MATRIX PENCIL: A NOVEL APPROACH FOR FAST SHUNT COMPENSATED CONTROLLED SWITCHING*

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Abstract

In this paper a novel and efficient approach is presented for power system switching transient analysis: The so-called Matrix Pencil Method MP, is based upon a modal decomposition of exponentially modulated sinusoids, formulated as a generalized eigen-value problem. In this paper we are interested in shunt compensated line transient voltages during auto re-closing operations. The MP method has readily proved it's efficiency in many areas (speech pattern recognition, aircraft, signal processing etc). Moreover, for such class of signals it shows better performance than many improved versions of the Prony's method such us Tufts and Kumaresan polynomial one. Many validation tests were carried out onto typical transient signals issued from EMTP/ATP simulations for different network configurations and show very satisfactory results.

1 Introduction

The fast and accurate estimation of transient signals is of great interest for control and protection applications in High Voltage power systems. In the recent years, a new strategy was developed to reduce over-voltages and/or over-currents generated by closing operations in HV power systems. This new approach called point on wave control (or controlled switching) is based upon the synchronization of a given reference signal in order to close the breaker contacts at an optimum target instant. For re-closing operations the optimum target is ideally at the zero voltage between circuit breaker opened poles.

According to the network configuration and their operating conditions, this task can be easier, or harder, to satisfy. For three-phase re-closing operation, in case of single-phase fault

that represents more than 90% of line faults types, at the breaker opening an important trapped charge may remain onto the healthy phases.

The particularity of shunt reactor compensated lines is that the discharge processes take a damped oscillating form with a main frequency typically between 25Hz-50Hz (about 50 to 90% of the network nominal frequency 50/60 Hz). Depending on the line compensation level, the circuit breaker voltage may show a more or less pronounced beat phenomenon. The optimal moment for re-closing is then at / near a minimum of the voltage beat.

Under these assumptions, if line voltages measurements are available, it is theoretically possible by a reliable algorithm to determine the optimal switching instant. Nevertheless, in practical situations, the line voltages waveforms are often distorted by high frequency noise of random nature, particularly under unbalanced operating conditions (such as unsymmetrical single-phase faults). Inter-phases coupling phenomena may also be induced. Both are unavoidable and may affect the computational accuracy of the re-closing instant prediction. Moreover, in real world power systems, the shunt compensation rate of a given line can vary at any moment to adapt with a line power flow variation. No 'a priori' information being available on the compensation level makes harder the controller task.

The controlled switching devices available today are often used for simple closing operation, where ''zero crossing technique'' are applied to source side voltages and seems to be sufficient for optimum closing moments prediction. For on-line application of controlled switching particularly when dealing with shunt compensated line a more sophisticated algorithm is required (no pre-established switching moments can be considered).

One suitable analysis tool for such transient signals is the Prony's method [7, 8]. This one is dedicated to the estimation of modal components (frequency, damping, phases and magnitude) of exponentially damped sinusoids. In power system area, the original method has been successfully applied for electromechanical mode identification from ambient data [4], analysis of power systems response data [3]. An improved version, more recently developed by Tufts and Kumaresan [1], was also applied for fault current transient analysis into Petersen-coils distribution network [8].

Based upon the basic assumptions, under noisy conditions, a more efficient approach is the Matrix Pencil method MP [9, 10]. Many research studies in signal processing and speech recognition areas has proved that MP method is more efficient in terms of accuracy and less sensitive to noise then the previous ones [9, 10].

The present paper is organized as follows. The considered problem is firstly formulated and basic hypothesis are settled down in Section 2. A brief description of the basic steps of the proposed algorithm is done through Section 3. Based upon many research investigations, some guidelines are given for optimal tuning of the algorithm in Section 4. Finally simulation tests were performed, in Section 5, firstly onto synthetic signals then onto more realistic transient data generated by EMTP (Electromagnetic transients program) simulator.

