CONTROL OF A LABORATORY HELICOPTER USING FEEDBACK LINEARIZATION

M. López-Martínez and F.R. Rubio

Escuela Superior de Ingenieros, Universidad de Sevilla
Camino de los Descubrimientos s/n, 41092-Sevilla (Spain)
fax:+34954487340
e-mail:(mlm,rubio)@cartuja.us.es

Keywords: Non-linear control, Multivariable control, Underactuated system, Robust control, Double Rotor.

Abstract

In this paper a control structure based on feedback (Input-State) linearization has been applied to the elevation subsystem of a laboratory double rotor helicopter. This system is multivariable with 2 inputs and 4 outputs, highly nonlinear and strongly coupled. This article focuses on the elevation subsystem which, in turn, is underactuated with 1 input and 2 outputs. A switching control law between exact and approximate input-state linearization is proposed, which presents good result.

1 Introduction

In this paper a study about feedback linearization applied to the elevation subsystem of a double rotor helicopter is made. The double rotor system is a highly nonlinear, multivariable, underactuated, strongly coupled and non-minimum phase system. The elevation system, in turn, is a nonlinear and underactuated system.

In previous works, see [7] and [8] for details, the control structure was based in partial feedback linearization. Concretely, the computed torque technique was used to linearize the slow dynamics of the system (the body dynamics)[2]. The rotor dynamics was considered to be fast enough to separate both dynamics. In this way the angular velocity of the rotor was considered as constant from the point of view of the body dynamics.

In this paper a complete linearization for the elevation subsystems is searched, taking into account ideas such as approximate linearization, exposed in [5].

This paper is structured as follows: In Section 1 a brief introduction is given. In section 2 the system is described and a model is presented. In the third section the control strategies carried out are described, that is, the full state linearization and an approximate input-state linearization. The next section shows a switching control based on the two linearization laws, in order to control the system in the whole working range. In section 5 simulation results are presented. The last section has concluding remarks and possible future developments.

2 System Description and Model

The laboratory helicopter consists of a 2 DOF mechanism thrusted by two rotors resembling a helicopter. The degrees of freedom are the orientation and the elevation angles. This equipment has the following characteristics: It is multivariable, underactuated, nonlinear, strongly coupled and with non-minimum phase behaviour.

Figure 1: Double Rotor Laboratory Helicopter

In this analysis, the orientation angle is fixed $(\theta = const)$, and the angular velocity of the tail rotor is null $(\omega_g = const = 0)$. The elevation movement will be controlled by the main rotor.

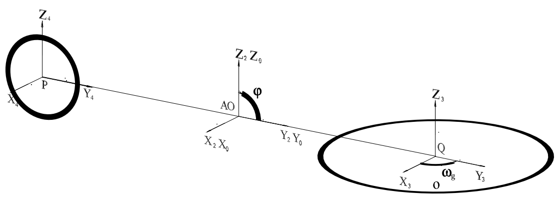

Figure 2: Elevation Subsystem

The equations of the elevation subsystem are as follows

$$I_\varphi \ddot{\varphi} + G_s S(\varphi) + G_c C(\varphi) + K_\varphi \cdot \dot{\varphi} = \hat{L}_g |\omega_g| \omega_g \tag{1}$$

$$I_g \dot{\omega}_g = P_m - (B_g + \hat{D}_g |\omega_g|) \omega_g \tag{2}$$

where:

φ:	Elevation Angle measured from the horizontal plane.
I_φ:	Inertia of the elevation system with respect to its rotation axis.
ω_g:	Angular Velocity of the main rotor.
I_g:	Inertia of the propeller with respect to its rotation axis.
$\hat{L}_g \omega_g$:	Torque due to the aerodynamic force of impulsion in main rotor.
$K_\varphi \cdot \dot{\varphi}$:	Friction Torque.
$G_s S(\varphi)$:	Gravity Torque 1. ($S(\varphi) = sin(\varphi)$)
$G_c C(\varphi)$:	Gravity Torque 2. ($C(\varphi) = cos(\varphi)$)
P_m:	Engine Torque.
B_g:	Friction constant of the engine.
$\hat{D}_g$:	Drag Constant of the propeller.

