

LINEAR IMPULSIVE DIFFERENTIAL EQUATIONS FOR HYBRID SYSTEMS MODELING

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Abstract

Hybrid systems are those that have a mixture of continuous and discrete valued state variables and a combination of continuous and discrete dynamics. The modeling framework of hybrid systems has received considerable attention over the past few years. The modeling framework is aimed to build a mathematical model which is suitable for complex dynamical analysis and control synthesis using hybrid systems. This paper shows that the linear impulsive differential equations may promote a unified framework for hybrid systems modeling.

1 Introduction

Physical systems are often subjected to disturbances, changing operation conditions and component failures, and in many cases, the changes take place in a short space of time. Examples are found for example in biological systems and mechanical systems subjected to shock. Such systems can be modeled by differential and/or difference equations which jump instantaneously from one state to another. If there is no jump over some time interval, then the mathematical model is described by the solution of a differential and/or difference equation. The analysis of an instantaneous change in the state of a system is much more complicated. Mathematical models of systems that undergo instantaneous changes in the state are called *impulsive systems*.

Impulsive dynamical systems are characterized by the occurrence of abrupt changes in the state of the system which occur at certain time instants over a period of negligible duration. The dynamic behaviour of such systems is more complex than the behaviour of dynamical systems without impulsive effects. The presence of impulses means that the state trajectory does not preserve the basic properties which are associated with non impulsive dynamical systems, such as existence, uniqueness and continuity with respect to initial conditions. Recently, impulsive differential equations have been used for hybrid systems modeling.

Hybrid systems are used for modeling and analyzing systems which have interacting continuous-valued and discrete-valued state variables. The continuous state variable may be the value of the state in continuous time, discrete time or a mixture of

the two. The mathematical model of the continuous state is described by a differential or difference equation. The discrete state variable is generally represented by a finite state digital automaton or an input/output transition system. The behaviour of the hybrid system is influenced by state variables which interact at an *event* (or *trigger*) time which occurs whenever the evolution of the system satisfies a particular condition which then initiates changes in the state variables.

Hybrid control systems are control systems where both plant and controller consist of continuous and discrete state variables. Recently, a common framework used for a hybrid control has been developed by separating the system components into three sub components; namely, a plant with a conventional controller, a discrete state variable controller and an interface [3]. The three layered configuration is shown in Figure 1. The plant and the conventional controller are usually modeled by differential or difference equations. The discrete state variable controller is designed via a rule based decision process which supervises the conventional controller. This interface facilitates communication between the plant and the discrete controller and simultaneously converts the continuous state to discrete state variables (C/D), and vice versa (D/C). Some examples of hybrid control systems are discussed in [3].

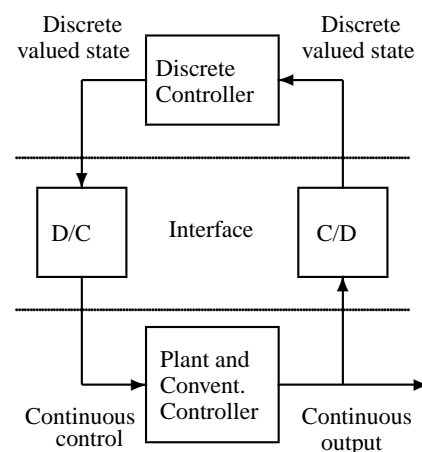


Figure 1: Three Layers Framework

The modeling framework for hybrid systems has received considerable attention over the past few years. The aim is to build a mathematical model which is suitable for complex dynamical

cal analysis and control synthesis using hybrid systems. The models proposed in the literature reflect a wide range of both applications and justifications. An overview of hybrid system modeling can be found in [7, 8]. In that paper, a unified approach is also proposed which is a generalization of five models of hybrid control systems developed from system and control perspectives found in [4, 5, 9, 21, 24]. The unified mathematical model tries to capture all possible important aspects of continuous and discrete valued variables as well as their interaction.

Currently, there are two paradigms in the theoretical framework of hybrid systems: *aggregation* and *continuation*. In the aggregation approach, the system state variables are treated as a discrete event dynamic system (or a finite automaton) by aggregating the continuous valued state variables via cell to cell partition. A theory of cell to cell mapping as a global method for analyzing nonlinear systems is available [14].