2 Problem formulation- Hypothesis

2.1 Problem statement

Many transient switching simulations and field tests [5], reveal that the line voltages during the ''dead time'', may have more than one frequency component. A fairly significant component may be found, with a frequency and magnitude depending onto the line transposition scheme. A less significant component with a higher frequency, induced by the faulted phase voltage's decay may also exist but is usually quickly damped. These investigations shown also that, for the beat signal to be synchronized (circuit breaker voltage) only two frequencies are relevant: the source side nominal frequency (50Hz) and the line side main frequency.

The algorithm to be designed has to estimate the line side voltage with sufficient accuracy and speed in order to pass the switching command within the time limits laid down by the network control requirements (typically about 300 ms). Taking into account the mechanical operating time required by the circuit breakers, this would leave approximately 150ms for the algorithm to estimate the frequency signals of between 25 and 50 Hz. It has also to be sufficiently robust with respect to "noise" originated by harmonics, transients from the network, non-linearity into instrument transformer's, or inside data acquisition chain for instance.

2.1 Control strategy

The phases to earth voltage quantities may, as shown in Figure.1, be strongly distorted by inter-phases coupling effects (mainly for the third phase to be opened). In order to reduce the coupling effects and the fault noise without lost of the most meaningful information, we choose as reference signal a composite signal issued from Clarke modal transform [6].

The global algorithm strategy to perform fast synchronous closing is:

- Identify the healthy phases from the faulted one by observing the residual current and each phase current,

- Compute the reference composite voltage from the three phases voltages,

- Detect its most dominant modal component and estimate the main frequency,

- Predict the optimum closing instants from the estimated frequency.
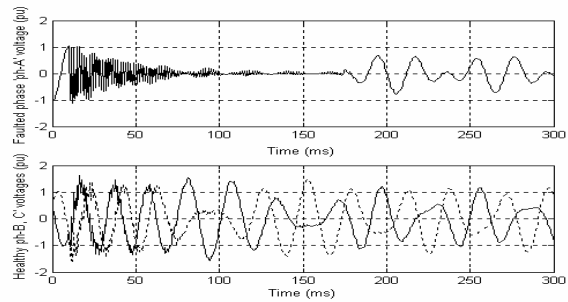
(a) Phases V_a, V_b, V_c line voltages (pu)
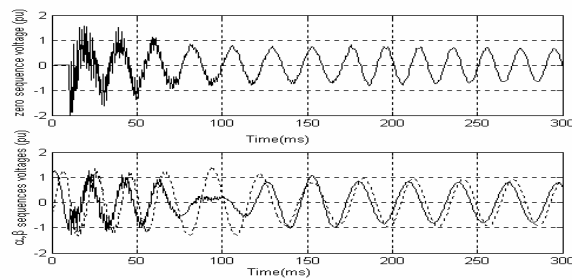
(b) Clarke components voltages V_0, V_α, V_β (pu)

Figure 1 Typical voltages at shunt compensated line de-energizing –Single phase A to ground fault**.**

In the proposed scheme the MP approach is used for main modes extraction and their estimation. An optional feature is also proposed to track eventual fluctuation in the signal's components that may affect the estimated model. The key idea [7] is to use an LMS approach recursive algorithm to adapt an AR (autoregressive) backward prediction model associated to the original signal's model. The advantage of this technique over a sliding window MP approach is making possible a real time application with a cheaper computation cost.

3. Principle of the Analysis Method

Let us consider an N sequence of noisy samples, of a given signal $y_r(n)$ which may be expressed as a sum of exponentially damped sinusoids:

$$y_r(n)=\sum_{i=1}^{p} A_i \, e^{\alpha_i \, nT_s} \, cos(2\pi f_i \, nT_s +\varphi_i \,)+\omega_r(n) \qquad (1)$$

where f_i , α_i , A_i , φ_i are the frequencies, damping, magnitudes, and initial phases of the ith modal component respectively. T_s is the sampling interval and $\omega(n)$ a white gaussian noise. By applying the Hilbert transform to the real sequence (eq.1), one obtains the complex model of this signal known as Prony's model [8]:

$$y(n)=\sum_{ki=1}^{p/2}\beta_i\,Z_k^n+\omega(n) \qquad (2)$$

where $\beta_i = A_i\,exp(j\varphi_i\,)$ is the complex amplitude,

$Z_k =exp(j2\pi f_i +\alpha_i\,)T_s$ the complex frequency.

