Norm based approaches for automatic tuning of Model Based Predictive Control
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Norm based approaches for automatic tuning of Model Based Predictive Control

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Abstract

In this paper a general procedure for tuning multivariable model predictive controllers (MPC) with constraints is presented. It has been applied to tune the control system of an activated sludge process control in a wastewater treatment plant. Control system parameters are obtained by solving a mixed sensitivity optimization problem, defined in terms of the H_∞ norms of different weighted closed loop transfer functions matrices of the system, and a set of constraints, some of them expressed using the l1 norm. The use of linear models for the control allows for the specification of many convex performance criteria to state stability conditions and some desired closed-loop behaviour. The mathematical optimization for tuning all controller parameters is tackled in two iterative steps due to the existence of integer and real numbers. First, integer parameters are obtained using a special type of random search, and secondly a sequential programming method (SQP) is used to tune the real parameters.

Keywords: Model predictive control, robust control theory, l1 norm, mixed sensitivity problem, constrained multiobjective optimization

1. Introduction

Model based predictive control (MPC or MBPC) has become the leading form of advanced multivariable control in the process industries. The popularity of MPC is due to the successful results, the natural way of incorporating constraints, and its simplicity for operators.

MPC controllers have been tuned traditionally through a number of different parameters including prediction horizon, number of computer input moves, input and output weights in the objective function, and, in some cases, artificially imposed input/output constraints. The tuning task can be difficult if the system is of multivariable nature since the whole set represents a formidable array of possible tuning combinations and also because many of these parameters have overlapping effects on the closed-loop performance and robustness. In these cases the advantages of using automatic MPC tuning methods is clear.

In the literature, many works dealing with the automatic tuning of MPC can be found. In [2], Ali and Zafiriou proposed an off-line procedure for tuning the parameters of a nonlinear predictive controller specifying time-domain performance criteria. For linear MPC, Al-Ghazzawi [1] has developed an on-line tuning strategy based on the parametric and linear approximation between the closed-loop predicted output and the MPC tuning parameters, but without considering output constraints on the on-line optimization step. Another approach is given by Li [6], which uses fuzzy decision criteria to determine optimal real MPC tuning parameters, but leaving apart the horizons. Frequency domain methods for tuning linear optimal controllers have been studied since the beginning of 1980's (see Doyle and Francis [3] for a review). Lee and Yu [5] presented tuning rules based on frequency analysis of the closed loop behaviour of MPC controllers.

In [4] we already have proposed a methodology for the on-line automatic tuning of the whole set of parameters of linear Model Based Predictive Control Systems, and it was carried out by minimizing the Integral Square Error (ISE) norm as performance index. An important drawback of this work is that within the optimization procedure dynamical simulations have to be carried, making the procedure extremely slow.

At the view of previous works, we propose a new approach for the optimal automatic tuning of MPC which is based on the frequency domain robust control theory [9], and the optimization theory. The more relevant aspects of the actual proposal are:

- It is based on the resolution of a mixed sensitivity optimization problem defined in terms of the H_∞ norms of different weighted closed loop transfer functions of the system and a set of controllability and operation constraints, expressed by means of the l1 norm. The optimal tuning parameters evaluation for a linear MPC control scheme is carried out by solving a MINLP/DAE optimization problem.

- The use of the proposed automatic tuning approach within an Integrated Design framework is straightforward which is very useful to perform at the same time the design of the optimal plant for activated sludge process and the optimal linear MPC for this process.

- The approach has been validated on a simulated example based on a real wastewater treatment plant. Real scenarios have been considered in the simulated model by means of real data records of the main disturbances to make a more realistic analysis of the results.

The paper is organized as follows. First, the method for automatic tuning of the MPC is posed and explained in detail. Second, the activated sludge process model, selected for validation, is described. Third, the control problem and the application of the tuning method is stated. Then, some results are presented, to end with conclusions.

2. MPC formulation

The MPC considered is based on a linear state space model of the plant and calculates manipulated variables by solving the following on-line constrained optimization problem subject to constraints on inputs, predicted outputs and changes in manipulated variables.

$$\min_{\Delta u} V(k) = \sum_{i=Hw}^{Hp} W_y \cdot (\hat{y}(k+i \mid k) - r(k+i \mid k))^2 + \sum_{i=0}^{Hc-1} W_u \cdot (\Delta \hat{u}(k+i \mid k))^2 \qquad (1)$$

where k denotes the current sampling point, $\hat{y}(k+i \mid k)$ is the predicted output vector at time $k+i$, depending of measurements up to time k, $r(k+i \mid k)$ is the reference trajectory, $\Delta \hat{u}$ are the changes in the manipulated variables, H_p is the upper prediction horizon, H_w is the lower prediction horizon, H_c is the control horizon, W_u is a vector representing the weights of the change of manipulated variables and W_y is a vector representing the weights of the errors of set-points tracking.

