Controlling of chaos in the process of crystallization of dibasic lead phosphite

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Abstract

In this paper three algorithms of control of chaos (destohastization, control with proportional feedback and control with time-delayed feedback) in the process of continuous mass crystallization of dibasic lead phosphite are offered.

The first method involves a corrective action in compliance with the required values of the dynamic variables and, therefore, involves a feedback as a necessary component of the system. An algorithm based on the Poincare cross section was developed by Ott, Grebogi, and Yorke, which is referred to as the acronym of their names (the OGY algorithm). For today the OGY-method has quite a number of modifications.

The second attractive for scientists method is the time-delayed feedback one. It was offered in 1992 by the Lithuanian physicist K. Pyragas. The main advantage of his idea is the continuity of the method. In other words, when one uses the OGY-method and its modifications the algorithm starts working when a system gets, for example, to the given area of an unstable fixed point, that takes some time, but when using the Pyragas' method the algorithm is supposed to be switched on any time when it is necessary due to a feedback principle. The efficiency of Pyragas' method and its modifications for the physical, chemical and a number of other systems was demonstrated.

Another approach to the stabilization of the stochastic behavior of dynamic systems involves external disturbances without feedback. This method of suppression of chaos is referred to as the destochastization method.

We show the possibility of using of each method for controlling chaos in the process of continuous mass crystallization of dibasic lead phosphate.

Keywords: chaos, feedback, time-delayed feedback, crystallization, OGY-method

1. Introduction

In this paper the results of computational modeling of chaos control in the process of continuous mass crystallization of dibasic lead phosphite are offered.

The first method involves a corrective action in compliance with the required values of the dynamic variables and, therefore, involves a feedback as a necessary component of the system. An algorithm based on the Poincare cross section was developed by Ott, Grebogi, and Yorke [1], which is referred to as the acronym of their names (the OGY algorithm). For today the OGY-method has quite a number of modifications, e.g. [2] - [7].

The second attractive for scientists method is the time-delayed feedback one. It was offered in 1992 by the Lithuanian physicist K. Pyragas [8]. The main advantage of his idea is the continuity of the method. In other words, when one uses the OGY-method and its modifications the algorithm starts working when a system gets, for example, to the given area of an unstable fixed point, that takes some time, but when using the Pyragas' method the algorithm is supposed to be switched on any time when it is necessary due to a feedback principle. The efficiency of Pyragas' method and its modifications for the physical, chemical and a number of other systems was demonstrated [10]-[20].

Another approach to the stabilization of the stochastic behavior of dynamic systems involves external disturbances without feedback. This method of suppression of chaos is referred to as the destochastization method. The theorem was proved that, for a set of one-parametric quadratic maps of logistic type:

$$x_{j+1} = \lambda x_j (1 - x_j), \tag{1}$$

there is a periodic parametric disturbance transferring map (1) from the set of stochastic maps to a set of regular maps. This theorem suggests that there are ways of chaos control without feedback. The concept of regions of parameters in which chaos can be controlled without feedback makes it possible to search for a representation of the controlling parameter in such a manner as to suppress stochastic oscillations. The most frequently used representation is

$$\lambda = \lambda_0 + \lambda_1 \cdot \sin\left(\frac{kw}{T}\right). \tag{2}$$

where λ_0 is the value of the controlling parameter of map (1) in the region of chaos ($|\lambda_1| << \lambda_0$) and w is the frequency. It was shown that the frequency w should be taken from intervals of intermittency in chaos.

2. Control with proportional feedback

Let's demonstrate the efficiency of control with proportional feedback concerning the logistic map (1). This method is a modification of the OGY-algorithm and it was worked out by authors [21].

The essence of the algorithm of proportional feedback control can be formulated as follows. Since the system behaves chaotically, at a certain instant of time, it will find itself in the vicinity of a stationary unstable point x_s (Fig. 1). If the controlling parameter of the system is changed at this instant of time, beginning from the next step in time, the state of the system will be directed toward the stationary unstable point (of the cycle of period-1) x_s (Fig. 1).

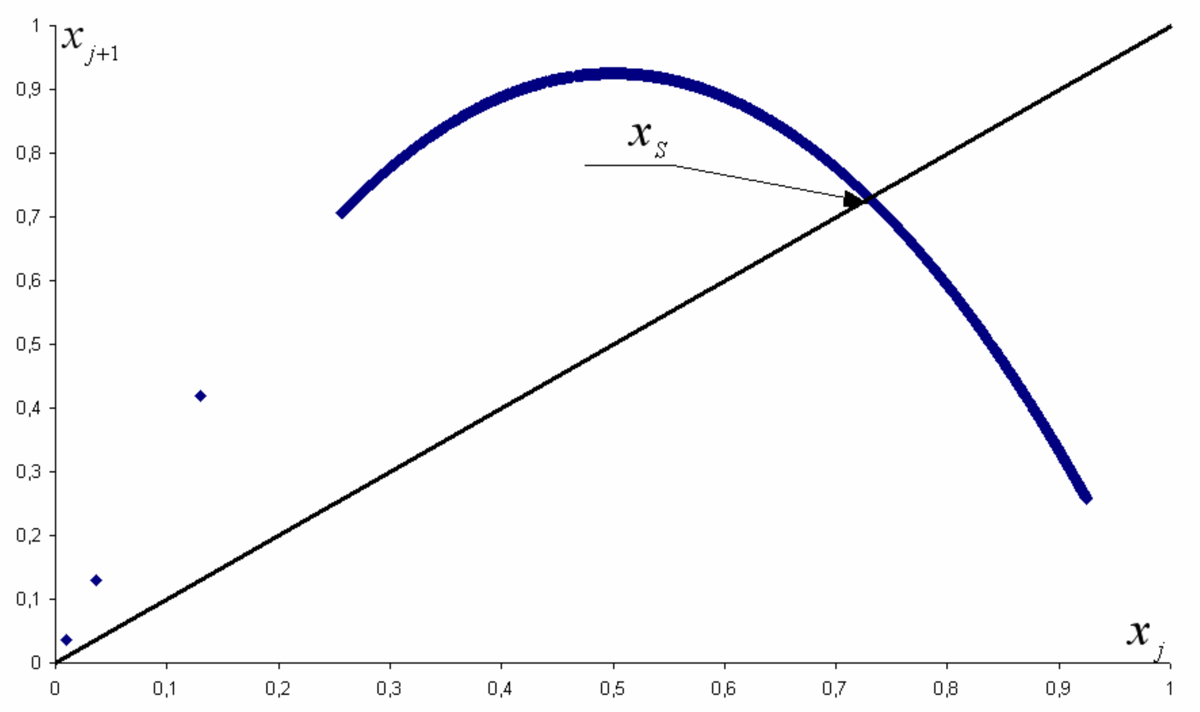


Fig. 1. Quadratic map of the logistic equation $x_{j+1} = \lambda x_j(1 - x_j)$, where x_s is the stationary unstable point.