$\omega(n)$ is then a complex white Gaussian noise.

3.1 Modal estimation by Matrix Pencil Approach

The Matrix Pencil approach, is based upon the following properties. Let us firstly define from the N available data samples two $(N-L)\times L$ linear prediction matrices as:

$$Y_1 =\begin{bmatrix} y(1) & \cdots & y(L) \\ \vdots & \ddots & \vdots \\ y(N-L) & & y(N-1) \end{bmatrix},$$

$$Y_2 =\begin{bmatrix} y(0) & \cdots & y(L-1) \\ \vdots & \ddots & \vdots \\ y(N-L-1) & \cdots & y(N-2) \end{bmatrix} \qquad (3)$$

where the parameter L refers to the pencil parameter which is restricted by $p\leq L\leq N-p$. Based upon the model signal specific form, this matrix pair may be decomposed in the following form:

$$Y_1 =Z_1 RZ_0 Z_2 \qquad (4)$$

$$Y_2 =Z_1 RZ_2 \qquad (5)$$

where: $Z_0 =\begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \ddots & z_p \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-L-1} & z_2^{N-L-1} & & z_p^{N-L-1} \end{bmatrix}$, $Z_2 =\begin{bmatrix} 1 & z_1 \cdots & z_1^{L-1} \\ 1 & z_2 & z_2^{L-1} \\ \vdots & \vdots \ddots & \vdots \\ 1 & z_p & z_p^{L-1} \end{bmatrix}$,

$$Z_0 =diag(z_1,z_2,\cdots,z_p)\,,\quad R=diag(B_1,B_2,\cdots,B_p\,)$$

So that the z_k , $k=1,\cdots,p$ signal poles are the generalized eigenvalues of the following matrices linear called Matrix pencil [9, 10]:

$$Y_1 -\lambda Y_2 =Z_1 R(Z_0 -\lambda I)Z_2 \qquad (6)$$

where: Y_1 , Y_2 are the data matrices defined above and λ a scalar parameter.

One may determine the signal poles by computing the matrix product $Y_1^+Y_2$. The superscript '+' refers to the Moore-Penrose Pseudo inverse. This matrix product is characterized by p non zero eigenvalues equals to the z_k , $k=1,\cdots,p$ (i.e related to the noiseless signal subspace) and L-p zero eigenvalues (i.e related to the noise subspace). Equivalently the product $Y_1^+Y_2$ has p non zero eigenvalues equal to the z_k^{-1} , $k=1,\cdots,p$ and L-p zero eigenvalues.

This may be easily shown, by replacing the Y_1 , Y_2 matrices by their equivalent expressions in (4), (5). Then the generalized eigenvalues problem may be resolved through the following steps:

Step1
Perform the singular value decomposition SVD of the matrix Y_2 as : $Y_2 =U\Sigma V^H$, where Σ is a diagonal matrix containing the singular values of Y_2 and U,V their corresponding left and right eigenvectors respectively.

Step2
To determine the optimal low rank order p of the model one may simply observe the singular values of Y_2 which are the diagonal elements of the matrix Σ . Only the most significant singular values, those which are above a given threshold ε_1 (in connection with the singular values discrepancy), represent the signal subspace and had to be retained. Their number corresponds to the estimated model order p, the L-p remaining one are set to zero in order to reduce the noise effects:

$$\Sigma=\begin{bmatrix} \sigma_1 & 0 & \cdots & & 0 \\ 0 & \ddots & & & \\ \vdots & & \sigma_p & & \vdots \\ & & & \ddots & 0 \\ 0 & & & 0 & \sigma_L \end{bmatrix},\quad \sigma_L\leq\cdots\leq\sigma_{p+1}\leq\varepsilon_1 <\sigma_p$$