Rewriting the equations 1 and 2

$$\begin{bmatrix} I_\varphi & 0 \\ 0 & I_g \end{bmatrix} \begin{bmatrix} \ddot{\varphi} \\ \dot{\omega}_g \end{bmatrix} + \begin{bmatrix} G_s S(\varphi) + G_c C(\varphi) + K_\varphi \cdot \dot{\varphi} - \hat{L}_g |\omega_g| \omega_g \\ (B_g + \hat{D}_g |\omega_g|)\omega_g \end{bmatrix} = \begin{bmatrix} 0 \\ P_m \end{bmatrix} \tag{3}$$

It can be seen that there is only an engine (P_m) and 2 DOF, the elevation angle (φ) and the angular velocity of the rotor (ω_g). Therefore it is an underactuated system.

3 Control Strategies: Input-State Linearization

In this section the control structure is presented, which is based on two control loops. The inner one will carry out a feedback input-state linearization in such a way that the resultant system is equivalent to three integrators. The outer loop has to fulfill the specifications imposed on the system.

In the development of the linearization loop it will be seen that such a law is not suitable near the static equilibrium point of the system. In order to control the system in a region around this point, the model of the system will be modified (see [5]) and a new law will be obtained, which will be suitable only in this region. Next, a switching control based on the two laws will be studied and applied depending on the working point.

On the other hand, the outer loop will be closed using an LQR controller that will be designed to control a chain of three integrators. In practice, due to uncertainties, the linearization is not exact. In this way a fourth integrator will be added to ensure the system to be, at least, of type one. In this situation the LQR controller will be designed for an augmented system (See figure 3).

To obtain the linearization law, and taking into account that this system is underactuated, the input-state linearization technique explained in [1] will be applied, which consists of:

1. Expressing the system in the form

$$\dot{X} = f_{(X)} + g_{(X)} \cdot u$$

2. Verifying that the system is input-state linearizable applying the following theorem:

<u>Theorem</u>: The nonlinear system

$$\dot{X} = f_{(X)} + g_{(X)} \cdot u$$

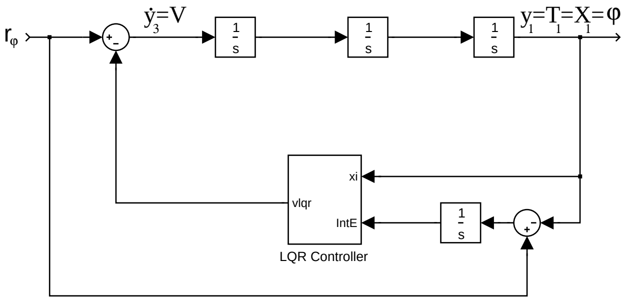

Figure 3: Resultant chain of Integrators controlled by a LQR controller with integral effect

with $f_{(X)}$, $g_{(X)}$ smooth vector fields and $f_{(0)} = 0$, is feedback linearizable *if and only if* there exists a region $\boldsymbol{D}$, containing the origin, in $\boldsymbol{R^3}$ in which the following conditions are satisfied:

(a) the vector fields $\{g, ad_f(g), ad_f^2(g)\}$ are linearly independent

(b) the set $\{g, ad_f(g)\}$ is involutive in $\boldsymbol{D}$, that is, all the Lie brackets of each pair of the vector fields of the set have to be a linear combination of themselves.

3. Finding a new state vector $T_{(x)} = [T_{1(x)}, T_{2(x)}, T_{3(x)}]^T$, that satisfies that the scalar products of the gradient of $T_{1(x)}$, $T_{2(x)}$ and $g_{(x)}$ are nulls, $< dT_1, g > = < dT_2, g > = 0$ with $< dT_3, g > \neq 0$, and such that a transformation as follows can be obtained:

$$u = \alpha_{(X)} + \beta_{(X)} \cdot V = \frac{- < dT_3, f >}{< dT_3, g >} + \frac{V}{< dT_3, g >}$$

where

- u: is the control signal applied to the actuator.
- V: is the control signal coming from the outer control loop and the input signal to the linearized system, which will be equivalent to a number of cascade integrators equal to the dimension of the state vector.