On the other hand, the continuation approach supposes the whole system to be described by either differential or difference equations. In this approach, the discrete valued state variables are considered as uncertainty or disturbances, or embedding them as jump actions in ordinary differential or difference equations. In the first case, an appropriate hybrid controller can be designed using robust control. In the second case, if the jump exhibits changing dynamics, the controller can be designed using a multi controller design approach or the jump linear quadratic (JLQ) method [18].

In applications, the progress of both paradigms have been impeded by the conservatism which is embedded in the design methodology [8]. The aggregation approach, on the one hand, is often faced with the problem of choosing an appropriate partition method and a non determinism of automata that lead to undecidability and computation complexity. On the other hand, the continuation approach is often limited by a compromise of design requirements found in conventional control design. Thus, verification is needed when the resulting controller from one approach is implemented in a real system. Effort is being made to bring about a unified design for both continuous and discrete valued controllers.

2 Systems with Impulsive Effects

There are two main approaches for studying the behaviour of impulsive differential systems. The first approach uses a *generalized function* to represent a *jump discontinuity* in the state with the help of the Dirac function. This approach was developed in [13]. In the second approach, the jump discontinuity is represented by an *impulsive vector* which was initiated by [19] (where the first stability results were obtained) and further developed in [6, 16, 17] and the references cited therein.

The impulsive vector representation provides a general characterization of external disturbances, perturbations or even impulsive controls. The impulsive ordinary differential equations of interest are those characterized by linear systems. The equa-

tions studied here are consequently referred to as linear impulsive differential equations, or simply as *linear impulsive systems (LIS)*. Applications of linear impulsive systems in control designs can be found in [15, 22]. The following example demonstrates the modeling of external disturbances and a mechanical system subjected to collision.

External Disturbance

Allowing the plant state to “instantaneously” change its value during the time period t_k to t_k^+ is a simple way to model “jump” disturbances. Consider a linear system subjected to external disturbances as follows

$$\dot{x}(t) = Ax(t) + Bu(t) + D \sum_k \alpha_k \delta(t - t_k) \quad (1)$$

where δ is the Dirac delta function; that is, $\delta(\sigma) = 0$ for all $\sigma \neq 0$, and

$$\int_0^\epsilon \delta(\sigma) d\sigma = 1 \quad \text{for all } \epsilon > 0 \quad (2)$$

Hence, the system representation (1) can be equivalently represented in the form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t); & t \neq t_k \\ x(t_k^+) &= x(t_k) + d(t_k) & t = t_k \end{aligned}$$

where $d(t_k) = D\alpha_k$. Here an impulsive disturbance at time t_k in (1) is equivalent to an instantaneous change in the state x at time t_k . This representation is known as a linear impulsive dynamical system.

Now consider the effect of a disturbance signal μ which acts on a system with state x according to

$$\dot{x}(t) = Ax(t) + Bu(t) + D\mu(t)$$

where the disturbance signal μ is the output of the linear finite dimensional system

$$\begin{aligned} \dot{x}_\mu(t) &= A_\mu x_\mu(t) + B_\mu \sum_k \alpha_k \delta(t - t_k) \\ \mu(t) &= C_\mu x_\mu(t) \end{aligned}$$

The two equations can be equivalently written more succinctly as

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{x}_\mu(t) \end{bmatrix} &= \begin{bmatrix} A & DC_\mu \\ 0 & A_\mu \end{bmatrix} \begin{bmatrix} x(t) \\ x_\mu(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t); \\ & t \neq t_k \\ \begin{bmatrix} x(t_k^+) \\ x_\mu(t_k^+) \end{bmatrix} &= \begin{bmatrix} x(t_k) \\ x_\mu(t_k) \end{bmatrix} + d(t_k); t = t_k \end{aligned} \quad (3)$$

where the impulsive vector $d(t_k)$ has the constrained form

$$d(t_k) \triangleq \begin{bmatrix} 0 \\ B_\mu \end{bmatrix} \alpha_k$$

A Mechanical System

Consider the linear differential equation studied in [25], which is used to model a multi-input/multi-output mechanical systems subject to collision or shock effects according to

$$M\ddot{y} + D\dot{y} + Ky = Lu + Pf \quad (4)$$

Here $y \in R^d$ is the system coordinate vector, $u \in R^p$ is the force vector, $M \in R^{d \times d}$ is the generalized positive definite inertia matrix, $D \in R^{d \times d}$ is the generalized structural damping matrix, $K \in R^{d \times d}$ is the generalized stiffness damping matrix, $L \in R^{d \times m}$ is the actuator force distribution matrix, and f is the external input which is defined as

$$f = \begin{cases} 0 & : \text{ free motion, } t \neq t_k \\ \Psi & : \text{ during collision, } t = t_k \end{cases}$$

