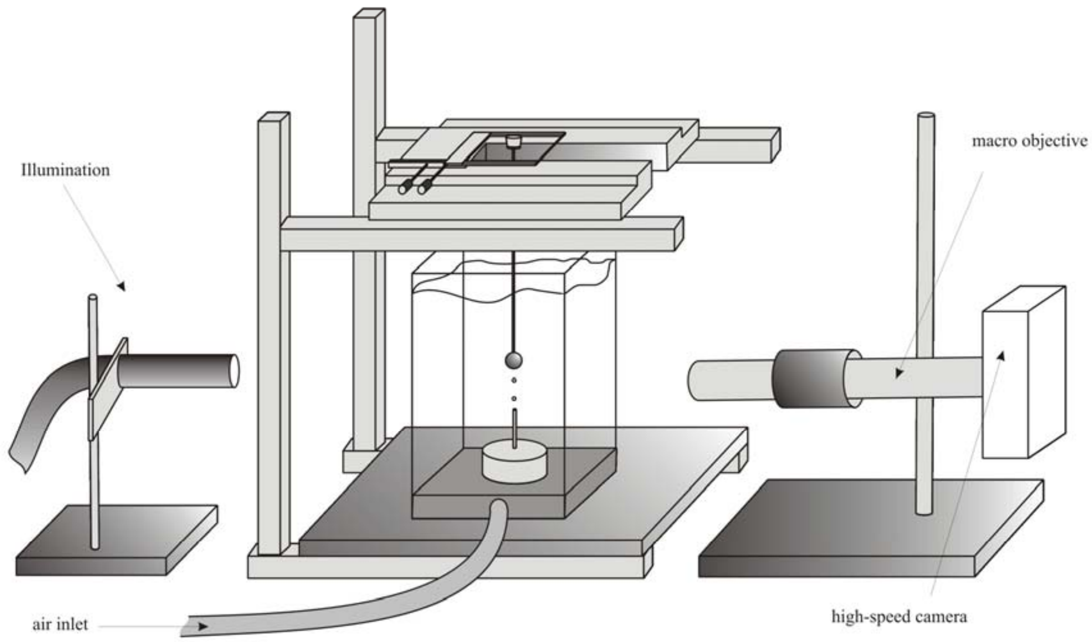

Fig. 2. The scheme of the experimental apparatus

3. Results and discussion

During the experimental measurements the bubble motion was captured to observe the subsequent bubble motion before the interaction with stationary spherical particle and on the surface of this particle. Following parameters were measured: particle radius r_p, bubble radius r_b and bubble rising velocity v_b. Furthermore the bubble centre coordinates (x_i, y_i) were evaluated together with the time information. The time interval between two subsequent images was 0,0020 s. The initial angle φ_0 and collision angle φ_c were evaluated from the captured images. The initial angle φ_0 characterizes the bubble position in big distance from the solid particle (see Figure 1). The collision angle φ_c is higher than φ_0 due to the deviation of the bubble trajectory.

As an example of experimental results, data on a bubble trajectory change while approaching the surface of a stationary spherical particle (d_p = 6,97 mm) are illustrated in the Figure 3. Bubble trajectories (r_b = 0,29 mm) for several initial positions of bubble centre from the vertical axis of the particle are given here. If the bubble is moving directly along the vertical axis, no deviation is observed. The trajectory deviation increases with the growing distance between the bubble centre and the particle vertical axis, thus with increasing initial angle φ_0. The theoretical maximum initial angle is 90^o that coincide with the situation when the distance of the bubble and the particle centres equals $r_p + r_b$.