The MPC prediction model used in this paper is a linear discrete state space model of the plant obtained by linearizing the model equations [8]. The reference trajectories *r(k)* approach the set-point trajectories exponentially from the current output values, with T_{ref} as the 'time constant' of the exponentials and T the sampling period.

When the MPC controller is linear and unconstrained, it can be represented with a transfer function K_{MPC}. The full closed loop system with measured disturbances has been represented in Figure 1.

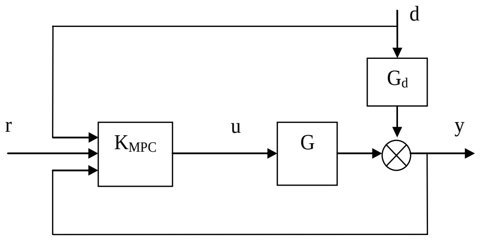

Figure 1 : Closed loop system with measured disturbances

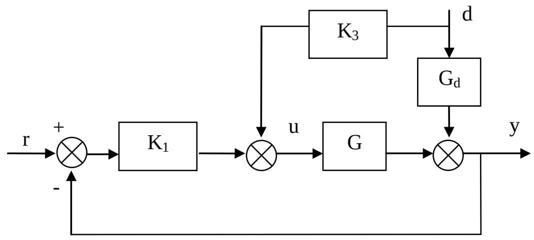

Figure 2 : Equivalent closed loop system

The controller block is multivariable, so the transfer function is

$$u = \begin{pmatrix} K_1 & K_2 & K_3 \end{pmatrix} \cdot \begin{pmatrix} r \\ y \\ d \end{pmatrix} = K_1 r + K_2 y + K_3 d \qquad (2)$$

where K_i are the transfer functions between the control signal and the different inputs (r,y,d) which depend on the control system tuning parameters (W_u, H_p, H_w,, H_c and T_{ref}). Particularly, in our MPC formulation $K_2 = -K_1$ (see [8]) and the block diagram of figure 1 can be transformed in the diagram of figure 2.

Consequently, taking into account control law and the transfer function of the open loop system, the closed loop response can be obtained from

$$y = \frac{GK_1}{1+GK_1} r + \frac{1}{1+GK_1} \tilde{d} \qquad (3)$$

where $\tilde{d}$ are the filtered disturbances

$$\tilde{d} = \left(GK_3 + G_d \right) d \qquad (4)$$

In order to define the automatic tuning problem, we define the sensitivity function *S'* between the load disturbances (*d*) and the outputs (*y*) and *M'* the Control Sensitivity transfer function defined between the load disturbances (*d*) and the control signals (*u*) when the reference is zero. Their calculation is straightforward applying block algebra to diagram of figure 2:

$$S' = \frac{y}{d} = \frac{K_3 G + G_d}{1 + GK_1} \qquad M' = \frac{u}{d} = \frac{K_3 - K_1 G_d}{1 + GK_1} \qquad (5)$$

3. Automatic tuning of MPC

3.1. Mixed sensitivity optimization problem

The problem of finding an optimal MPC is stated as a mixed sensitivity optimization problem that takes into account both disturbance rejection and control effort objectives, in the same tuning function. The problem definition is then

$$\min_{K_1, K_3} \left\| \begin{matrix} W_p S' \\ W_{esf} s \cdot M' \end{matrix} \right\|_\infty = \min_{K_1, K_3} \left\| N \right\|_\infty \qquad (6)$$

subject to the set of constraints explained below.

K_1 and K_3 are the MPC control compensators (see block diagram of Figure 2) which depend on the tuning parameter vector defined by $c = (H_p, H_c, W_u)$. W_p and W_{esf} are suitable weights for optimization. Note that control efforts rather than magnitudes of control are included in the objective function by considering the derivative of the transfer function *M'*.

3.2. Performance constraint

In order to ensure that disturbances are properly rejected we impose

$$\left\| W_p \cdot S' \right\|_\infty < 1 \qquad (7)$$

W_p is selected for the specification of load disturbances rejection, what means that its inverse must be smaller in magnitude than the inverse of disturbances spectrums.