  In order to achieve this, the first state $T_{1(x)}$ has to be found, which will be obtained from the solution with the following equations:

$$\begin{aligned} < dT_1, g > &= 0 \\ < dT_1, ad_f g > &= 0 \\ < dT_1, ad_f^2 g > &\neq 0 \end{aligned} \tag{4}$$

  The new state vector will be created using the first state.

$$T_{(x)} = [T_{1(x)}, T_{2(x)}, T_{3(x)}] = [T_{1(x)}, Ł_f T_{1(x)}, Ł_f^2 T_{1(x)}]$$

The problem can be solved following the steps mentioned before:

1. Definition of the state vector:

$$X = \begin{bmatrix} \varphi - \varphi_{eq} \\ \dot{\varphi} \\ \omega_g \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{5}$$

2. Expressing the equation (5) as

$$\dot{X} = f_{(X)} + g_{(X)} \cdot u$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{-G_s S(\varphi_{eq}+x_1) - G_c C(\varphi_{eq}+x_1) - K_\varphi \cdot x_2 + \hat{L}_g |x_3| x_3}{I_\varphi} \\ \frac{-(B_g + \hat{D}_g |x_3|) x_3}{I_g} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{I_g} \end{bmatrix} P_m \tag{6}$$

3. Verify that the feedback linearization is possible. In order to do this, the cases in which x_3 is positive or negative will be studied separately.

case $x_3 > 0$:

$$ad_f^0(g) = g$$

$$ad_f(g) = \begin{bmatrix} 0 \\ -\frac{\hat{L}_g}{I_g I_\varphi} 2x_3 \\ \frac{1}{I_g^2}(B_g + 2x_3 \hat{D}_g) \end{bmatrix}$$

and

$$ad_f^2(g) = \begin{bmatrix} \frac{2\hat{L}_g}{I_g I_\varphi} x_3 \\ -\frac{2K_\varphi \hat{L}_g}{I_g I_\varphi^2} x_3 - \frac{2\hat{L}_g \hat{D}_g}{I_g^2 I_\varphi} x_3^2 \\ -\frac{2\hat{D}_g}{I_g^3}(B_g + \hat{D}_g x_3) x_3 + \frac{1}{I_g^3}(B_g + \hat{D}_g 2x_3)^2 \end{bmatrix}$$

It can be easily seen that the vector fields $\{g, ad_f(g), ad_f^2(g)\}$ are linearly independent.

Next, the Lie brackets of each pair of the vector fields of the set $\{g, ad_f(g)\}$ will be verified to be a linear combination of themselves, and therefore the set $\{g, ad_f(g)\}$ is involutive in $\boldsymbol{D}$.

$$[g, ad_f g] = \left(\frac{\partial}{\partial x}(ad_f g)\right) g - \frac{\partial g}{\partial x} ad_f g = \lambda_1 g + \lambda_2 ad_f g$$

and

$$\frac{\partial g}{\partial x} = \mathbf{0}$$

yields

$$\left(\frac{\partial}{\partial x}(ad_f g)\right) g = \begin{bmatrix} 0 \\ -\frac{2\hat{L}_g}{I_g^2 I_\varphi} \\ \frac{2\hat{D}_g}{I_g^3} \end{bmatrix} = \lambda_1 \begin{bmatrix} 0 \\ 0 \\ \frac{1}{I_g} \end{bmatrix} + \lambda_2 \begin{bmatrix} 0 \\ -\frac{\hat{L}_g}{I_g I_\varphi} 2x_3 \\ \frac{1}{I_g^2}(B_g + 2x_3 \hat{D}_g) \end{bmatrix}$$

It can be seen that there exists the pair λ_1 and λ_2 when $x_3 \neq 0$, and therefore the set of vector fields is involutive if and only if $x_3 \neq 0$.

Since the two conditions are satisfied, it can be said that the system is input-state linearizable, except next to a region in which $x_3 = 0$.

4. Find a function $T_{(X)}$:

Substituting in the system equations (4) is obtained:

$$< dT_1, g >= \frac{\partial T_1}{\partial x_1} \cdot 0 + \frac{\partial T_1}{\partial x_2} \cdot 0 + \frac{\partial T_1}{\partial x_3} \cdot \frac{1}{I_g} = \frac{\partial T_1}{\partial x_3} \cdot \frac{1}{I_g} = 0$$

$$< dT_1, ad_f g >= \frac{\partial T_1}{\partial x_1} \cdot 0 + \frac{\partial T_1}{\partial x_2} \cdot ad_f g \rfloor_2 + 0 \cdot ad_f g \rfloor_3 = 0$$

$$< dT_1, ad_f^2 g >= \frac{\partial T_1}{\partial x_1} \cdot ad_f^2 g \rfloor_1 + 0 \cdot ad_f^2 g \rfloor_2 + 0 \cdot ad_f^2 g \rfloor_3 \neq 0$$