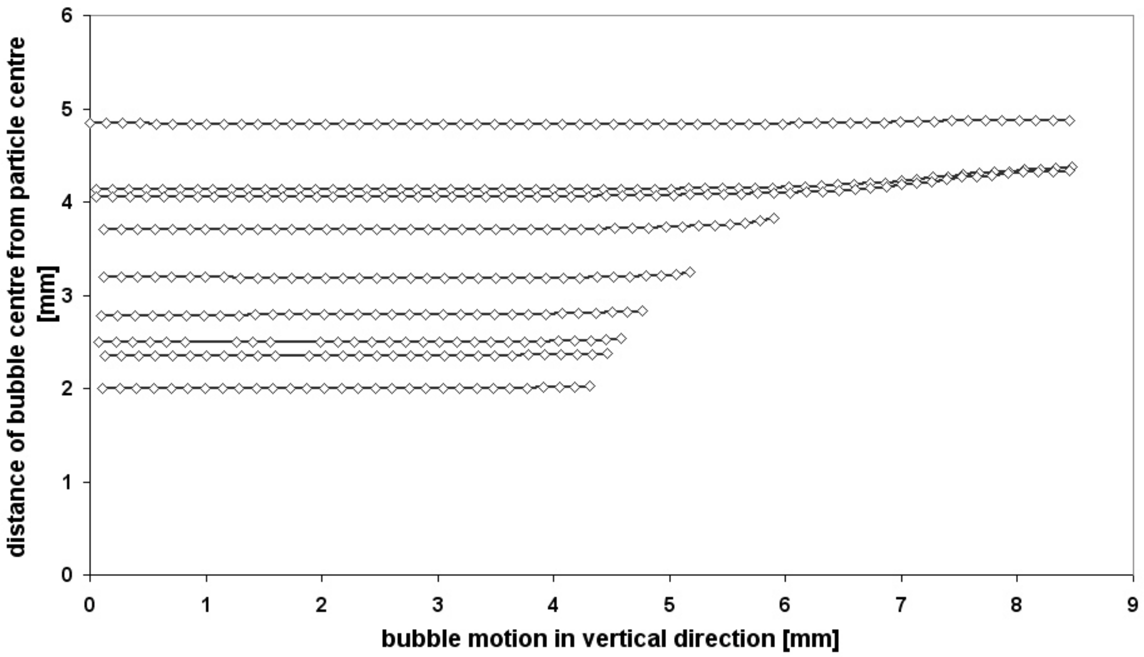

Fig. 3: The trajectory deviation of the bubble centre for several initial positions of bubble centre (r_b = 0,29 mm) from the vertical axis of the particle (d_p = 6,97 mm), turned over 90^o.

The change of bubble trajectory is not significant, because around stationary particle we consider narrow hydrodynamic zones. In case of small bubbles (r_b < 0,5 mm), only very small stream field could be observed. Therefore, the streamline influences the bubble motion immediately before an interaction with the particle. This fact could be illustrated if the dependence between initial angle and collision angle is plotted. The experimental data are given in the Figure 4. All data for different bubble sizes and 3 particle sizes (d_p = 6,97 mm, 10,0 mm and 15,0 mm) are gathered here.