Let's consider the stabilization of the cycle of period-1 of the system (1) if the λ = 3.7. Uncontrolled behavior of the system (1) is shown in Fig.2 and the result of the control is shown in Fig. 3a.

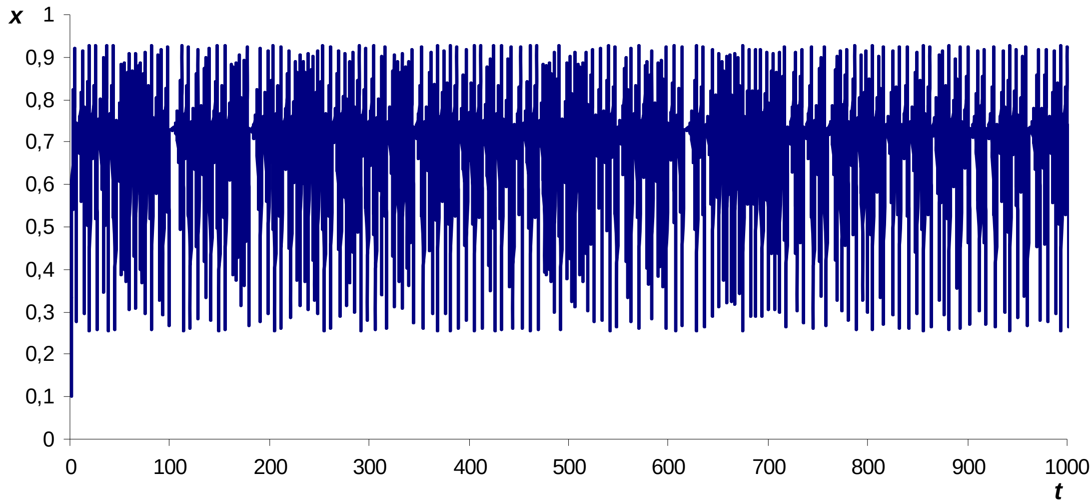


Fig. 2. The behavior of the system (1) at a value of controlling parameter λ =3.7.

The system got into the given neighborhood of the fixed point at t = 100. Then the algorithm of the control was switched on and the system was stabilized near the fixed point x_s = 0.7297 at t = 118. The control was specially switched off at t = 600 and the system came back to the chaotic behavior.

3. Control with time-delayed feedback

In this part, the application of the extended method of time-delayed feedback [22], which is the modification of Pyragas' method to control chaotic oscillations is considered. The method is offered by E.S. Socolar, D.W. Sukow, D.J. Gauthier in 1994 and gives more possibilities in comparison with the [8] at the expense of using the information about previous states of the system.

According to [22] the controlled logistic map (for the stabilization of cycle of period-1) could be presented as

$$x_{j+1} = (\lambda_0 + \varepsilon^*) x_j (1 - x_j), \tag{3}$$

where

$$\varepsilon^* = \gamma(x_j - x_{j-1}) + R\varepsilon_{j-1} \tag{4}$$

γ, R – the parameters of a time-delayed feedback function.

Let's demonstrate the work of the extended method of time-delayed feedback for stabilization of the logistic map (1) at a value of the controlling parameter λ = 3.7. (Fig. 3b). As we can see, the system demonstrates the chaotic behavior till an iteration 50. Then, the control, realizing an algorithm of the extended method of time-delayed feedback was switched on, and the stabilization of cycle of period-1 has occurred (unstable fixed points are x_s=0.7297). The values of the parameters of the time-delayed feedback function are γ= 3.5, R = 0.5. It is clear that the extended method of time-delayed feedback is more efficiency that the proportional feedback one.

4. Control with destohastization algorithm

The method of destohastization gives possibility of suppression of chaotic oscillations without using of feedback. The following regularity are using for suppression of chaos more often

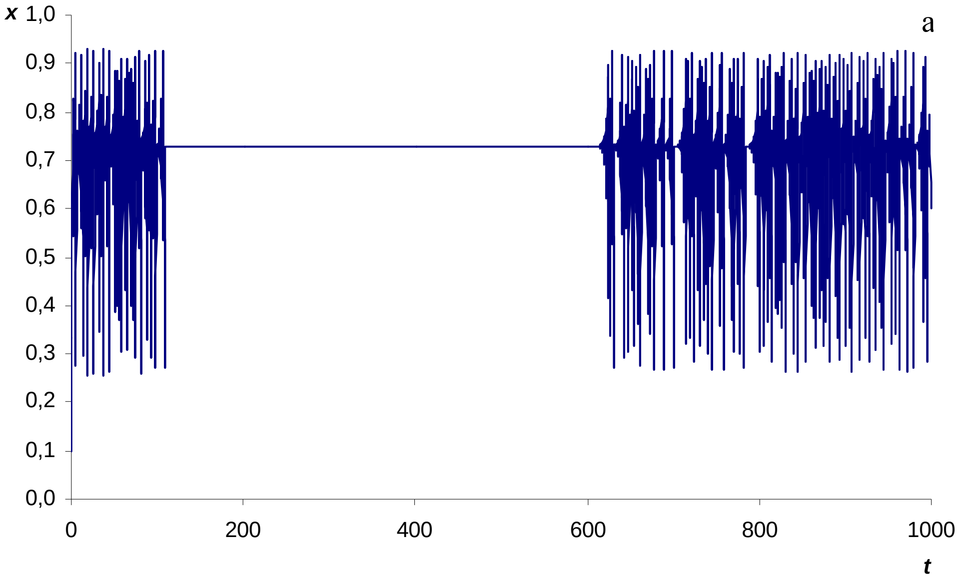
$$\lambda = \lambda_0 + \lambda_1 \cdot \sin\left(\frac{kw}{T}\right). \tag{5}$$

In this case the controlled logistic map is

$$x_{j+1} = \left(\lambda_0 + \lambda_1 \cdot \sin\left(\frac{kw}{T}\right)\right) \cdot x_j \cdot (1 - x_j). \tag{6}$$

As is seen from the bifurcation diagram (Fig. 4), between the stages of chaotic behavior there are the intermittency windows (for example, there is the window with period-6 at λ = 3.84).

A value of λ_0 corresponding to λ = 3.7 (the region of chaos) was considered as corresponding to an unperturbed state of the parameter; the perturbation period was set equal to intermittency window period – 6. These values specified the amplitude λ_1 in (6) so is to maintain λ = 3.7 and, at the same time, to obtain regular oscillations. The regularization occurred at λ_1 = 0.335. The regimes without destohastization and with destohastization being switched on t = 20 are shown in Fig. 5. The oscillations after t = 20 corresponds to the cycle of period-6.