Step3
From the most significant singular values σ_k , $k=1,\cdots,p$ and their associated left and right eigenvectors $U=[u_1,\cdots,u_p]$, $V=[v_1,\cdots,v_p]$, one may then compute the $(p\times p)$ square matrix G_0 defined as:

$$G_0 =\widetilde{\Sigma}^{-1}U_p^H Y_1 V_p \qquad (7)$$

having as eigenvalues the estimated signal poles inverse z_k^{-1} , $k=1,\cdots,p$. One may note that by performing the truncation procedure before computing G_0 acts as a filtering process for the noise present in Y_1 which is highly correlated with that in Y_2 (another SVD of does not seem necessary).

Step 4

Once the signal poles are estimated, one may form a Van Der Monde linear system:

$$y_n = V\beta \tag{8}$$

where: $V = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & & z_p \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-1} & z_2^{N-1} & & z_p^{N-1} \end{bmatrix}$,

$$\beta = [\beta_1, \beta_2, \cdots, \beta_p]^T, \; y_n = [y(0), \cdots, y(N-1)]^T$$

V is called the Van Der Monde Matrix.

The solution for the complex amplitude vector β in the total least square sense may be expressed as:

$$\hat{\beta} = (V^T V)^{-1} V^T y_n \tag{9}$$

Under the assumption that the signal has time invariant parameters over the considered time interval ($t_n = nT_s$, $n = 0, \cdots, N-1$), these four steps allow a sufficiently accurate estimation of the signal model. However, the Matrix Pencil algorithm performance rapidly decreases in case of no-stationary signals. In order to check the validity of the estimated model and track eventual slow change in the estimated parameters one may use a sliding time MP version but this is too much time consuming and unsuitable for on-line applications. Another approach would be the use of an iterative algorithm to adapt the recurrence relation (AR-prediction model) of order 'L' satisfied by the signal using a backward version of the least mean squares (LMS) technique.

3.3 Model tracking by backward adaptive LMS

At this stage an initial backward linear prediction vector may be computed from:

$$Y_0 = -\widetilde{Y}_2 \, h_0 = -V_p \widetilde{\Sigma}_p^{-1} U_p^H h_0 \tag{10}$$

$$h_0 = [y(0), \cdots, y(n-L-1)]^T$$

Step 5

The prediction error $e(n)$ is defined as:

$$e(n-1) = y(n-1) - \hat{y}(n-1) = y(n-1) + \sum_{j=1}^{L} b_j(n-1) \, y(n-1+j) \tag{11}$$

The linear prediction vector is updated according to:

$$b(n) = b(n-1) - \mu e(n-1) y_n \tag{12}$$

$$y_n = [y(n), \cdots, y(n+L-1)]^T, \; b(n) = [b_1(n), \cdots, b_L(n)]^T,$$

μ is the fixed step of the LMS algorithm.

By monitoring the backward prediction error it is possible to detect wide changes in the signal. In such case the MP estimation procedure (steps 1- 4) may be re-initialized using a new N data set, from $t_n = nT_s$ when the absolute value of $e(n)$ surpasses a given threshold ε_2. One has to note however, that some inaccuracies may be introduced if the signal components change occurs during the initialisation step spanned by the first N considered samples.

4. Analysis of MP performance

4.1 First order perturbation analysis

As shown above, the performance of the MP algorithm is mainly dependent upon the estimation accuracy of the poles inverse z_k^{-1}, $k = 1, \cdots, p$ eigen values of the matrix G_0, or the matrix product $Y_1^+ Y_2$ (see section 3.1). Under white Gaussian additive noise assumption, a first order perturbation analysis has been conducted by Hua [9] and leads to the following results:

(i) $var\{(\delta(z_k^{-1})\} = 2var\{(\delta(2\pi f_k)\} = 2var\{\delta(\alpha_k)\}$, does not depend on the initial phases φ_k,

(ii) $var\{(\delta(z_k^{-1})\}$ is proportional to the inverse of the ratio $SNR_k = |B_i|^2 / \rho^2$ and independent from others SNR_j for $j \neq k$,

(iii) For a given N, the perturbation $\delta(z_k^{-1})$ is symmetrical about $L = N/2$ and its variance is minimal between $L \in]N/3, 2N/3[$.