That demonstrate that $T_{1(X)} = T_{1(x_1)}$

Choosing the simplest function $T_{1(X)} = x_1$, yields

$$< dT_{3(X)}, g >= \nabla T_3 \cdot g = \text{Ł}_g T_3 = \text{Ł}_g \text{Ł}_f^2 T_1 = \frac{\hat{L}_g}{I_\varphi I_g} 2x_3 \neq 0 \Longleftrightarrow x_3 \neq 0$$

$$\begin{aligned} < dT_3, f >= \nabla T_3 \cdot f = \text{Ł}_f T_3 = \text{Ł}_f^3 T_1 = \\ = \left(\frac{K_\varphi \left(G_s S(\varphi_{eq} + x_1) + G_c C(\varphi_{eq} + x_1)\right)}{I_\varphi^2}\right) + \\ + \left(\frac{G_c S(\varphi_{eq} + x_1) - G_s C(\varphi_{eq} + x_1)}{I_\varphi} + \left(\frac{K_\varphi}{I_\varphi}\right)^2\right) \cdot x_2 - \\ - \left(\frac{\hat{L}_g}{I_\varphi}\left(\frac{K_\varphi}{I_\varphi} + \frac{2(B_g + \hat{D}_g x_3)}{I_g}\right)\right) \cdot x_3 \cdot x_3 \end{aligned} \tag{7}$$

Next, the general case will be written including the case $\boldsymbol{x_3 < 0}$. The equations results:

$$< dT_{3(X)}, g >= \nabla T_3 \cdot g = \text{Ł}_g T_3 = \text{Ł}_g \text{Ł}_f^2 T_1 = \frac{\hat{L}_g}{I_\varphi I_g} 2|x_3| \neq 0 \Longleftrightarrow |x_3| \neq 0$$

$$\begin{aligned} < dT_3, f >= \nabla T_3 \cdot f = \text{Ł}_f T_3 = \text{Ł}_f^3 T_1 = \\ = \left(\frac{K_\varphi \left(G_s S(\varphi_{eq} + x_1) + G_c C(\varphi_{eq} + x_1)\right)}{I_\varphi^2}\right) + \\ + \left(\frac{G_c S(\varphi_{eq} + x_1) - G_s C(\varphi_{eq} + x_1)}{I_\varphi} + \left(\frac{K_\varphi}{I_\varphi}\right)^2\right) \cdot x_2 - \\ - \left(\frac{\hat{L}_g}{I_\varphi}\left(\frac{K_\varphi}{I_\varphi} + \frac{2(B_g + \hat{D}_g |x_3|)}{I_g}\right)\right) \cdot x_3 \cdot |x_3| \end{aligned} \tag{8}$$

5. Finally, the control signal to apply to the actuator is obtained through the law:

$$u = \frac{- < dT_3, f >}{< dT_3, g >} + \frac{V}{< dT_3, g >} = \frac{V - \text{Ł}_f^3 T_1}{\text{Ł}_g \text{Ł}_f^2 T_1}$$

It can been seen that the diffeomorphism obtained is well-defined for all values of the state X except for the value $x_3 = 0$. Therefore, the control law will be well-defined if and only if $< dT_3, g >$ is well-defined for all values of the state X, which is not satisfied when $x_3 = 0$. In this way, the linearization that has been obtained is not global for all X. So a region around the $x_3 = 0$ has to be studied and find what values of $x_3 = \omega_g$ make $\text{Ł}_g \text{Ł}_f^2 T_1$ be too small and make **u** be too high, and therefore cause a saturation phenomenon in the actuator without having null velocity.

The minimum velocities of the rotors that do not cause saturation of the law have to be determined.

6. The outer controller will have to be designed for a linear system equivalent to three cascade integrators.

Approximate Input-State Linearization

In this section a simplified model of the aerodynamic forces applied to the system has been used. It is well-known, from the dimensional analysis of these forces, that they vary proportionally with the square of the angular velocity. The simplification consists in linearizing this force in a region that contains $x_3 = 0$, that is, linearize the force when the angular velocity is next to zero. In this region, the constant L_g will have a value proportional to the medium value of the angular velocity, concretely, the angular velocity when switching between both laws. This fact will be demonstrated in next section.