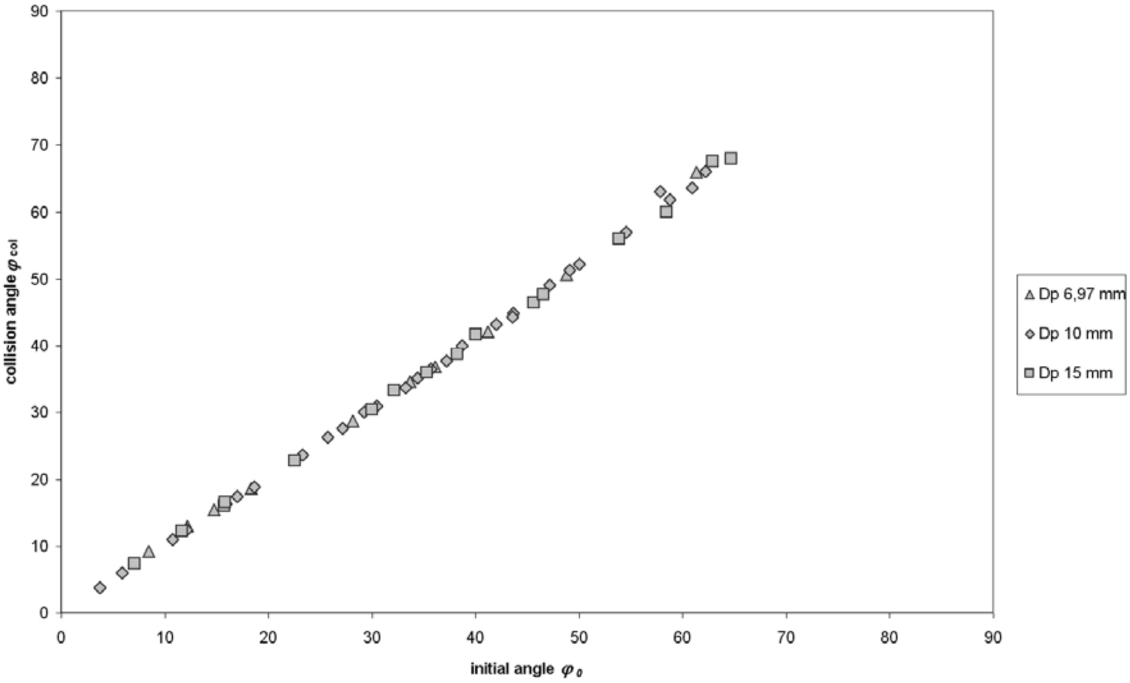

Fig. 4: The dependence of the collision angle on the value of the initial angle

A linear dependence of the parameters is obvious from the measured experimental data. The proportion constant is equal to 1,04. The trajectory deviation caused by the presence of a stationary solid spherical particle is very small and it is observed only in the immediate vicinity of the surface. These results are in good correspondence with theoretical assumptions. The solid particle is stationary and the liquid medium does not move around it. The bubbles are generated with sufficient distance preventing following mutual interference. Unfortunately, the obtained results could not be compared with any published data. Sutherland's model (Schulze, 1984) as well as generalized Sutherland model (Dai et al., 1998; Dai et al., 1999) requires motion of both bubble and particle. Only Wang (Wang et al., 2003) studied interactions between stationary bubble and smaller falling particle. In our study, however, the diameters of the particle and the bubble differ significantly (Wang: $d_b/d_p \equiv 1000$, here $d_b/d_p \equiv 0{,}1$).

According to our experimental results, no collision occurs for initial angles greater than 75°. For such initial angles the streamlines deviate significantly from the linear direction and consequently the distance between the particle and bubble centre is greater than r_b+r_p during whole bubble motion. The encounter efficiency is defined by eq. [7] using the limit initial angle $\varphi_{lim,o}$. This initial angle is identical with the angle which corresponds to the grazing trajectory defined as the trajectory separating trajectories that encounter the particle from those that do not. Therefore, the limit initial angle is the maximal angle when the bubble encounters the particle. The detailed visualization near the grazing trajectory is illustrated in the Figure 5. Here, the particle diameter was 10 mm and bubble diameter 0,9 mm. On this diagram are given several bubble trajectories with different initial position of bubble centre with respect to the particle vertical axis. Bigger points illustrate the situation when the bubble interacted with the particle surface.

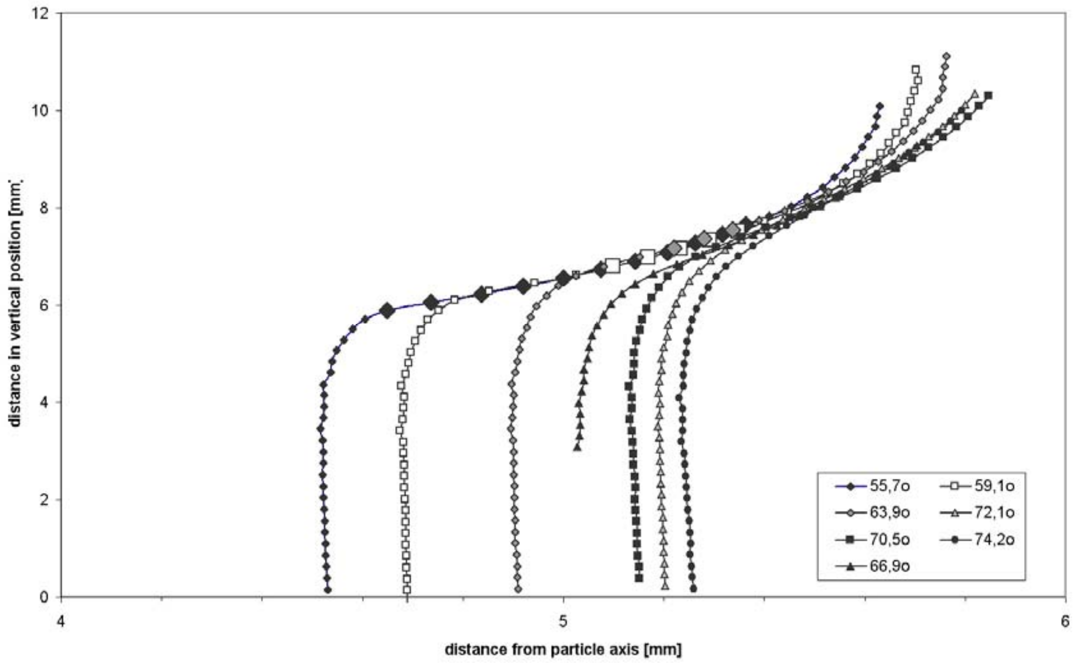

Fig. 5: Bubble trajectories with different initial position of bubble centre with respect to the particle vertical axis (d_p = 10 mm, d_b = 0,9 mm)