(iv) For each of these optimal values, $\delta(z_k^{-1})$ is inversely proportional to N^2.

The above important results are very helpful for a right selection of the algorithm parameters in real time application.

4.2 Tuning of the algorithm parameters

There are four main parameters to be adjusted in order to perform an efficient real time implementation of the Matrix pencil algorithm. First, the number of considered samples N, is of major importance in the estimation step. In fact, even if an increasing number of samples, allows a more accurate estimation [7], the singular value decomposition of the data matrix has a computational cost of N^2. Moreover,

the delay between an eventual change in the signal and its detection by the LMS technique is on the order of N periods. The second one which is the pencil parameter L, is optimal between $N/3$ and $2N/3$, in order to ensure good level of accuracy in the poles estimates as shown by Hua in [9, 10]. On the other hand the bias introduced in the estimates during the LMS tracking procedure increases when the prediction order L is decreased as shown by Tufts and Kumaresan in [1]. For real-time application the lower optimal bound $N/3$ seems to be more suitable. The third main parameter is the sampling step which is not only limited by the Shannon limit but provides better estimates when being higher then quarter the barycentre of the p frequencies inverses [7, 9]. Finally, the choice of the threshold in the SVD truncation is based upon a prior knowledge about its characteristics so that, the dominant singular value (lying to the signal subspace) are proportional to the signal mode amplitudes while the lowest singular values are proportional to the noise variance.

5. Application to power system transient - discussion

5.1 Validation on synthetic test signals
Numerical tests were carried out firstly to evaluate the Matrix Pencil algorithm performance in retrieving the transient voltage main frequency, of a multi-mode stationary signal in presence of white Gaussian noise. To assess the noise effects onto the frequency estimates about 100 independent realizations for various Signal to noise ratios (from 10 to 40 dB).
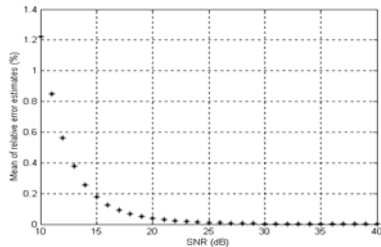

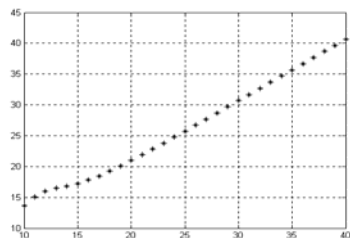**Figure 2.** *Variance in (dB) of the relative error of 'f₁' estimate (%) versus SNR (dB)*

Figure 3. *Variance's inverse of 'f₁' (dB) versus SNR (dB)*

The corresponding means of the relative errors err_f_1 (in %) on the main mode estimates and the corresponding var_f_1 inverse's variance are computed from:

$$err\_f_1 = 100.\frac{\hat{f}_1 - f_1}{f_1} , \quad var\_f_1 = 10\,log(1/var(\hat{f}_1)) .$$

Both figures 2 and 3 show the great performance of the MP algorithm. For the considered example, the mean relative error don't exceeds 1.2% (SNR= 10dB), and decreases rapidly above a threshold of SNR=18 dB, from $err\_f_1 = 0.1\%$ to about $err\_f_1 = 0.0025\%$ for SNR=20dB, which is a very accurate result.

The variance which is about $var(\hat{f}_1) = 0.0398\ Hz$ for SNR=10dB and about $var(\hat{f}_1) = 0.0001\ Hz$ for SNR=40dB illustrate the robustness of the method.

