ENERGY SAVING BENEFITS OF PATH PLANNING FOR AUTONOMOUS UNDERWATER VEHICLES IN MARINE ENVIRONMENTS WITH EDDIES OF VARIABLE SIZE

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Abstract

This work explores the benefits, in terms of energy cost, of path planning in marine environments showing certain spatial variability. Specifically, extensive computations have been carried out to calculate, by means of dynamic programming, optimal paths on ocean environments with eddies. The length scales of eddies are different for each environment and range from scales of tens to hundred meters. To get statistical confidence, different realizations of the eddy field and starting-ending points of the path have been considered for each environment. Results indicate that energy costs of planned paths decreases if the length scale of the eddies is increased. Substantial energy savings of planned paths compared to straight line trajectories are obtained when eddy structures are of 40 m size or greater.

1 Introduction

Autonomous underwater vehicles (AUVs) must frequently operate in ocean environments characterized by complex spatial variability [5]. This spatial complexity is induced by the turbulent nature of the ocean, described by the continuos change of a wide range of spatial and time scales. This variability, ranging from scales order of centimetres up to large scale ocean currents, can strongly modify the energy consumption of AUVs motions [3].

Numerical ocean models are employed to provide nowcasts and forecasts of ocean variability [4]. A typical numerical ocean model consists of finite difference equations representing the momentum, heat and salt balance in a determined area. These equations are integrated forward in time to predict the evolution of the current field, temperature and salinity at different depths, given the wind stresses and buoyancy forcing at the sea surface. Numerical ocean models can span a considerable part of the large scale ocean variability, while small scale ocean variability of the same spatial scale as AUVs ($\sim 1\ m$-10 m) is usually missed due to computing limitations. This lack of information constitutes a serious problem when strong momentum exchange between the AUV and small scale ocean structures occurs [2].

Predictions of large scale current fields constitute a valuable information when vehicles have to operate energy-exhaustive missions in environments characterized by comparatively strong currents. In such cases, information of the environmental current field can be incorporated into existing path finding algorithms to plan safety routes with minimum energy cost [1]. On the other hand, the knowledge of the current field, to be included in the optimal planning process, has a cost in itself, that may be due to several factors, as availability of nowcast and forecast data, sophistication of the oceanographic model, available computational time. So it becomes of interest to establish the amount of energy saving that maybe expected through optimized planning as related to the oceanic variability.

This work explores the dependence of energy saving of a planned path with the spatial scale of the ocean structures existing in the environment. Section II defines the path planning problem to be solved. Section III displays the results obtained from navigating an hypothetical AUV following optimal trajectories through ocean environments characterized by dynamical structures of different size. Section IV concludes the work.

2 The path planning problem

Consider a two-dimensional underwater environment discretized in space over an $n \times p$ regular grid along the cartesian directions. Let $\Delta x, \Delta y$ be the gridding intervals in the x, y axis respectively. Any point in the grid defines a node $\mathbf{x} = (h, k)$, $0 \leq h < n$, $0 \leq k < p$. A path Γ between a starting node **s** and a destination node **d** is defined through a sequence of nodes $\Gamma = \{\mathbf{s}, \cdots, \mathbf{x}_i, \mathbf{x}_{i+1}, \cdots, \mathbf{d}\}$, and it is made by straight-line segments connecting any two adjacent nodes $\mathbf{x}_i, \mathbf{x}_{i+1}$. In practice, it is assumed that the AUV navigation is defined through via-points that are the nodes of the grid. A current velocity vector $\mathbf{v}_c(x, y) = (\dot{x}_c, \dot{y}_c)$ is defined at any point in space. Within this setting, the path planning problem can be enunciated as follows: given a start node **s**, a destination node **d** and a current velocity field, find a path such that the energy cost required for a vehicle travelling along the path at a constant speed c is minimum, subject to the constraints that the path does not intersect any solid obstacle. For simplicity, we will assume that $n = p$ and that the start and destination nodes define the beginning and ending coordinates in the x-axis, i.e., $\mathbf{s} = (0, \cdot), \mathbf{d} = (n-1, \cdot)$. All paths will be considered strictly monotone with respect to the x-coordinate, and such that any two adjacent nodes $\mathbf{x}_i, \mathbf{x}_{i+1}$ satisfy the relation $h_{i+1} = h_i + 1$. This implies that each admissible path is a sequence of m nodes.