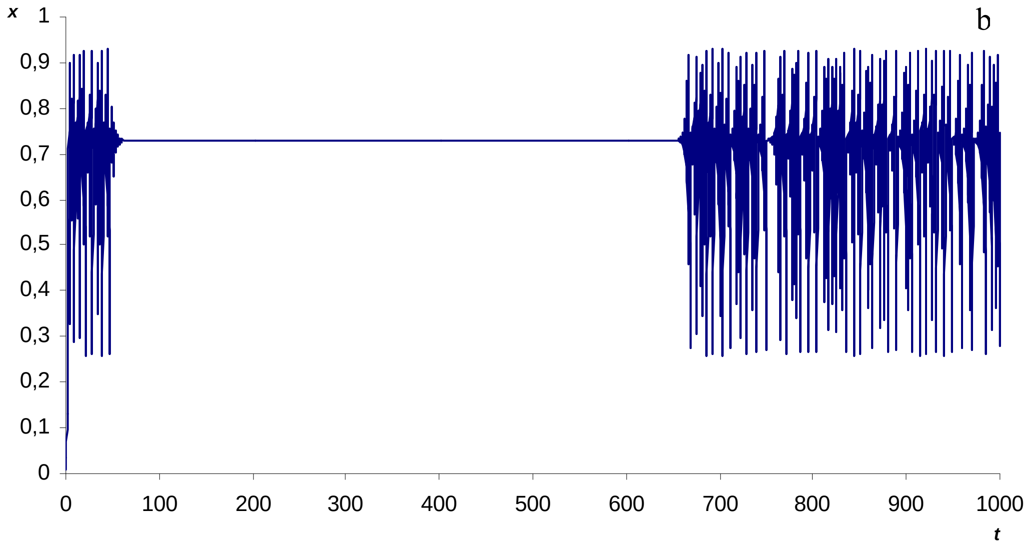


Fig. 3. Control of chaotic oscillations of logistic map (1) by means of (a) proportional feedback method and (b) extended time-delayed feedback method.

Externally forced oscillations are superimposed on the natural oscillations of the system; the result is the regularization of chaotic oscillations.

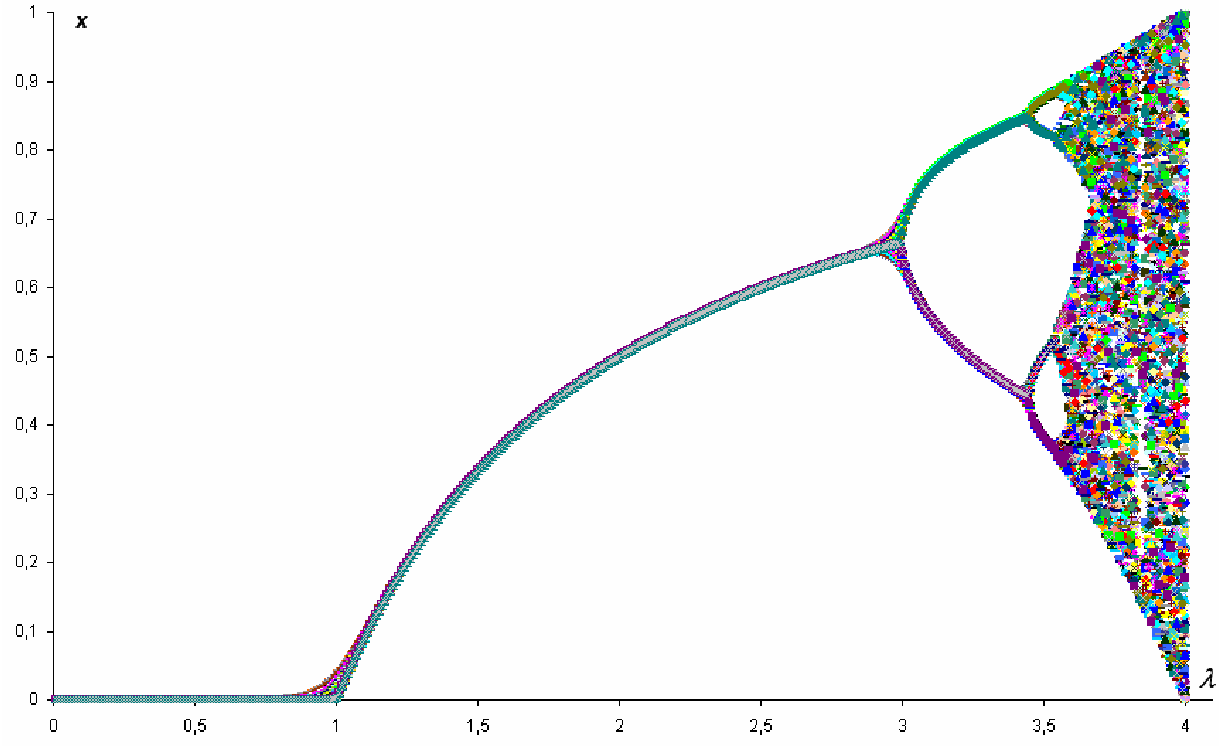


Fig. 4. The bifurcation diagram of the logistic equation.

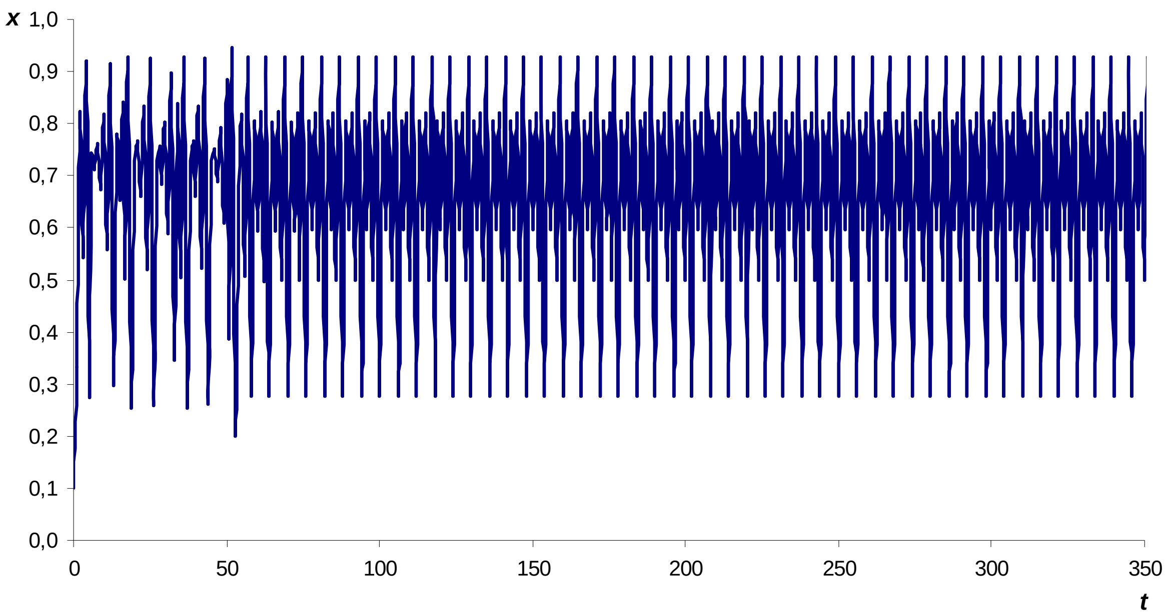


Fig. 5. Control of chaotic oscillations of logistic map (1) by means of destochastization method.