5.2 Validation on power system simulated data
In order to show the capability of the method for real world transient voltages it is necessary to test it on more realistic data. A study case was simulated by mean of the EMTP/ATP simulator (see figure 4).
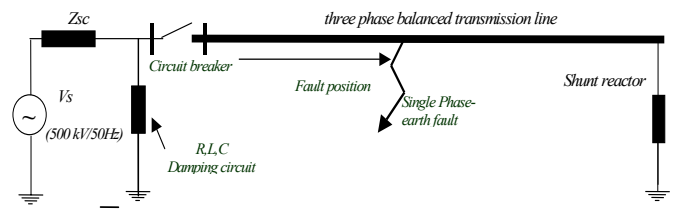

Figure 4. Single-phase equivalent system diagram

This system consists of the following elements. A voltage source is modelled by a three phase ideal sinusoidal source, with a rated voltage of 500 kV, and a nominal frequency of 50 Hz, in series with an impedance Z1 = 0.5+25.1 ohm, chosen to yield a short circuit current of 10 kA. A three phases balanced transmission line, modelled by distributed line, with constant parameters and 400 km length. A shunt compensation load modelled by a three-phases balanced inductive load is also connected at the line receiving side. The circuit breaker used for energizing /de-energizing the line is herein modelled by an ideal switch.

For the simulations, three compensation rates were considered (30%, 50%, 80%). A single-phase asymmetrical fault modelled by an ideal switch between one phase (phase A) and the ground in series with a resistance of $R_{fault} = 1\Omega$. This is to take into account the high frequency transient phenomenon induced by tripping operations, in presence of fault. Five typical fault positions were considered: 0, ¼, ½, ¾, 1, of the line length. The reference signal chosen for this analysis is the composite voltage V_β defined in Section 4.2.

From Table 4.1, one may see the consistency of a dominant low frequency component at about 26.63Hz-26.67 Hz. This clearly indicates that a same oscillating process occurs at the

breaker poles opening which is independent from the fault position. This may be expected because no significant change has occurs in the system parameters constant line's parameter and compensation load (30%). On the other hand on may observe the presence of a higher frequency mode between 156Hz-366Hz, which is inversely proportional to the fault location (except for at the line sending terminal). This mode is due to the discharge process in the fault loop, which is mainly related to the system neutral capacitance, the relevant line inductance and the fault resistance. As the fault distance increases the line capacitance and inductance in the faulted loop also increase resulting into a lower natural frequency. Due to the constant fault resistance value 1 ohm the corresponding damping is quite consistent ($14s^{-1} - 16s^{-1}$).

Further simulations, which are not reported here, for lack of space show that the corresponding damping and phase results are probably not very accurate but may be improved by choosing an optimum window length and initial time to start the MP analysis. This is not the aim of the present work, where we are mainly focused onto the frequency estimation. Table 4.2, shows the close relation between the shunt compensation level and these main mode frequency. It shows also that the fault characteristics and the low oscillating process are completely decoupled. In fact, for the same fault position (at ¼ of the line length) the higher frequency (210Hz) is identical for three different compensation rates.

Several simulations show that, for reconstruction purpose, this approach may be applied to phase to earth voltages separately but no significant gain in the considered application is achieved.

An example, where the root locus of the estimated signal poles, the backward prediction error along with the estimated and the analysed signal are given in Appendix B (figure b.1, b.2). In figure b.3, the circuit breaker voltages of the healthy phases B-C are described. As seen at the predicted re-closing moment for both phases the breaker voltage are under the acceptable limit of 0.5 pu that ensure no prospective over-voltage at the line switching.

4 Conclusion

The Matrix Pencil method allows a fast and accurate detection and estimation of transient recovery voltage's provided that the SNR of the measured signal is above an acceptable level. Due to the presence of harmonic components and noise measurement in HV power systems a pre-filtering process seems to be unavoidable for best efficiency of the proposed algorithm. Besides, in case of some slow fluctuation of the analysed signal the use of an adaptive least square approach allows a track and the relevant changes in the signal modal components. Under assumption, that the transient voltage analysed signal is quite stationary in the window of interest, a fast and an accurate prediction of the optimum targets for shunt compensated line synchronous re-closing is made possible.