A dynamic programming approach (as in [1]) was employed to compute optimal paths. The energy cost required by a given path is evaluated computing and adding up the energy required to overcome the drag generated by the current field in each segment constituting the path. Consider the i-th segment $X_{i-1}X_i$ connecting the nodes $\mathbf{x}_{i-1}, \mathbf{x}_i$ of any arbitrary path; let d_i indicate its length, and let $\mathbf{e}_i$ be a unitary vector oriented along the segment $X_{i-1}X_i$ in the direction of desired motion of the vehicle. Since it is required that the vehicles moves along the segment at the nominal speed c, at any point (x, y) along the segment the vehicle must have a velocity $\mathbf{v}_i(x, y)$ given by:

$$\begin{aligned} \mathbf{v}_i(x,y) &= c\mathbf{e}_i - \mathbf{v}_c(x,y) \\ (x,y) &\in X_{i-1}X_i \end{aligned} \tag{1}$$

Consider the quantity:

$$J_i = \int\int_{X_{i-1}X_i} \|\mathbf{v}_i(x,y)\|^3 dxdy \tag{2}$$

then the energy cost W_i for the i-th segment is given by the expression:

$$W_i = \frac{\rho J_i}{c} \tag{3}$$

where ρ is a constant depending on the dimensions of the vehicle and water properties. The total cost of a given path is finally given by the summation $\sum_i^m W_i$. Note that W_i is the cost-to-go from node X_{i-1} to node X_i to be computed at each iteration by dynamic programming. In practical implementation, the computation of J_i is carried out as a finite summation over a grid of points that may or may not coincide with the grid of via-point nodes.

Different current fields have been defined on a grid of 100×100 points. The distance between grid points corresponds to $3\ m$, so that the total system size is $L = 300\ m$. The currents were obtained from a streamfunction field $\Psi(x, y)$ randomly generated from a specific isotropic power spectrum with random phases. The spectrum is peaked at a determined spatial scale in order to obtain a field of eddies with homogeneous length scale. The velocity field is obtained from the streamfunction field from the relations:

$$\dot{x}_c = -\frac{\delta\Psi}{\delta y} \tag{4}$$

$$\dot{y}_c = \frac{\delta\Psi}{\delta x} \tag{5}$$

Maximum velocity of the flow in all generated current fields is $0.4\ m/s$.

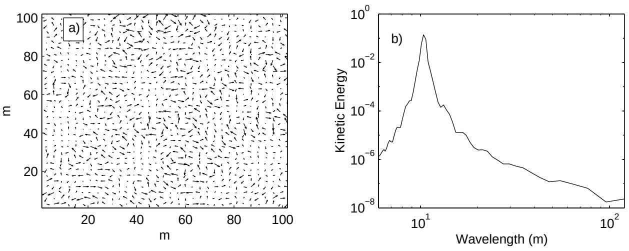

Figure 1: a) A realization of a randomly generated current field with eddy structures of $\approx 10\ m$ size. Only a section of $100 \times 100\ m^2$ is shown for clarity. b) Power spectrum of the kinetic energy of the current field.

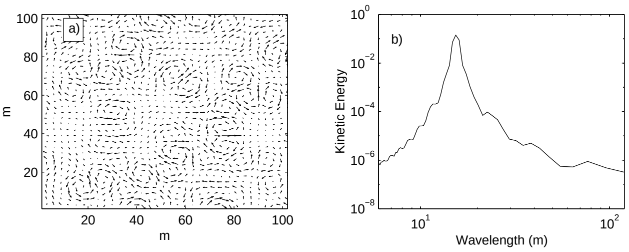

Figure 2: a) A realization of a randomly generated current field with eddy structures of $\approx 15\ m$ size. Only a section of $100 \times 100\ m^2$ is shown for clarity. b) Power spectrum of the kinetic energy of the current field.