5. The model

Let's consider now a system of nonlinear discrete equations, representing the mathematical model of the continuous mass crystallization of dibasic lead phosphite synthesized in the course of the following chemical reactions

$$PbO + 2CH_3COOH = Pb(CH_3COO)_2 + H_2O,$$
$$2PbO + Pb(CH_3COO)_2 + 0.5H_2O = 2PbO\cdot Pb(CH_3COO)_2\cdot 0.5H_2O,$$
$$Na_2HPO_3 + 2PbO\cdot Pb(CH_3COO)_2\cdot 0.5H_2O = 2PbOPbHPO_3\cdot 0.5H_2O + 2CH_3COONa.$$

The given model is offered in [23] and describes the entire spectrum of the behavior (both stationary states and regular and chaotic oscillations during the change of a value of a bifurcation parameter of the system – the flow rate of a sodium phosphite solution v_q)

$$\begin{gathered}
\frac{c_1^{j+1}-c_1^j}{\Delta t}=\frac{v_q}{V}(c_1^0-c_1)-k_1\gamma_1 c_1^j c_2^j,\\
\frac{c_2^{j+1}-c_2^j}{\Delta t}=\frac{v_q}{V}(c_2^0-c_2)-k_1\gamma_2 c_1^j c_2^j,\\
\frac{c_4^{j+1}-c_4^j}{\Delta t}=k_1\gamma_1 c_1^j c_2^j-\frac{v_q}{V}c_4^j,\\
\frac{c_3^{j+1}-c_3^j}{\Delta t}=k_1\gamma_3 c_1^j c_2^j-\rho_3^0 k_0 r_0\left(\frac{c_3^j-c_3^S}{c_3^S}\right)^2-\frac{v_q}{V}c_3^j,\\
\frac{n^{j+1}-n^j}{\Delta t}=k_0\left(\frac{c_3^j-c_3^S}{c_3^S}\right)-\beta_1(n^j)^2-\sum_2^M N_k^j\eta_k^j\Delta k-\frac{v_q}{V}n^j,\\
\frac{N_k^{j+1}-N_k^j}{\Delta t}+\frac{N_k^{j+1}\eta_k^{j+1}-N_{k-1}^{j+1}\eta_{k-1}^{j+1}}{\Delta k}=-\frac{v_q}{V}N_k^j\\
\eta_k^j=\beta_2 n^j k^{\beta_3},
\end{gathered}\tag{7}$$

$$t=0,\ c_1(0)=c_1^0,\ c_2(0)=c_2^0,\ c_3(0)=0,\ n(0)=0,\ N(0,k)=0,\ \eta(0,k)=0,$$

where c_i is the concentration of i-th reagent; γ_1,γ_2 and γ_3 are the ratios of the masses of the components; v_q is the flow rate of the sodium phosphite solution; V is the volume of solution; ρ_3^0 is the density of lead phosphite crystals; r_0 is the volume of the cluster; c_3^S is the equilibrium concentration of dibasic lead phosphite; k_1 is the rate constant of the chemical reaction; k_0 is the rate constant of the cluster formation; n is the number of clusters in the solution; β_1 is the rate nucleation constant; β_2 is the crystal growth rate constant; β_3 is the exponent in the power dependence of the crystal growth rate η on the number of clusters; $\eta(k)$ is the crystal growth rate (the change in the number of clusters in the crystal per unit time); k is the number of clusters in the growing crystal; t is the time; and $N(k)dk$ is the number of crystals containing from k to $k+dk$ clusters.

Indexes are designated: 1 – dibasic lead diacetate; 2 – sodium phosphite; 3 – dibasic lead phosphite; 4 – sodium acetate.

The equation of the changes in the dibasic lead phosphite concentration (c_3) and the equation of the changes in the number of dibasic lead phosphite clusters (n) form a system of two coupled oscillators. The equation of the changes in the concentration (c_3) is the driving oscillator and the equation of the changes in the number of clusters (n) is the driven oscillator. These equations can be transformed by linear rearrangements to the form of the logistic equation (1) with the following variable controlling parameters

$$\lambda_c = 1 + \Delta t \sqrt{\left(\frac{v_q}{V}\right)^2 + \frac{4\rho_3^0 k_0 r_0}{c_3^S}\left(\frac{k_1 \gamma_3 c_1^j c_2^j}{c_3^S} - \frac{v_q}{V}\right)},$$

$$\lambda_n = 1 + \Delta t \sqrt{\left(\frac{v_q}{V} + \sum_k N_k^j dk \beta_2 k^{\beta_3}\right)^2 + 4\beta_1 k_0 \left(\frac{c_3^j - c_3^S}{c_3^S}\right)^2}. \qquad (8)$$

The flow rate of the sodium phosphite solution, as mentioned above, is the bifurcation parameter of the concerned system. With v_q increase the system passes through period-doubling bifurcation to a chaotic behavior and then through windows of intermittency to a chaotic behavior again (see the table 1). The equations (8) connect the flow rate of the sodium phosphite solution with the controlling parameters of the equations of the changes in the dibasic lead phosphite concentration and the equation of the changes in the number of dibasic lead phosphite clusters, which were transformed to the form of the logistic equation (1).

Table 1. Order-chaos transition during the variation of the flow rate v_q

v_q, l/h	Cycles in c_3	Cycles in n	v_q, l/h	Cycles in c_3	Cycles in n
3,600	1	1	13,248	Chaos	Chaos
5,400	1	1	13,284	Chaos	Chaos
7,200	2	2	13,320	Chaos	Chaos
7,236	2	2	13,500	Chaos	Chaos
7,992	2	2	13,680	Chaos	Chaos
9,000	2	2	13,860	Chaos	Chaos
10,800	2	2	13,932	Chaos	Chaos
11,700	4	4	13,968	6	6
12,600	8	8	14,004	6	6
12,672	16	16	14,040	12	6
12,960	32	32	14,090	Chaos	Chaos
13,032	64	64	14,400	Chaos	Chaos
13,068	64	64	14,500	Chaos	Chaos
13,176	Chaos	Chaos	14,600	Chaos	Chaos
13,212	Chaos	Chaos	14,700	Chaos	Chaos

c_3 is the concentration of dibasic lead phosphite, n is the number of clusters.