The equations of the elevation subsystem are the following:

$$I_\varphi \ddot{\varphi} + G_s S(\varphi) + G_c C(\varphi) + K_\varphi \cdot \dot{\varphi} = L_g \omega_g \quad (9)$$

$$I_g \dot{\omega}_g = P_m - (B_g + D_g)\omega_g \quad (10)$$

where the constants L_g and D_g are different from those of the quadratic forces $\hat{L}_g$ and $\hat{D}_g$.

Applying the same procedure than before, it is noticed that the vector fields $\{g, ad_f(g), ad_f^2(g)\}$ are linearly independent and also constant. Therefore, the Lie brackets of each pair of vector fields are nulls, and can be expressed as a linear combination of themselves. In this way, the set $\{g, ad_f(g)\}$ is involutive in $\boldsymbol{D}$.

Since the two conditions are satisfied, it can be said that the system is input-state linearizable.

Choosing the simplest function $T_{1(X)} = x_1$, yields

$$< dT_{3(X)}, g >= \frac{L_g}{I_\varphi I_g} \neq 0$$

as desired.
On the other hand

$$< dT_3, f >= \left(\frac{K_\varphi \left(G_s S(\varphi_{eq} + x_1) + G_c C(\varphi_{eq} + x_1) \right)}{I_\varphi^2} \right) + \\ + \left(\frac{G_c S(\varphi_{eq} + x_1) - G_s C(\varphi_{eq} + x_1)}{I_\varphi} + \left(\frac{K_\varphi}{I_\varphi} \right)^2 \right) \cdot x_2 - \\ - \left(\frac{L_g}{I_\varphi} \left(\frac{K_\varphi}{I_\varphi} + \frac{B_g + D_g}{I_g} \right) \right) \cdot x_3 \quad (11)$$

Finally, the control signal to apply to the actuator is the following one:

$$u = \frac{- < dT_3, f >}{< dT_3, g >} + \frac{V}{< dT_3, g >}$$

It can be noticed that this diffeomorphism is well-defined for all value of the state vector X. Furthermore, the control law is also well-defined due to the fact that the value of $< dT_3, g >$ is a non-null constant. Therefore, the obtained linearization is global for all X.

4 Switching between Exact and Approximate Input-State Linearization

In previous sections, exact feedback linearization has been demonstrated to be valid far from the point of static equilibrium of the system, in which engine saturation ensues. It has been also shown that in a region next to this point an approximate linearization law is valid using a linear model of impulsion.

To follow up, in this section, a switching law between the exact and the approximate laws, will be carried out. The election of the switching velocity depends only on the saturation phenomenon, taking into account a non-abrupt switching.

Next, the conditions to ensure a soft switching will be developed.

Linear Impulsion Model

To sum up, the control law applied to the actuator will follow the law

$$u_1 = \frac{V - Ł_f^3 T_1}{Ł_g Ł_f^2 T_1} = \frac{V - \alpha_1}{\beta_1} = \frac{V}{\beta_1} - \xi_1$$

where

$$\beta_1 = \frac{L_g}{I_\varphi I_g} \quad (12)$$

$$\alpha_1 = K_1 + K_2 - \left(\frac{L_g}{I_\varphi} \left(\frac{K_\varphi}{I_\varphi} + \frac{B_g + D_g}{I_g} \right) \right) \cdot x_3 \quad (13)$$

and

$$K_1 = \left(\frac{K_\varphi \left(G_s S(\varphi_{eq} + x_1) + G_c C(\varphi_{eq} + x_1) \right)}{I_\varphi^2} \right) \quad (14)$$

$$K_2 = \left(\frac{G_c S(\varphi_{eq} + x_1) - G_s C(\varphi_{eq} + x_1)}{I_\varphi} + \left(\frac{K_\varphi}{I_\varphi} \right)^2 \right) \cdot x_2 \quad (15)$$

therefore

$$\xi_1 = \frac{\alpha_1}{\beta_1} = \frac{K}{L_g} - M x_3 - B_g x_3 - D_g x_3 \quad (16)$$

with

$$K = (K_1 + K_2) I_\varphi I_g \quad (17)$$

$$M = \frac{I_g K_\varphi}{I_\varphi} \quad (18)$$

Quadratic Impulsion Model

$$u_2 = \frac{V - Ł_f^3 T_1}{Ł_g Ł_f^2 T_1} = \frac{V - \alpha_2}{\beta_2} = \frac{V}{\beta_2} - \xi_2$$