Fault position	f(Hz)	α	A(pu)	φ(rds/s)
x=0	327.0448	-16.6409	0.0406	-1.5624
	26.6720	-0.3681	1.1888	1.0092
	33.5008	0.6993	0.1063	-0.4830
x = ¼	366.7894	-23.8823	0.0822	0.7545
	26.6329	-0.1093	1.2169	1.0161
	33.2090	0.5255	0.2097	-0.1749
x= ½	343.6607	-16.5825	0.1186	2.6544
	33.8549	-1.8724	0.2313	-0.4361
	26.6138	-0.7945	1.2948	1.0500
x=¾	210.9595	-15.0715	0.1748	1.1800
	26.6608	-0.4261	1.2720	0.9311
	33.5688	-0.2490	0.2288	-0.3030
x=1	158.6893	-14.8835	0.1747	0.5870
	26.6618	-0.1493	1.2533	0.9217
	33.5193	1.0850	0.2243	-0.2195

Table 4.1. Estimated modal components for different fault positions- 30% compensation rate

Compensation level	f(Hz)	α	A(pu)	φ(rds/s)
30%	210.9595	-15.0715	0.1748	1.1800
	26.6608	-0.4261	1.2720	0.9311
	33.5688	-0.2490	0.2288	-0.3030
50%	210.8066	-15.2170	0.1784	1.8794
	42.4962	-0.6652	0.2921	0.1199
	34.1562	-0.3003	1.1414	1.3074
80%	210.8605	-14.0728	0.1758	1.8705
	52.7571	-1.4741	0.3534	0.4657
	42.7623	-0.6714	1.0872	1.6484

Table 4.2. Estimated modal components for different compensation rates- fault at ¼ line length.

Acknowledgements

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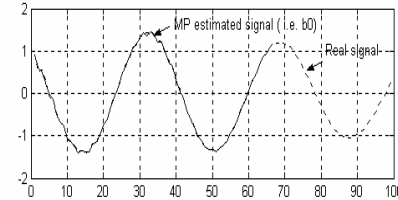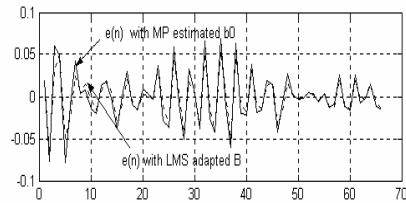[10] Y. Hua, T. K. Sarkar. "Matrix Pencil Method and its performance". *Proceeding ICASSP,* April 1988.

Appendix A

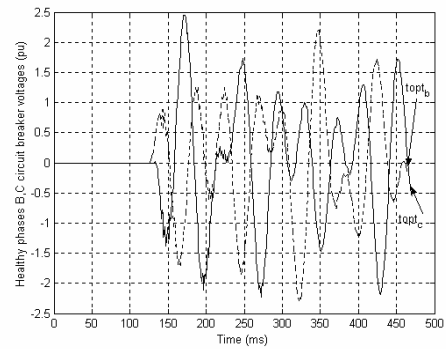| Mode	$f(Hz)$	α	$A(pu)$	$\varphi(rds/s)$
1	26.7238	-1.2112	1.2545	1.0241
2	34.8058	-9.9065	0.1283	-1.1119

Appendix B
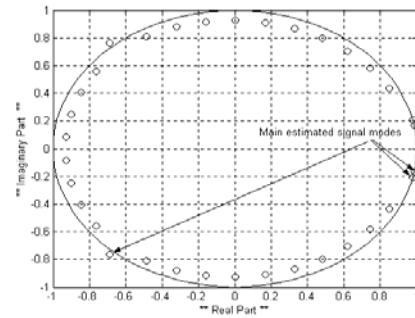
(b.1)   Estimated signal poles root locus

(b.2)   Real and MP estimated signals

(b.3) Breaker B,C voltages - 4th predicted target