6. Control of chaotic oscillations in the process of crystallization via proportional feedback algorithm

For the proportional feedback algorithm to be applicable to a system with chaotic behavior, the equations describing of the system (or its maps in the Poincare cross-section) should be transformable to logistic equation of form (1). Therefore, the proportional feedback algorithm is applicable to system (7).

However, when constructing maps of the mathematical model (7) for the crystallization of dibasic lead phosphite, we found that, in maps of first order, fixed points are absent (Fig. 6a, 6b). The existence of the quadratic map with a region of windows can be explained as follows. Once the limiting supersaturation is attained [due to reaction (3)], the formation of clusters leads to a decrease in the concentration of dibasic lead phoshite. Thus, it takes a time for the specified supersaturation degree to be attained in the system due to reaction (3). Therefore, we solved the problem of stabilization of stationary unstable points of the cycle of period-2. From Fig. 6c and 6d, it is seen that the map of the second order has windows instead of a stationary point (cycles of period-1); however, the stationary points of the cycle of period-2 are present, i.e., the points that were stabilized in this work. A map of the second order characterizes, for example, the relation between the concentrations of dibasic lead phosphite at the jth and $(j + 2)$th steps in time (Fig. 6c). The bisector intersects the map of the second order to give four stationary points, two of which can be stabilized (Fig. 6c, points c_{s1}, c_{s2}). That means that a certain control of the system can stabilize the cycle of period-2.

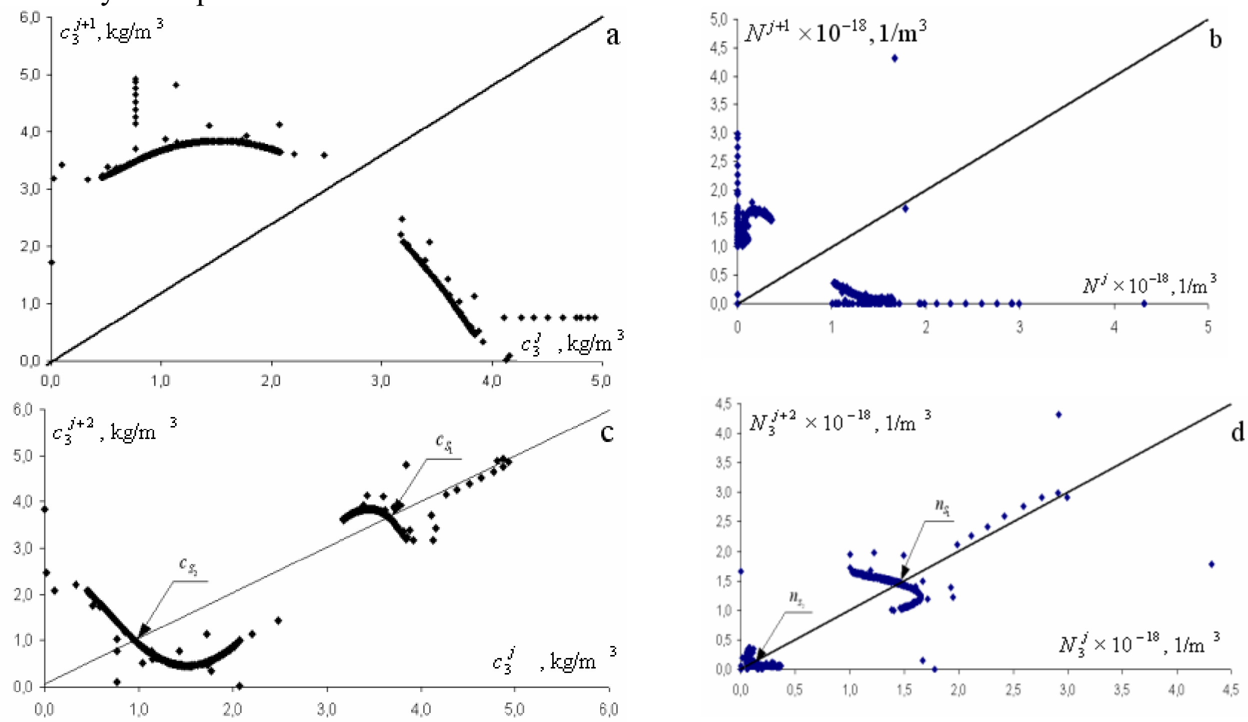


Fig. 6. Maps of the (a, b) first and (c, d) second orders for the equations describing variations in the concentration of dibasic lead phosphite and in the number of clusters at a flow rate of v_q = 13,26 l/h: c_{s1} and c_{s2} are the stationary points of the map of the second order for the concentration equation, n_{s1} and n_{s2} are the stabilized points of the map of the second order for the number of clusters, c_s and n_s are the stabilized points of the maps of the first order for these equations.

Let us construct a modification of the proportional feedback algorithm. The equation of the system (7) which describes the changes in the lead phosphite concentration caused by the chemical reaction and formation of clusters can be transformed to the form

$$c_{j+1} = F(v_q, c_j). \tag{8}$$

The cycle of period-2 takes place if all the points with odd indices tend to the point c_{s2}; the points with even indices, to c_{s1}. If this is the case, the following relations are valid

$$c^{j+1} = c_{S2} + \left.\frac{dF}{dc}\right|_{c_{S1}} (c^j - c_{S1}), \tag{9}$$

$$c^{j+2} = c_{S1} + \left.\frac{dF}{dc}\right|_{c_{S2}} (c^j - c_{S2}). \tag{10}$$

When the controlling parameter v_q is changed by a value δv_q, relations (9) and (10) take the form

$$c^{j+1}(v_q + \delta v_q) = c_{S2}(v_q + \delta v_q) + \left.\frac{dF}{dc}\right|_{c_{S1}} (c^j - c_{S1}(v_q + \delta v_q)), \tag{11}$$

$$c^{j+2}(v_q + \delta v_q) = c_{S1}(v_q + \delta v_q) + \left.\frac{dF}{dc}\right|_{c_{S2}} (c^j - c_{S2}(v_q + \delta v_q)). \tag{12}$$

In the first order of smallness, we cam write

$$c_{S1}(v_q + \delta v_q) = c_{S1}(v_q) + \frac{dc_{S1}}{dv_q}\delta v_q, \tag{13}$$

$$c_{S2}(v_q + \delta v_q) = c_{S2}(v_q) + \frac{dc_{S2}}{dv_q}\delta v_q,. \tag{14}$$