where

$$\beta_2 = \frac{2 \hat{L}_g |x_3|}{I_\varphi I_g} \quad (19)$$

$$\alpha_2 = K_1 + K_2 - \left(\frac{\hat{L}_g}{I_\varphi} \left(\frac{K_\varphi}{I_\varphi} + \frac{2(B_g + \hat{D}_g |x_3|)}{I_g} \right) \right) \cdot x_3 \cdot |x_3| \quad (20)$$

and K_1 y K_2 are the same that in the fore mentioned case, and therefore

$$\xi_2 = \frac{\alpha_2}{\beta_2} = \frac{K}{2\hat{L}_g|x_3|} - \frac{M}{2}x_3 - B_g x_3 - \hat{D}_g|x_3|x_3 \qquad (21)$$

with K y M the same mentioned before.

Non-Abrupt Switching Conditions

In order to ensure a soft switching between both laws, the following condition has to be imposed, that is, $u_1 = u_2$ at the switching instant. Similarly V will be imposed to be the same at the switching instant. Due to this, it can be obtained that

$$u_1 = \frac{V}{\beta_1} - \xi_1 = u_2 = \frac{V}{\beta_2} - \xi_2$$

then

$$-\frac{\beta_1 - \beta_2}{\beta_1\beta_2}V = \xi_1 - \xi_2$$

Since this equality has to be valid for all V, the following two conditions are taken out.

$$\beta_1 = \beta_2 \quad \Longrightarrow \quad \xi_1 = \xi_2$$

Analyzing these conditions, the following relations are obtained. From the first condition,

$$L_g = 2\hat{L}_g|x_3|$$

From the second condition,

$$D_g = \hat{D}_g|x_3| - \frac{M}{2}$$

5 Simulation Studies

Switching Linearizing Laws with an external LQR

Taking into account the non-abrupt switching conditions, an appropriate value of ω_s has to be chosen in such a way that the exact linearizing law does not generate a control signal u that makes the engine saturate. The value of $\omega_s = 0.08{\cdot}\omega_{max}$ has been chosen, where ω_{max} is the maximum velocity of the rotor.

Figures 4, 5 and 6 show respectively the signals V, u and φ of this controller.

Figure 5 shows the quadratic linearization signals u_2 versus the linear one u_1. It can be seen that the linear one is smooth near the zero and sharp for higher values. In the quadratic one the opposite occurs. Due to this fact the resultant switching signal U is always smooth, as can be seen in figure 6 and the same happens to the angular velocity of the rotor w.

Figure 7 shows the system response in the elevation angle using a square wave as reference, which serves to demonstrate the quality of the control performance achieved.