Define $x \triangleq [y^T \quad \dot{y}^T]^T$. Then (4) can be written in the state space form

$$\dot{x}(t) = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ M^{-1}L \end{bmatrix} u(t); t \neq t_k \quad (5)$$

$$x(t_k^+) = x(t_k) + \begin{bmatrix} 0 \\ M^{-1}P \end{bmatrix} \Psi; t = t_k$$

At time t_k , the position y is continuous but the velocity \dot{y} is discontinuous. In this model, the force Ψ represents an impulsive force during collision that results in a jump discontinuity in the velocity components of the states. The mathematical model in (4) could be used in a number of applications such as to model a rigid bar hinged at one end which is subjected to an impact load at time t_k at the other end, or a constrained manipulator [10].

3 Impulsive Differential Equations

Consider a minimal order continuous time linear time invariant plant with state $x(t) \in R^n$, output $y(t) \in R^l$, and input $u(t) \in R^m$ subjected to impulsive vectors $\{d(t_k); k \in Z^+\}$ on the plant state as described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \\ x(t_k^+) &= x(t_k) + d(t_k); \quad t \geq 0 \end{aligned} \quad (6)$$

where both the times t_k and values $d(t_k)$ are unknown. The impulsive vector may represent the external disturbances, failure of the system's components or an impulsive control.

The solution of systems with impulsive effects (6) in the extended state space begins from the initial condition (t_0, x_0) and moves along the trajectory $(t, x(t))$. If at time instants $t_k \geq t_0$ there is an impulsive jump $d(t_k)$, then the state is instantaneously changed to the new state $x(t_k^+) = x(t_k) + d(t_k)$. The state then follows the trajectory with the new initial condition

$x(t_k^+)$ until the occurrence of the next transition time instant at time t_{k+1} . That is, the solutions of impulsive systems are characterized by three components: the dynamics of ordinary differential equations, the transition time instants $\{t_k\}$ and the impulsive vectors $\{d(t_k)\}$.

In order to make the standard notation conform to commonly used notation in system and control theory, some new definitions that correspond to the occurrence of an impulsive jump and the nature of an impulsive jump are considered in the following.

3.1 Impulsive Vectors

1. Open loop impulsive vector

In this case, the impulsive vector $d(t_k)$ is an arbitrary vector in R^n which occurs at time $t_k, \forall k \in Z^+$ which is independent of either the state or the output of the system. This so called *open loop* impulsive vector arises in mathematical models of physical systems as a result of: exogenous disturbances, failure of system components or open loop impulsive controls. Applications of open loop impulsive control can be found in drug management in the human body [23] and optimal space trajectories problems [11].

2. Closed loop impulsive vector

In this case, the impulsive vector $d(t_k)$ is dependent on the state or the output of the system, and can be written in the form

$$d(t_k) = \psi x(t_k)$$

where ψ is either a constant or time varying matrix. This type of impulsive vector is usually found in controlling the behaviour of a linear system in which information about the system is being used to effect the trajectory. Applications of closed loop impulsive vector can be found in the pulse frequency modulation systems studied in [22] and in the recently developed Hybrid Reference Control (HRC) in [15].

3.2 Transition Time Instants

The transition time instants are defined as the times when impulsive vectors occur. The occurrence of impulsive vectors may be the result of external disturbances, system component failures, clock timing, or a logical decision. The time instant also defines an "event" or a "trigger" time which represents a discontinuity in the state of the system. In addition, an event may be used to cause another event at some time in the future. Following [2], the time instants will be classified in two ways.

1. Scheduled time instants

The sequence of the scheduled time instants t_k is given by

$$t_k = t_{k-1} + \tau_k; \quad \tau_k > 0$$

where the value of τ_k is *known* a priori for all $k \in Z^+$. In the simplest case, $\tau_k = \tau$ is constant for all $k \in Z^+$ that leads to *uniform* time instants similar to uniform sampling in digital systems.