Taking into account that, after the reaction of a control (v_q + δv_q), the point $c_{j+2}(v_q + \delta v_q)$ will coincide with the stationary point c_{s1} (i.e., $c_{j+2}(v_q + \delta v_q) = c_{s1}$), and substituting Eq. (13) in Eq. (11) and Eq. (14) and (11) in (12), we obtain the expression for the increment of the controlling parameter

$$\delta v_q^j = \frac{\left.\frac{dF}{dc}\right|_{c_{S2}} \left.\frac{dF}{dc}\right|_{c_{S1}}}{\left[\left.\frac{dF}{dc}\right|_{c_{S2}} \left.\frac{dF}{dc}\right|_{c_{S1}} - 1\right]\frac{dc_{S1}}{dv_q^j}} (c^j - c_{S1}). \tag{15}$$

In this case, the controlling parameter v_q required for determining the concentration c_{j+1} at the (j + 1)th step in time is determined by the relation

$$v_q^{j+1} = v_q^j + \delta v_q^j \tag{16}$$

where δv_q^j is calculated by (15) and v_q^j is the value of the controlling parameter at the *j*th step in time.

From relation (15), it follows that to construct a control algorithm of a cycle of period-2 it is necessary to determine the slopes *dF*/*dc* of map (8) at the stationary points c_{s1} and c_{s2} (Fig. 7a), as well as the parametric sensitivity of the stationary point c_{s1} to changes in the controlling parameter v_q (dc_{s1}/dv_q). The variation of the concentration of dibasic lead phosphite is shown in Fig. 7a. The control algorithm based on Eq. (15) was set into operation 20 min after the beginning of the process; as a result, the cycle of period-2 was stabilized. Since the second oscillator (the number of clusters in the solution) is the driven oscillator (the number of clusters in the solution) is the driven oscillator after the control is set into operation, with the number of clusters oscillating between n_{s1} and n_{s2} (Fig. 7b).

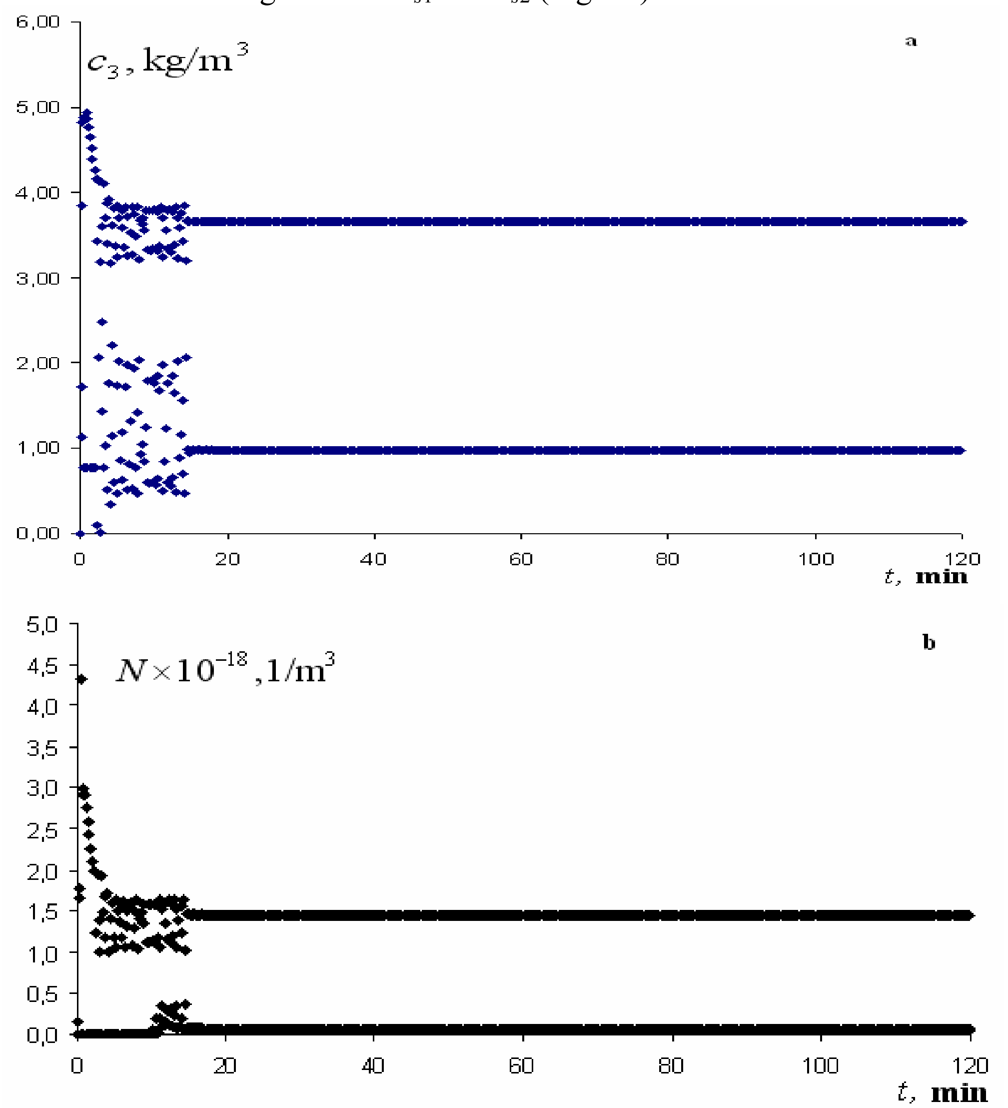


Fig. 7. Stabilization of the cycle of period-2 via proportional feedback method: the time dependences of (a) the concentration and (b) the number of clusters of dibasic lead phosphite (v_q = 13.26 l/h) before and during the operation of control.