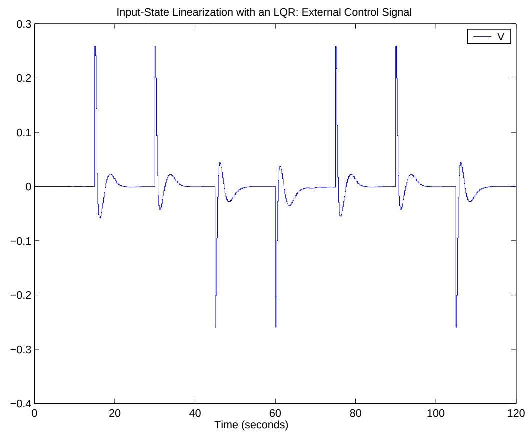

Figure 4: Control signal generated by the external controller

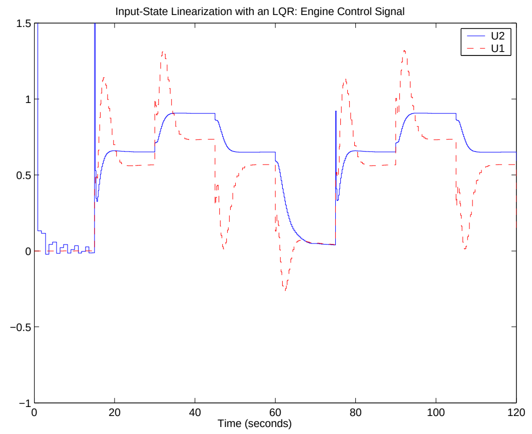

Figure 5: Quadratic linearizing signal versus the linear one

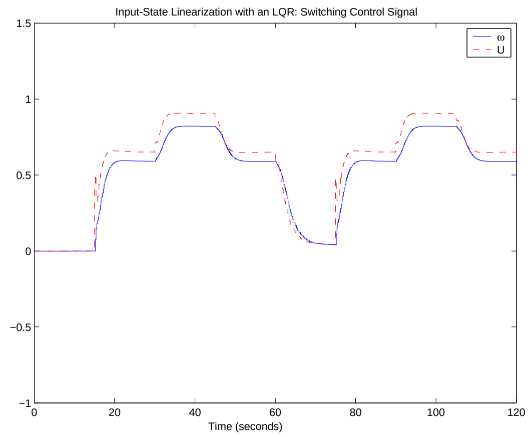

Figure 6: Switching linearizing control signal

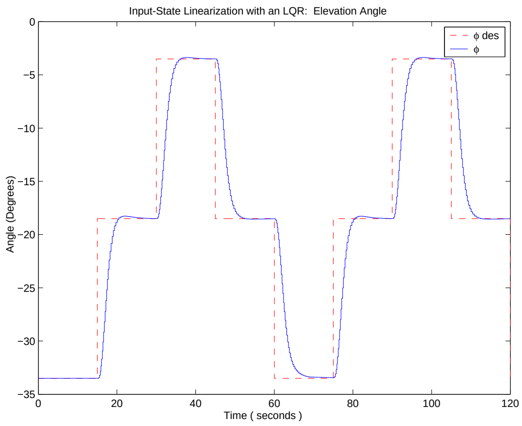

Figure 7: Elevation angle of the system

Switching Linearizing Controller with external integral-LQR

The difference with respect to the previous case is the inclusion of an additional integral term of the error in the outer loop to ensure that the steady state error is null. This structure is necessary when the linearization law is applied to the real set, where, due to uncertainties in the model, there exists a non-null steady-state error. In simulation the results obtained in this case are similar to those of the previous section, so the figures will be omitted.

6 Conclusions

In this paper an input-state linearization law has been applied to the elevation subsystem of a laboratory double rotor helicopter. As the exact input-state linearization provides a law that cannot be applied in the whole operating range, a switching law has been developed. The second law applied, has been obtained using an approximation of the model in the working range, in which the exact law made the engine sature. Both laws have been simulated using external LQR and integral-LQR controllers designed for a chain of three integrators. The best results have been obtained with the switching law.

As a possible future development, a suitable linearizing law will be searched out for the complete laboratory helicopter, using new results for non-minimum phase MIMO systems and taking into account [3] and [4].

Acknowledgements

The authors would like to acknowledge CICYT for funding this work under grant DPI2001-2424-C02-01.

References

[1] Spong, M.W. and Vidyasagar, M., (1989) “Robot Dynamics and Control. Chapter 10: Feedback Linearization”. New York, John Wiley and Sons. ISBN:0-471-50352-5

[2] Spong, M.W “Underactuated Mechanical Systems”

[3] Isidori, Alberto, (1996) “Nonlinear Control Systems”. London, Springer-Verlag. ISBN:3-540-19916-0

[4] Sastry, Shankar. (1999) “Nonlinear Systems: Analysis, Stability and Control. Springer-Verlag, New York 1999. ISBN:0-387-98513-1

[5] Hauser, J., Sastry, S. and Kokotovic, P., (1992) “Nonlinear Control Via Approximate Input-Output Linearization: The Ball and Beam Example”. IEEE Transactions on Automatic Control, Vol 37, No.3.

[6] Mullhaupt, P., Bonvin, B., (1996) “A Discrete-Time Decoupling scheme for a differentially cross-coupled system”. 13th IFAC Triennial World Congress, San Francisco, USA.

[7] López-Martínez, M., Castaño, F. and Rubio, F.R., (2000) “Optimal Control of a 2-DOF Double Rotor System” Controlo’00: 4^o Portuguese Conference on Automatic Control. ISBN: 972-98603-0-0

[8] López-Martínez, M., Ortega, M.G. and Rubio, F.R., (2001) “Control Robusto de un Sistema de dos Rotores en Cuadratura”. XXII Jornadas de Automática. Barcelona. ISBN: 84-699-4593-9