2. Conditioned time instants

Conditioned time instants t_k occur if either the time t or the state $x(t)$ satisfies a particular condition. The condition can be defined, for example, as

$$t_k = \{t : \varsigma(t) = \epsilon, t \in R^+\}; \quad \text{or}$$

$$t_k = \{t : \varsigma(x(t)) = \epsilon, (t, x(t)) \in R^+ \times R^n\}$$

Alternatively, the condition can also be given in term of a set $S \subset R^n$ by $t_k = \{t : x(t) \in S, S \subset R^n\}$.

4 Impulsive Dynamical Systems

4.1 Hybrid Systems Representation via LIS

LIS modeling can be extended to cover the problem of systems with switching dynamics which is commonly found in hybrid control system design. The major development of impulsive dynamical systems is to capture the behaviour of an instantaneous jump of state of a dynamical system. However, impulsive vectors may also be used to describe a dynamical system subjected to the output of a higher order model.

From a system and control perspective, it is common for the design to be carried out using the continuation approach, since control system design requires a tractable evolution for both synthesis and assessment of the controller. In terms of discrete variables, the continuation model can be characterized as follows:

1. Autonomous (or controlled) switching

In this case, the vector field of the continuous dynamics changes discontinuously when the state satisfies some constraints (or in response to a control command)

2. Autonomous (or controlled) impulses

In this case, the state of the system jumps discontinuously on the satisfaction of some given constraints (or in response to a control command)

In practice, there might only be one type of discrete variable present, If both types are found, we have the so called *full power* modeling of hybrid systems [7, 8].

To illustrate, consider the dynamics of a system that consists of switching between two dynamic systems according to

$$\dot{x}(t) = A_i x(t); \quad i = 1, 2$$

This system can be written in the form

$$\begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}$$

$$\begin{bmatrix} z_1(t_k^+) \\ z_2(t_k^+) \end{bmatrix} = \begin{bmatrix} z_1(t_k) \\ z_2(t_k) \end{bmatrix} + \begin{bmatrix} d_1(t_k) \\ d_2(t_k) \end{bmatrix}$$

$$x(t) = z_1(t) + z_2(t)$$

where either $d_1(t_k) = -z_2(t_k)$ or $d_2(t_k) = -z_1(t_k)$. This choice of decision vectors then implies that either $x(t) = z_1(t)$

or $x(t) = z_2(t)$ for $t_k < t \leq t_{k+1}$. Modeling switching systems via systems with impulsive effect representation was first observed in [8]. An autonomous switching can be viewed as a special case of autonomous impulse by embedding the discrete state into a larger continuous state via the universal extension property of the phase space R^n [20].

Recent papers on hybrid control design are related to problems of controlling systems with switching dynamics. So far, little attention has been paid to the problem of systems which are subjected to an instantaneous change in the state. The capability of an impulsive differential equation to capture the modeling of switching systems may provide a unified framework for hybrid systems, and promote a new direction in analyzing and synthesizing hybrid control systems.

4.2 Fundamental Properties

Impulsive differential equations are basically piecewise differential equations where the discontinuities in the system state are caused by jumps in the solutions. Most results of the theory of impulsive ordinary differential equations have been developed in [6, 17] (and the references cited therein) but the investigation has been limited to the case where the impulsive vectors have a closed loop representation.

4.2.1 Existence, Uniqueness of Solutions

Let $\Omega \subset R^n$ be an open set, and consider a LIS of the form

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t); & t \neq t_k \\ x(t_k^+) = x(t_k) + d(t_k); & t = t_k \\ x(t_0^+) = x_0 \end{cases} \quad (7)$$

where the solution $x(t) = x(t; t_0^+, x_0) \in \Omega$ for $t \geq t_0$. The impulsive vector $d(t_k)$ and time instants t_k , for each $k \in Z^+$, are defined in the domain Ω which contains the set

$$\mathcal{G} = \{(t, x) \in R \times \Omega : t_k^+ < t \leq t_{k+1}, \forall k \in Z^+ \text{ and } x \in \Omega\}.$$

- Autonomous Linear Impulsive Systems

The system (7) is called an autonomous system if $Bu(t) = 0$ for all $t \geq 0$ and $x \in R^n$.

- Non autonomous Linear Impulsive Systems

The system (7) is called a non autonomous systems if $Bu(t) \neq 0$ for all $t \geq 0$ and $x \in R^n$.