7. Control of chaotic oscillations in the process of crystallization via extended method of time-delayed feedback

The value of the bifurcation parameter of the mathematical model at the (j + 1)-th step of time is determined by relation

$$v_q^{j+1} = v_q^0 + \delta v_q^j, \tag{17}$$

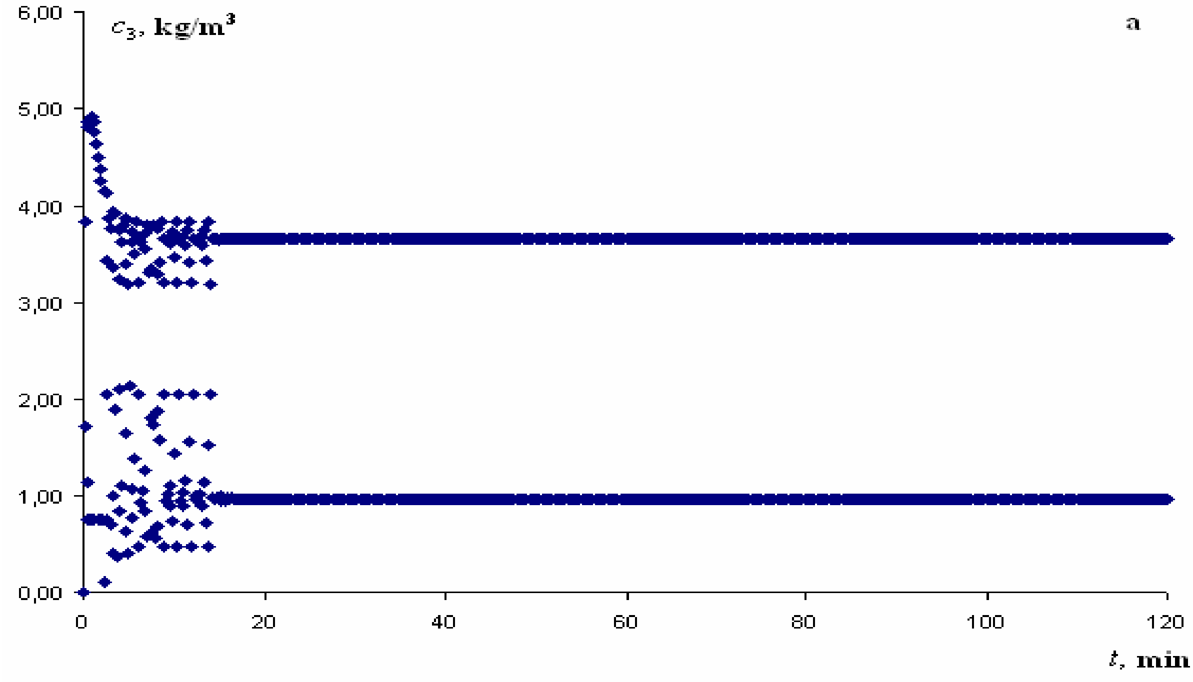
where v_q^0 is the initial value of the flow rate of the sodium phosphite solution; δv_q^j is the increment of the flow rate of the sodium phosphate solution on the next step.

Let's write down the increment of the flow rate of sodium phosphite, e.g., for the case of stabilization of the cycle of the period-2. Similarly to (4) and in accordance with the extended method of time-delayed feedback we have

$$\delta v_q^j = \gamma(x_j - x_{j-2}) + R\varepsilon_{j-2}, \tag{18}$$

where ε_{j-2} corresponds to the increment of the flow rate of the sodium phosphite solution, which was calculated on the (j – 4)th step in time.

Further, we can calculate a new value of the dibasic lead phosphite concentration by substituting the value of the flow rate of the sodium phosphite solution, which was calculated from the equation (17), in the equation (8).



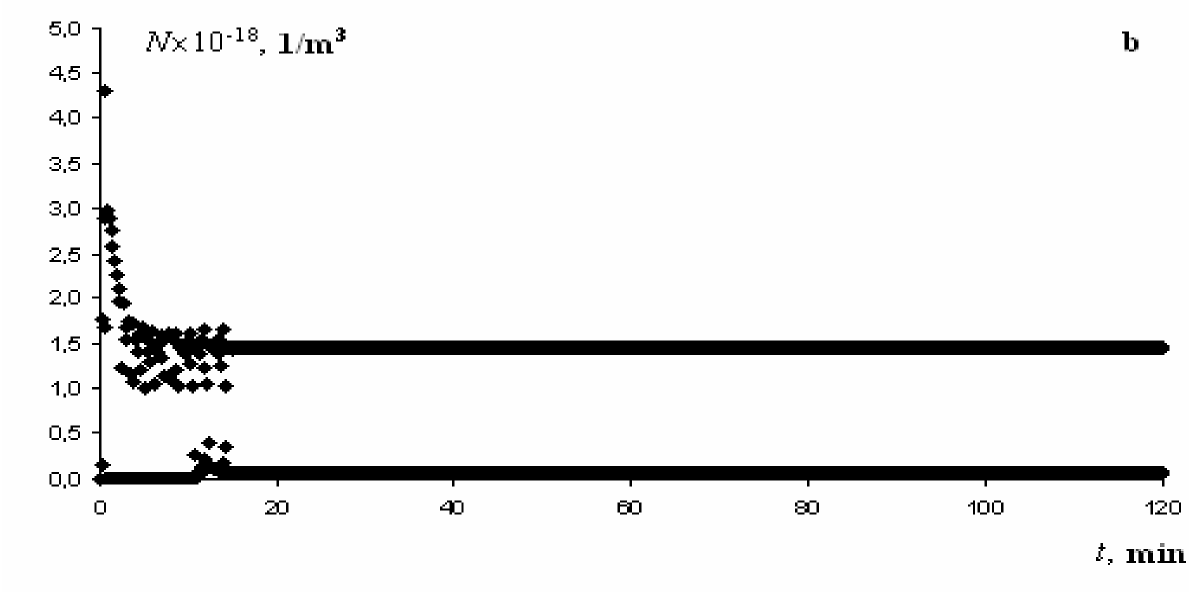


Fig. 8. Stabilization of the cycle of period-2 via extended method of time-delayed feedback: the time dependences of (a) the concentration and (b) the number of clusters of dibasic lead phosphite (v_q = 13.26 l/h) before and during the operation of control.

Let's turn to the example. The realization of the considered algorithm is shown in the Fig. 8. We were working in the first chaotic domain (marked by the extraheavy line in the table 1) at the value of the flow rate of the sodium phosphite solution v_q = 13.26 l/h. The system demonstrated the chaotic behavior. We switched on the controlling algorithm on the 14th minute and the system was stabilized near fixed points c_{s1}=3.657 kg/m^3 and c_{s2}=0.973 kg/m^3 on the 16th minute, which corresponds to the cycle of the period-2 (Fig. 8a). Since the second oscillator (the number of clusters in solution) is the driven one, is adjusting itself to the cycle of the period-2 of the driving oscillator (Fig. 8b). It was also found that stabilization near these fixed points is real when the parameters of the time-delayed feedback function take on the values from the following intervals: $\gamma \in [2.607; 3.000]$, $R \in [0; 0.99]$. It should be noted that the system is characterized by the heightened sensibility to the change of the parameters of the time-delayed feedback function.