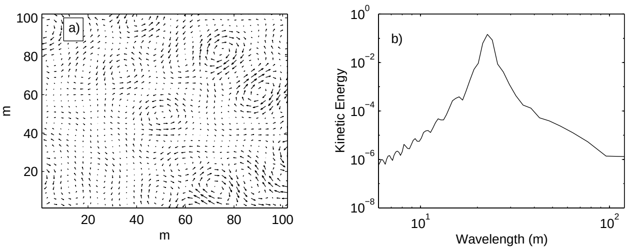

Figure 3: a) A realization of a randomly generated current field with eddy structures of $\approx 20\ m$ size. Only a section of $100 \times 100\ m^2$ is shown for clarity. b) Power spectrum of the kinetic energy of the current field.

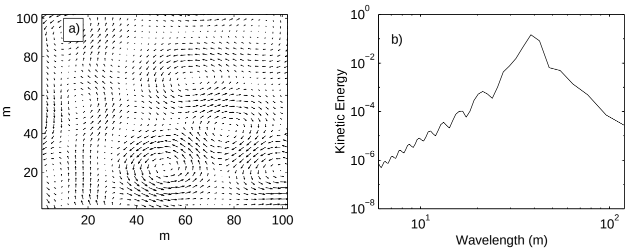

Figure 4: a) A realization of a randomly generated current field with eddy structures of $\approx 40\ m$ size. Only a section of $100 \times 100\ m^2$ is shown for clarity. b) Power spectrum of the kinetic energy of the current field.

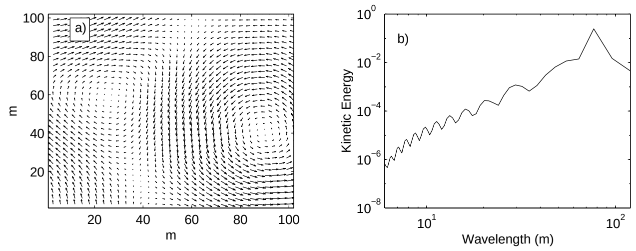

Figure 5: a) A realization of a randomly generated current field with eddy structures of $\approx 75\ m$ size. Only a section of $100 \times 100\ m^2$ is shown for clarity. b) Power spectrum of the kinetic energy of the current field.

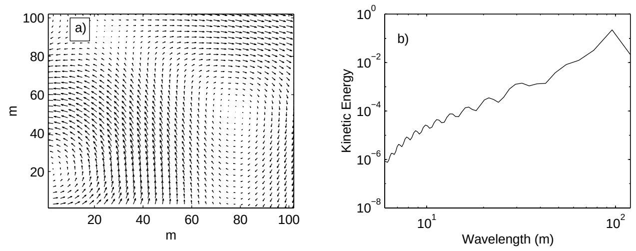

Figure 6: a) A realization of a randomly generated current field with eddy structures of $\approx 100\ m$ size. Only a section of $100 \times 100\ m^2$ is shown for clarity. b) Power spectrum of the kinetic energy of the current field.

3 Results

Six cases of ocean environments with typical eddy sizes of $10\ m$ (Figures 1a and b), $15\ m$(Figures 2a and b), $20\ m$ (Figures 3a and b), $40\ m$ (Figures 4a and b), $75\ m$(Figures 5a and b) and $100\ m$ (Figures 6a and b) have been considered. An ensemble of five realizations of the eddy field has been carried out for each eddy size. For each realization, five optimal paths have been computed from initial locations with y-coordinates $y = 45, 105, 135, 195$ and $255\ m$, to ending points at the opposite side of the simulated ocean basin and y-coordinates equal to the starting point(Figure 7). Consequently, a total of twenty-five optimal paths have been computed for each eddy size. The dimensions of the AUV ($1\ m$) are considered much smaller than the dimensions of the ocean basin. Thus static route planning, which does not account for the AUVs dynamics and described in previous section, is appropiate. The required navigation speed, c and drag coefficient were arbitrarily fixed to $0.5\ \frac{m}{s}$ and $0.004\ s^{-1}$, respectively.