3.3. Limits on control and output variables

The maximum value of the control (u_{max}) and the output variable (y_{max}) for the worst case of disturbances can be constrained to be less than certain limits by means of its l1 norm and the following conditions:

$$\left\| M' \right\|_1 < u_{\max} \quad \left\| S' \right\|_1 < y_{\max} \qquad (8)$$

3.4. Multiobjective optimization approach

The optimization problems for optimal automatic tuning can be stated as multiobjective optimization problems by considering constraints (8) as objectives f_i together with constrained optimisation of $\left\| N \right\|_\infty$. Then the multiple objectives are:

$$f_1 = \left\| N \right\|_\infty ;\; f_2 = \left\| M' \right\|_1 ;\; f_3 = \left\| S' \right\|_1 \qquad (9)$$

In this method the objectives must approach fixed goals, giving with these parameters different importance to every objective.

3.5. Algorithm description and implementation

The main problem when solving this optimization problem is that involves real and integer variables. In this work we propose a two iterative steps algorithm that combines a random search based on the Solis method [10] for tuning the horizons, and the classical Sequential Quadratic Programming (SQP) for tuning weights W_u.

The controller implementation is based on the MPC Toolbox of MATLAB® and some modifications of Maciejowski [8]. The real part of the optimization problem is tackled with the goal attainment method, implemented in MATLAB function *fgoalattain*.

4. Activated sludge process and model predictive controller

4.1. Plant description

The plant layout is represented in Figure 3, consisting of one aerobic tank and one secondary settler [11]. The basis of the process lies in maintaining a microbial population (biomass) into the bioreactor that transforms the biodegradable pollution (substrate) when dissolved oxygen is supplied through aeration turbines. Water coming out of the reactor goes to the settler, where the activated sludge is separated from the clean water and recycled to the bioreactor to maintain there an adequate concentration of microorganisms.

The whole set of variables is presented in Figure 3. Generically, "*x*" is used for the biomass concentrations (mg/l), "*s*" for the organic substrate concentrations (mg/l), "*c*" for the oxygen concentrations (mg/l) and "*q*" for flow rates (m^3/h).

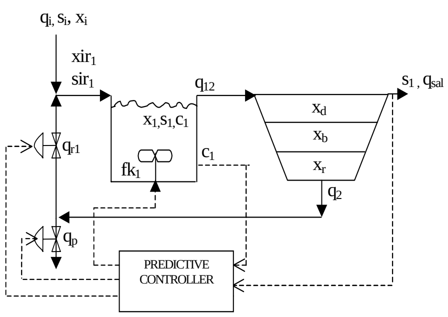

Figure 3: Plant and controller layout

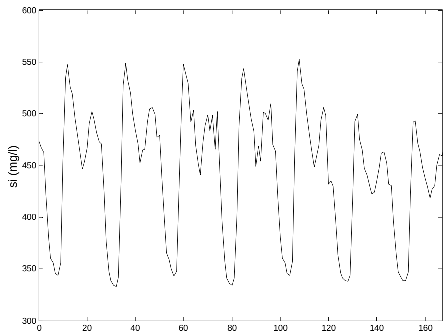

Figure 4: Substrate disturbances at the influent (s_i)

4.2. Control problem

The control of this process aims to keep the substrate at the output (s_1) below a legal value despite the large variations of the flow rate and the substrate concentration in the incoming water (q_i and s_i), which are the input disturbances and one of the main problems when trying to control the plant properly. Another control objective is to keep dissolved oxygen concentration (c_1) around 2 mg/l, concentration that is necessary for the proper working of activated sludge process.

The set of disturbances used in dynamic simulations (Figure 4) has been determined by COST 624 program and its benchmark.

The general structure of a multivariable controller applied to the activated sludge process can be seen in figure 5. Three manipulated variables are considered: recycling flow (qr_1), purge flow (q_p) and aeration factor (fk_1); and three outputs: substrate (s_1), biomass (x_1) and dissolved oxygen (c_1) in the reactor. Here the biomass is only a constrained variable for a good performance of the process and it is not controlled. In this work we will focus on substrate control, although the methodology proposed is general and could be also extended to oxygen control.