Experimentally measured collision efficiencies and their dependence on bubble size are given in the Figure 6. According to our observations there is no significant influence of the particle size. This fact is valid for situation where $d_b/d_p \cong 0{,}1$. For smaller bubbles (d_b < 0,7 mm) collision efficiency does not change significantly and its average value is 0,91. For bigger bubbles having higher rise velocity the collision efficiency decreases with increasing bubble size. This observation is in accordance with theoretical assumptions (Nguyen and Schulze, 2004). When the classical flotation is considered, the size of solid particles is much lower than the bubble size. Usually, the bubble diameters are considered in millimetres and particle diameters in micrometers. Furthermore, the motion of bubble and particle is usually considered during the deduction of model theories. Most of the authors substitute particles with mass points (Schulze, 1984; Nguyen and Schulze, 2004; Dai et al., 1998; Dai et al., 1999; Dai et al., 2000; Wang et al, 2003). All relations expressing encounter efficiency take into account these considerations. The ratio d_p/d_b occurs in all equations. In classical flotation this ratio is much lower than 1. In our arrangement is $d_p/d_b>1$. Therefore, it is not possible to use published theoretical relations for efficiency calculation otherwise the efficiency will be grater than 1.

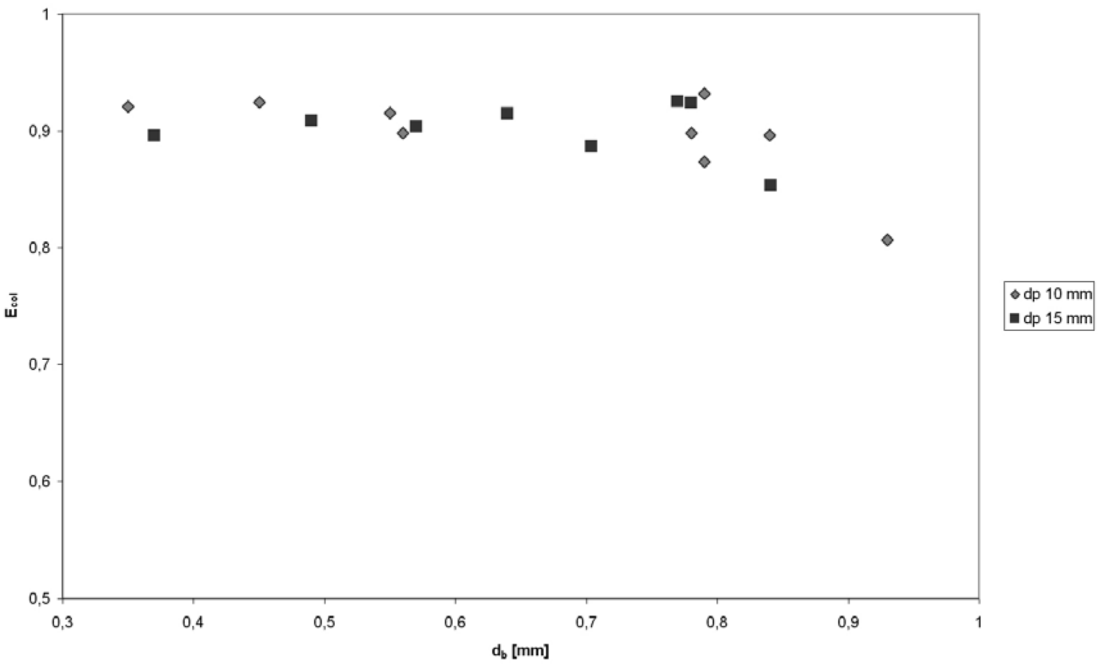

Fig. 6: Collision efficiencies and their dependence on bubble size

During the experiments the interaction between single bubble and inclined plane was studied also. The bubble diameter ranged from 0,4 to 0,9 mm. For identical experimental arrangement (bubble size, angle of inclination) the bubble trajectory was calculated using the theoretical model (eq. 12). Selected results (d_b = 0,6 mm) are illustrated in the Figure 7. The experimental bubble trajectory before an interaction is highlighted with empty points and the theoretical trajectory is demonstrated with the full line. The last point corresponds to the interaction with the inclined plane. The theoretical model is capable to determine the point where the plane begins influence the bubble motion. From this distance the bubble trajectory is deviated. The agreement of experimental and theoretical data is excellent for interactions where the inclination angle was 13.5^o and 19.5^o. For higher inclination angle the calculated deviation of bubble trajectory is higher then the experimentally measured one.