Figure 8 summarizes the results obtained from the different simulations. Specifically, it displays the mean and variance of energy consumption of an hypothetical AUV navigating through ocean enviroments with different eddy size and following optimal paths(dashed-circle), the mean and variance of energy consumption if trajectories are straightlines joinning starting and ending points (dashed-star) and the energy cost of a control case represented by straight trajectories in an ocean environment without eddies (dashed-dot). Two remarkable features should be pointed out. First, mean energy costs following optimal paths show an exponential-like behaviour, with an initial fast decreasing of energy consumption when increasing eddy size and an ending tail. Differences on energy costs between optimal and straight trajectories can be considerable in ocean environments populated with large and energetic eddy structures. In the present simulations, energy savings as much as 70% have been found. Efficiency of path planning, is reduced when eddy sizes are few tens the AUV dimensions.

A second remarkable result is related to statistical robustness of optimal paths. Robustness is measured by the variance around mean value. Figure 8 shows that energy costs between optimal paths computed for different random realizations and/or starting-ending points, are relatively similar. This behaviour is not observed when straight trajectories are choosen for AUV navigation. In this case, variance increases when increasing eddy size. This feature comes from increasing spatial inhomogenity when increasing the eddy size for fixed crossing distance. When eddies are large relatively to the crossing distance, cases with currents favouring straightline trajectories strongly differ in energy cost from cases where the currents are opposing the motion. Finally, it is important to notice that ocean spatial variability benefits large range AUV motions in the ocean if real-time ocean modelling and path planning capabilities are available.

4 Conclusions

A systematic simulation study of the energy savings obtained through optimal AUV path planning taking into account the current velocity field in the ocean has been presented. The study had the objective of putting in relation the size of the current field variability with the expected energy saving due to the optimized path planning. A substantial energy saving, as compared to straight line paths, is obtained for ocean eddy structures of size of $40m$ or greater.

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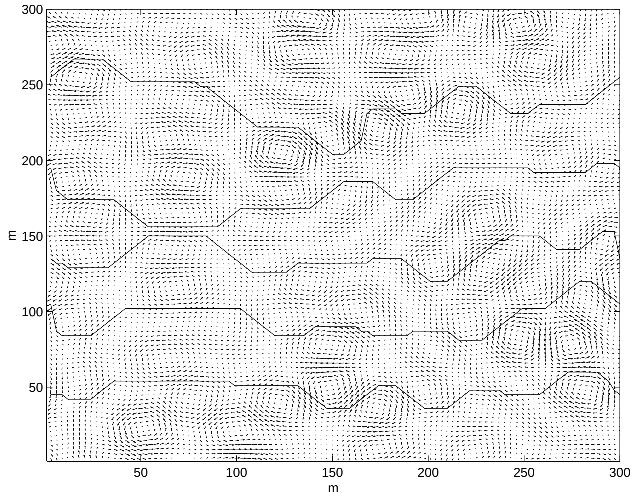

Figure 7: Optimal paths computed from five different starting points in a realization of an ocean environment with $\approx 40\ m$ eddy size

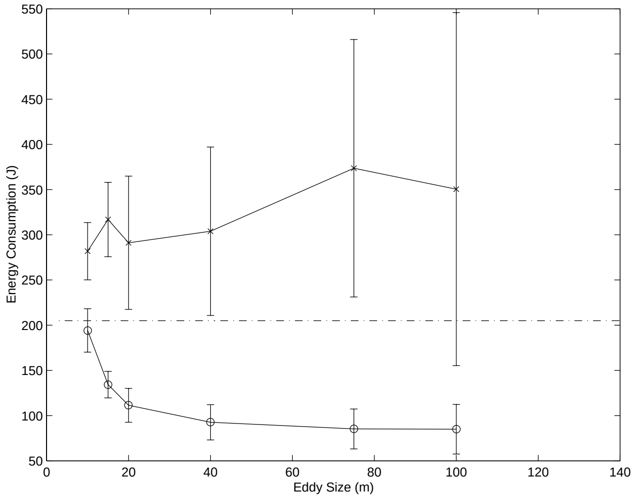

Figure 8: Mean and variance of energy consumption versus eddy size when following an optimized path (circle), straight trajectory (star) and moving on a straight line in absence of currents (homogeneous ocean) (dashed-dot line)

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