Notice that the initial condition $x(t_0^+) = x_0$ is used rather than $x(t_0) = x_0$. If the time t_0 corresponds to a transition time instant then $x(t_0^+)$ is understood to be the initial condition of the ordinary differential equation. The time evolution of linear impulsive systems consists of *continuous* and *jump discontinuous* functions.

Condition 1 Let Ω be an open set where $\Omega \subset R^n$ and

1. $0 \equiv t_0 < t_1 < t_2 < \dots < t_k < \dots$
2. $d(t_k)$ are bounded impulsive vectors such that $x(t_k^+) = x(t_k) + d(t_k) \in \Omega$

Then the nonempty \mathcal{G}_k is defined by

$$\mathcal{G}_k = \{(t, x) \in R \times \Omega : t_k < t \leq t_{k+1}, x \in \Omega\}$$

The solution x of (7) is continuous from the left on each interval $(t_k, t_{k+1}]$ and has right-hand and left-hand limits, $x(t_k^+)$ and $x(t_{k+1})$ respectively. This set defines the solution of impulsive systems in each region $(t_k, t_{k+1}) \times \Omega$. Therefore

$$\mathcal{G} = \bigcup_k \mathcal{G}_k$$

The presence of impulsive vector $d(t_k)$ at time t_k , means that the impulsive system (7) is non autonomous.

For any $t > t_m > 0$, the solution $x(t) = x(t; t_0^+, x_0)$ can be written in the integral form

$$\begin{aligned} x(t; t_0^+, x_0) &= e^{A(t-t_0)}x(t_0) + \int_{t_0}^t e^{A(t-\sigma)}Bu(\sigma-t_0) d\sigma \quad (8) \\ &+ \int_{t_0}^t e^{A(t-\sigma)} \sum_{k=0}^m d(t_k)\delta(\sigma-t_k) d\sigma \end{aligned}$$

The transition equation (8) gives the state $x(t)$ at time t in term of the state $x(t_0)$ at time t_0 , the input over the time interval $[t_0, t)$ and a sequence of impulsive vectors $\{d(t_k)\}$ occurring at time instants $t_k, k = 0, 1, \dots, m$ where $0 \equiv t_0 < t_1 < \dots < t_m < t$. The uniqueness of the solution is given by the following theorem which follows immediately from the existence and uniqueness of solutions of linear ordinary differential equations (ODE), see for example [12].

Theorem 1 *Consider the linear time invariant impulsive system (7). Suppose Condition 1 holds. Then for a given initial value $(t_0, x_0) \in \mathcal{G}$, there exist a unique solution given by equation (8) which is defined for all $t \in R$.*

4.2.2 Continuity of Solutions

Unlike the ODE, the continuity of the solution of (8) with respect to its initial condition cannot solely be guaranteed by the initial condition $x(t_0^+) = x_0$. The solution of (8) on the interval $(t_k, t_{k+1}]$ with the initial condition $x(t_k^+)$ is given by

$$x(t) = e^{A(t-t_k)}x(t_k^+) + \int_{t_k}^t e^{A(t-\sigma)}Bu(\sigma-t_k) d\sigma$$

The solution follows the initial value problem of ordinary differential equations for each interval $(t_k, t_{k+1}]$.

For the case of events which occur at time instants $\{t_k\}$ such that

$$0 \equiv t_0 < t_1 < \dots < t_k < t_{k+1} < \dots \quad (9)$$

the existence of a solution as in (8) is guaranteed. However, a problem can arise when condition (9) cannot be guaranteed. That is, suppose the initial condition $x(t_0^+) = x_0$ at time t_0^+ is not on the hyperplane $\varsigma_j(x(t))$

$$t_j = \{t : \varsigma_j(x(t)) = \epsilon, (t, x(t)) \in R^+ \times R^n\}$$

Suppose also at the time instant t_j , the solution (t_j, x_j) meets the hyperplane $\varsigma_j(x(t))$ such that the solution $(t_j^+, x(t_j^+))$ lies on the hyperplane $\varsigma_j(x(t))$. Hence, the part of the solution on the interval $t_j < t \leq t_{j+1}$ is also the solution of the interval $t_j \leq t \leq t_{j+1}$.