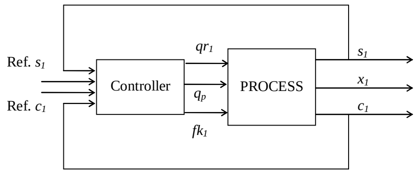

Figure 5: General controller structure

5. Tuning Results

The controller considered is a linear MPC with constraints applied to the nonlinear plant model, with sample period of 0.5 hours, suitable for representing the process dynamics. Disturbances s_i and q_i are assumed to be measured and scaled to make methodology improvement clearer. Biomass concentration x_1 is only a constrained variable. The selected plant is fixed with dimensions V_1=7668 m^3 and A=2970.88 m^2; and a steady state point defined by s_1=58.445 and qr_1=220.

5.1. H_∞ mixed sensitivity problem considering objectives f_1 and f_2

In table I the numerical results for three cases are presented. First a comparison of results with two different weights W_p (cases 1 and 2 of table I) is presented in figure 6, keeping W_{esf} constant.

$$W_{p1} = \frac{26.6s + 32}{s + 0.0001} \quad W_{p2} = \frac{8s + 9.6}{s + 0.0001} \qquad (10)$$

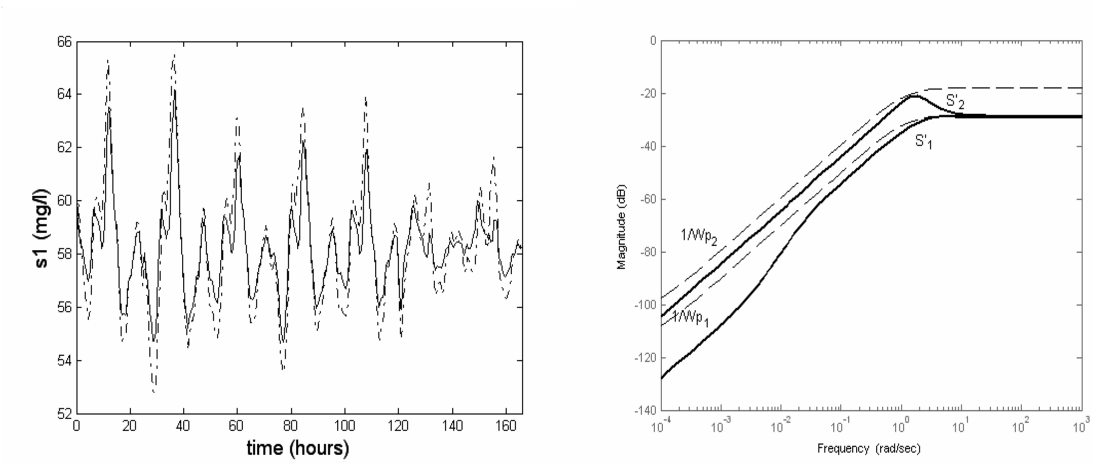

Figure 6: Comparison of substrate responses and sensitivity functions *S'* for two weights W_{p1} (solid line) and W_{p2} (dashed dotted line) (cases 1 and 2)

In this figure can be seen that disturbance rejection is better when using W_{p1} because the closed loop system bandwidths allowed are larger than with W_{p2}.

A comparison with two W_{esf} weights (cases 2 and 3 of Table I) is presented in figure 7, keeping now W_p constant. In this figure can be seen that for case 3 the control efforts are more relaxed than for case 2, producing a better disturbance rejection although with more energy consumption.

$$W_{esf\,2} = \frac{0.2s + 0.02}{s^2 + 5s + 0.0001} \quad W_{esf\,3} = \frac{0.02s + 0.002}{s^2 + 5s + 0.0001} \qquad (11)$$