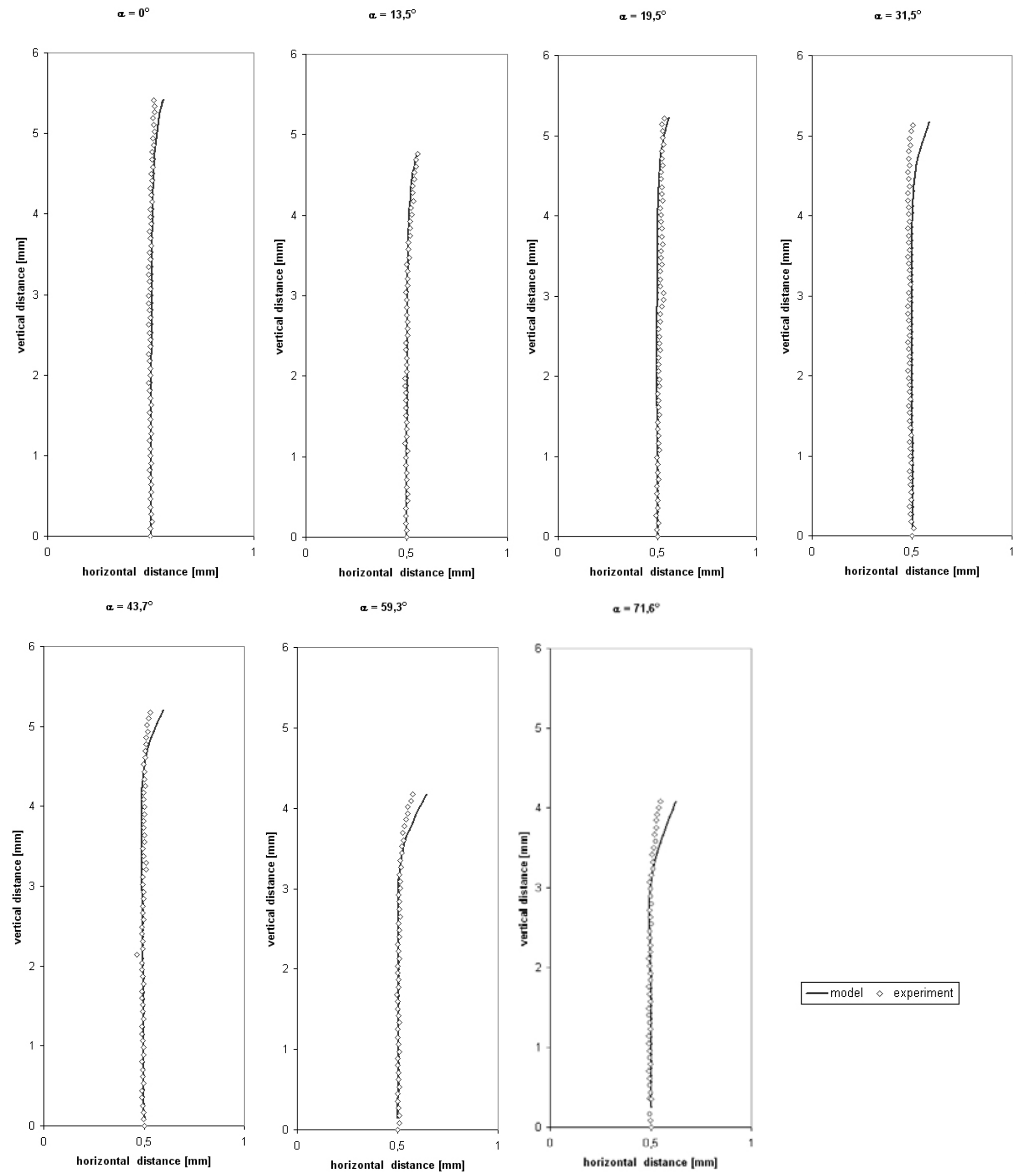

Fig. 7. Trajectory of a rising bubble and its interaction with the inclined plane (inclination angles 0^o – horizontal position, 13.5^o, 19.5^o, 31.5^o, $43{,}7^o$, $59{,}3^o$ and $71{,}6^o$, respectively). Comparison of experimental and theoretical data.

4. Conclusions

This paper was focused on the study of interaction between bubble and solid particle. We started our research with interactions between stationary spherical particle and single rising bubble. In our experiments, we studied the bubble motion and collision of the bubble. We found that the trajectory deviation caused by the presence of a stationary solid spherical particle is very small. This is observed only in the immediate vicinity of the particle surface. No collision occurs for initial angles greater than 75°. For such initial angles the streamlines deviate significantly from the linear direction and consequently the distance between the particle and bubble centre is greater than r_b+r_p during whole the bubble motion. For smaller bubbles (d_b < 0,7 mm) collision efficiency does not change significantly and its average value is 0,91. For bigger bubbles (d_b > 0,7 mm) the collision efficiency slightly decreases with increasing bubble size. No significant influence of particle size was observed.

Acknowledgements

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