For example, suppose the plant output of the LIS (7) is given by

$$y(t) = Cx(t) \quad (10)$$

and the transition time instant t_k is defined by the condition:

$$c_m^T x(t_k) = \delta_m$$

where c_m^T is the m -th row of C in (10). Then from (7), $c_m^T d(t_k) = 0$ implies $c_m^T x(t_k^+) = \delta_m$ in which case the solution of the LIS does not leave the hyperplane at $t = t_k$. Mathematically, this implies the solution x for $t > t_k$ is not defined. Moreover, if $c_m^T d(t_l) = 0$ for all $l > k$, then the solution is also not continuable to the right of $t > t_k$.

If $c_m^T d(t_k) = 0$, the existence question can be resolved [1] by assuming that an appropriately small time delay Δt occurs between the occurrence of an event at time t_k and the change in the state of the reference model at time t_k^+ such that the trajectory moves off the hyperplane before switching. That is, suppose $t_k^+ = t_k + \Delta t$ and

$$c_m^T e^{A\Delta t} x(t_k) \neq \delta_m.$$

Then $x(t_k^+) = x(t_k + \Delta t) + d(t_k) = e^{A\Delta t} x(t_k) + d(t_k)$ so that $c_m^T d(t_k) = 0$ implies $c_m^T x(t_k^+) = c_m^T e^{A\Delta t} x(t_k) \neq \delta_m$. Another way to avoid the possibility of the (mathematical) non-existence of a solution is to impose the constraint on the decision vector such that

$$c_m^T d(t_k) \neq 0.$$

The nonexistence problem can also be resolved by using the so called *beating condition* that is commonly assumed in the study of impulsive dynamical systems, see [6, 17]. The beating condition assumes that the solution meets one hyperplane not more than once in order to guarantee the existence of the solution. Instead, here it is assumed that the solution will (eventually) leave the hyperplane once the solution reaches it, possibly avoiding the fast occurrence of impulsive vectors (known as *chattering* in control theory).

More specifically, given a solution $x(t)$ of (8) which is defined on $[t_0, t_0 + \alpha)$ where $\alpha > 0$, a solution $\tilde{x}(t)$ is a continuation to the right of $x(t)$ if, for $\beta > \alpha$, the solution $\tilde{x}(t)$ is defined on $[t_0, t_0 + \beta)$ and $x(t) = \tilde{x}(t)$ for all $t \in [t_0, t_0 + \alpha)$. If $x(t)$ is defined over the interval $[t_0, t_0 + \alpha)$ and no such continuation is possible for $t > t_0 + \alpha$, then the interval $[t_0, t_0 + \alpha)$ is called the *maximal existence* of a solution $x(t)$.

Let $J^+(t_0^+, x_0)$ be the maximal interval from (t_0^+, ω) in which the solution $x(t; t_0^+, x_0)$ is defined. The following result is the reformulation of the condition in [6, 22] which summarizes the continuity of the solution of linear impulsive systems.

Theorem 2 Consider the linear time invariant impulsive systems (7). If the following conditions

1. $0 \equiv t_0 < t_1 < t_2 < \dots < t_k < \dots$
2. $x(t) \in \mathcal{Q}$ for $t \in J^+(t_0^+, x_0)$ where \mathcal{Q} is a compact subset of Ω

hold. Then $J^+(t_0^+, x_0) = (t_0^+, \infty)$

Proof. Condition 2 implies that there exists a finite $\eta \in \mathcal{Q}$ and $(\gamma, \eta) \in R^+ \times \Omega$. Suppose on the contrary, it is assumed that $J^+(t_0^+, x_0) = (t_0^+, \gamma)$ and $\gamma < \infty$. Then it follows that $\lim_{t \rightarrow \gamma} |x(t)| = \infty$ provided Condition 1 is satisfied. But, this contradicts the assumption that the limit $\lim_{t \rightarrow \gamma} |x(t)| = \eta$ exists and is finite.

5 Conclusions

In this paper, the use of linear impulsive differential equations for hybrid systems modeling has been discussed. The unique characteristic of the linear impulsive differential equations is the complexity introduced by the presence of impulsive vectors in relation to time instants of occurrence to the states of the system. Properties of the impulsive effects which are described by impulsive vectors has also been presented. Since the impulsive vector is not restricted to be a function of any variables in the system which is usually assumed in the literature, a more general result has been obtained. The full power modeling of hybrid systems via linear impulsive differential equations has been shown by using simple examples that show this modeling approach may promote a unified framework for analysis and control design of hybrid systems.

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