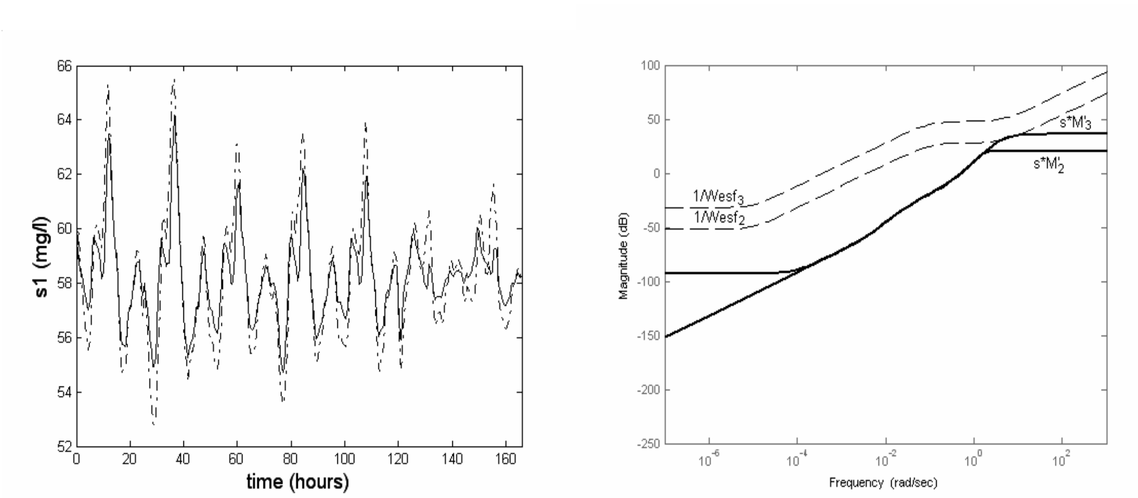

Figure 7: Comparison of substrate responses and sensitivity to control efforts functions $s{\cdot}M'$ for two weights W_{esf2} (dashed dotted line) and W_{esf3} (solid line) (cases 2 and 3)

TABLE I
RESULTS SUMMARY FOR MPC TUNING (5.1)

	Case 1	Case 2	Case 3
W_u	0.0023	0.0118	0.0011
H_p	9	8	6
H_c	2	3	2
$max(qr_1)$	1179.5	1092	1232
$max(s_1)$	64.2	65.52	63.94
$\|N\|_\infty$	1.44	1.04	0.73
$\|M'\|_1$	3995	1946	4416
$\|S'\|_1$	6.47	14.44	5.39
$\|W_p S'\|_\infty$	1.00	1.00	0.72
Weights	W_{p1}	W_{p2}, W_{esf2}	W_{esf3}
Computational time (min)	3.44	0.81	1.66

TABLE II
RESULTS SUMMARY FOR MPC TUNING (5.2)

	Case 4	Case 5
W_u	0.0019	0.0091
H_p	9	10
H_c	2	4
$max(qr_1)$	1185.5	1096.6
$max(s_1)$	64.17	65.33
$\|N\|_\infty$	4.41	1.87
$\|M'\|_1$	4053.5	1983.7
$\|S'\|_1$	6.24	13.1
$\|W_p S'\|_\infty$	0.95	0.97
Weights	W_{p1}	W_{p2}
Computational time (min)	10.8	5.89

5.2. H_∞ mixed sensitivity problem considering objectives f_1 and f_3

The second approach consists of considering objective f_3 instead of f_2. A comparison of substrate outputs with the previous two W_p weights is presented in figure 8. In

Table II numerical results are presented. For case 4 the disturbance rejection is better than for case 5 because W_{p1} is more restrictive than W_{p2}

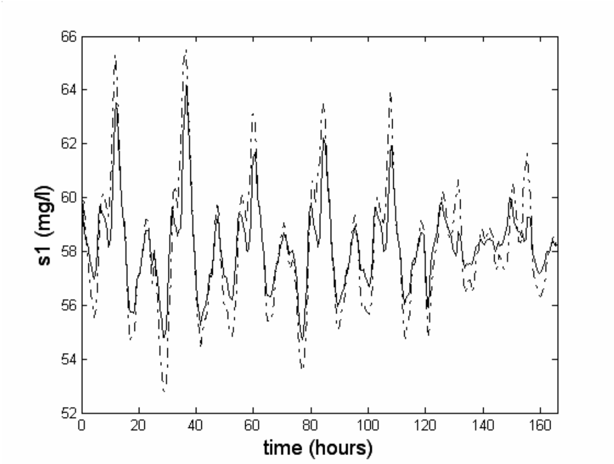

Figure 8: Comparison of substrate responses for two weights W_{p1} and W_{p2}

In the last rows of Tables I and II can be seen the computational time referred to a Pentium IV 2,4 GHz computer. This time is relatively low because nonlinear simulations are not needed to calculate system norms. The cost bound for the convergence criteria is 0.001, enough to assure no further controller adjustment in the optimization.

6. Conclusions

In this work a method for tuning model predictive controllers has been developed, based on several plant dynamical performance indexes. This method has been tested in MPC applied to the activated sludge process, and the closed loop responses for substrate concentration in the reactor show that obtained controllers are properly tuned, taking into account the large magnitude of influent disturbances.

The methodology proposed here is a general one, and any other performance criteria can be considered. The use of linear models also allows for the specification of convex performance criteria within an LMI framework.

Finally it is important to show that the developed method is particularly suitable for its inclusion in the resolution of the Integrated Design optimization problem, which determines the optimum controller and the optimum plant at the same time.

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