8. Control of chaotic oscillations in the process of crystallization via destohastization algorithm

To apply the destohastization method for chaos control to system (7), it was necessary to derive an explicit between the bifurcation parameter v_q of the system and the bifurcation parameter λ_c [see the first Eq. of system (8)] of logistic equation [of form (1)] derived from the equation of the system (7) which describes the changes in the lead phosphite concentration. An analysis of the terms of Eq. (8) showed that the terms containing v_q in explicit form are insignificant. Using the equations of (5) which describes changes in the concentrations of the reagents for the steady-state concentrations of sodium phosphite and dibasic lead diacetate (unlike the concentration of dibasic lead phosphite, the concentrations of these components tend to steady-state values with time), we obtained the following relation for $c_1^j c_2^j$

$$c_1^j c_2^j = \left(\frac{c_2^0}{k_1 \gamma_1} \frac{v_q}{V} \right)^{1/2} \frac{\gamma_1}{\gamma_2} c_2^0 . \tag{19}$$

Substituting (19) in (8) and disregarding insignificant terms, we obtained

$$\lambda_c = 1 + \Delta t \left[4 \frac{\rho_3^0 k_0 r_0}{c_3^s} \frac{k_1 \gamma_3 \Delta t}{c_3^s} \left(\frac{c_2^0 v_q}{k_1 \gamma_1 V} \right)^{1/2} \frac{\gamma_1}{\gamma_2} c_2^0 \right]^{1/2} . \tag{20}$$

Equation (20) yields v_q in the form

$$v_q = A(\lambda_c - 1)^4 , \tag{21}$$

where $A = \left[\frac{\left(c_3^S\right)^2 \gamma_2}{4 \rho_3^0 k_0 r_0 \gamma_3} \right]^2 \frac{V}{\left(c_2^0\right)^3 (\Delta t)^6 \gamma_1 k_1}$.

The parameter λ_c in the equation for the concentration of dibasic lead phosphite was periodically perturbed according to the formula

$$\lambda_c = \lambda_c^0 - a_0 \sin\left(\frac{2\pi}{T} \right) t, \tag{22}$$

where λ_c^0 is the unperturbed value of λ_c (corresponding to the region of chaos), T is the perturbation period (intermittency window period), and a_0 is the perturbation amplitude. Hence, using (21), we obtained the destochastization algorithm in the form

$$v_q = A \left[\left(\lambda_c^0 - 1 \right) - a_0 \sin\left(\frac{2\pi}{T} \right) t \right]^4 . \tag{23}$$

As is seen from the table, between the two stages of the stochastic behavior of the concentration of dibasic lead phosphite, there are two intermittency windows (passing one into another) with periods of 6Δt (v_q = 13.956 – 14.047 l/h) and 12Δt (v_q = 14.04 – 14.083 l/h) at Δt = 6 s.

A value of λ_c corresponding to a flow rate of v_q = 14.184 l/h (the region of chaos, see table) was considered as corresponding to an unperturbed state of the parameter; the perturbation period was set equal to the intermittency window period (6Δt = 36 s). These values specified the amplitude a_0 in (23) was varied so as to maintain v_q = 14.184 l/h and, at the same time, to obtain regular oscillations. The regularization occurred at a_0 = 0.008. The regimes without destochastization and with destochastization being switched on 20 min after the beginning of the crystallization process are shown in Fig. 9. As can be seen, the concentration of dibasic lead phosphite oscillates to form a cycle with a period equal to 6. Calculations show that the driven oscillator (the number of clusters in solution) is adjusted to the driving oscillator and also oscillates at a period of $6\Delta t$. Externally forced oscillations [the reagent flow rate oscillated according to relation (23)] are superimposed on the natural oscillations of the system; the result is the regularization of chaotic oscillations.

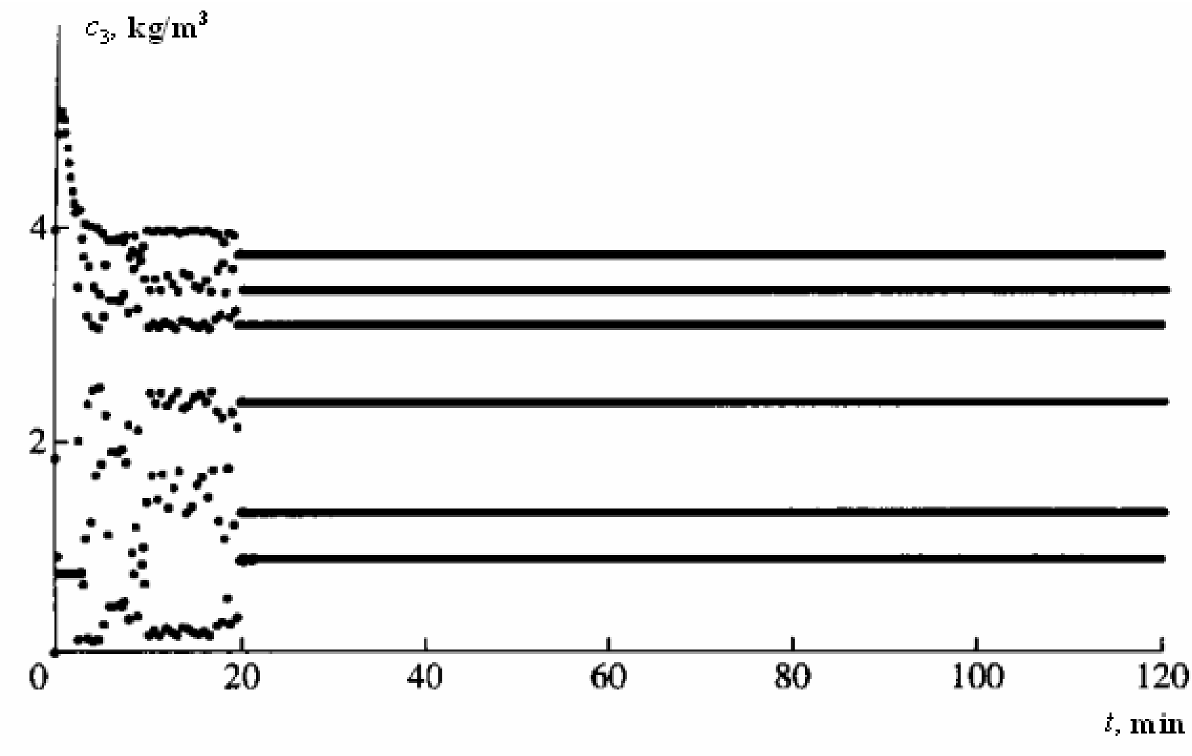


Fig. 9. Stabilization of the cycle of period-6 via destohastization method: the time dependences of the concentration of dibasic lead phosphite (v_q = 13.26 l/h) before and during the operation of control.

9. Conclusion

Thus, we considered three methods for controlling chaotic oscillations of the concentration and the number of clusters of dibasic lead phosphite during its crystallization. Note that the extended method of time-delayed feedback is more sensitive and offers a finer control in comparison with proportional feedback method and destochastization method.

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