A NEW APPROACH TO H_∞ SUBOPTIMAL MODEL REDUCTION FOR SINGULAR SYSTEMS

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Abstract

In this paper, the H_∞ suboptimal model reduction for singular systems is investigated. An optimal model reduction algorithm is designed for obtaining a stable reduced model. A necessary and sufficient condition for the existence of a stable reduced-order system is given and this criterion can be verified numerically. Also, a numerical algorithm is presented for obtaining such a reduced order stable system.

1 Introduction

In recent years, singular systems have been investigated extensively due to their broad applications in modelling and control of electrical circuits, power systems and economics, etc. Some important characteristics of singular systems include combined dynamic and static solutions, impulsive behaviors and large dimensionality. Thus model reduction is vital for analysis and controller design for such systems [4, 6].

The initial investigation of model reduction for singular systems was the chained aggregation approach proposed in [7]. The authors there developed a generalized chain-aggregation algorithm and gave an intuitive interpretation of the exact aggregation conditions for singular systems. The aim of the proposed method is to remove the unobservable subspace. Initial behavior of singular systems was also taken into consideration while performing model reduction. However, as pointed out in [8], its main drawback is its high computational costs.

Perev and Shafai [8] considered model reduction for singular system via balanced realizations and gave a model reduction algorithm. Unfortunately, their approach ignored the impulsive behavior which is of paramount importance to singular systems. With this reduction algorithm, the reduced order model may be a normal state space system, which has no impulsive behavior and does not track the original system response properly as demonstrated in [5]. Liu and Sreeram [5] proposed a new reduction algorithm via the Nehari's approximation algorithm and overcome this issue. With the approach in [5], the reduced-order model will be a really singular system and the approximation has been obtained as desired. For discrete singular systems, Zhang et al. [4] discussed the same problem with the approximation in H_2 norm and some results obtained. Moreover, Zhang et al.[9] discussed the H_∞ suboptimal model reduction problem for singular systems. In [9], it requires the transfer matrix of the error system to be rational in order to guarantee that H_∞ norm exists. However, the existence of the reduced-order system was not solved there and has remained open. Also the H_∞ model reduction problem for discrete singular systems was investigated in [3] with the restriction of admissible property.

In this paper, we will tackle the model reduction problem for singular systems and will present a new approach for the H_∞ suboptimal model reduction. In order to preserve the impulsive nature of singular systems, we will use reduced-order fast sub systems to approximate the fast sub systems as proposed in [5, 9]. However, a necessary and sufficient condition has been obtained for the existence of a stable reduced-order system in this paper. Further, an algorithm has been designed for the $\mathcal{H}_\infty$ suboptimal model reduction and this algorithm can be easily implemented via Matlab software .

The organization of this paper is as following. In section 2, a system transformation and suboptimal model reduction problem will be presented. In section 3, the Silverman-Ho algorithm will be given. In section 4, the main results about the $\mathcal{H}_\infty$ suboptimal model reduction will be given and a detailed algorithm will be illustrated in section 5. Conclusions will be presented in section 6.

2 Preliminaries

Consider the following singular systems

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) \quad x(0-) = x_0 \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $u(t) \in \mathcal{R}^q$ is the input vector and $y(t) \in \mathcal{R}^m$ is the output vector. $E \in \mathcal{R}^{n\times n}$, $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times q}$, $C \in \mathcal{R}^{m\times n}$ are constant matrices with E possibly singular. Assume that the matrix pair (E, A) is regular (i.e.,$|sE-A| \not\equiv 0$). In this paper, the realization quadruple (E, A, B, C) is used to represent the system (1). All matrices in this paper are assumed to have appropriate dimensions.

From [2], it is known that there exist two square nonsingular matrices Q and P such that system (1) can be transformed into the Weierstrass form

$$\begin{aligned} \dot{x}_1(t) &= A_1x_1(t) + B_1u(t) \quad x_1(0-) = x_{1,0} \\ y_1(t) &= C_1x_1(t) \\ N\dot{x}_2(t) &= x_2(t) + B_2u(t) \quad x_2(0-) = x_{2,0} \\ y_2(t) &= C_2x_2(t) \end{aligned} \tag{2}$$

where $x_1(t) \in \mathcal{R}^{n_1}$, $x_2(t) \in \mathcal{R}^{n_2}$, $n_1 + n_2 = n$, $N \in \mathcal{R}^{n_2\times n_2}$ is nilpotent, and

$$\begin{aligned} &QEP = diag(I, N), \quad QAP = diag(A_1, I) \\ &CP = [C_1 \;\; C_2], \quad P^{-1}x(t) = [x_1^T(t) \;\; x_2^T(t)]^T \\ &QB = [B_1^T \;\; B_2^T]^T, \quad y(t) = y_1(t) + y_2(t) \end{aligned}$$

System(1) is called system restricted equivalent(s.r.e) to the system(2). The transfer function matrix $G(s)$ is invariant under such s.r.e. transformation, i. e. ,

$$\begin{aligned} G(s) &= C(sE - A)^{-1}B \\ &= CP(sQEP - QAP)^{-1}QB \\ &= C_1(sI - A_1)^{-1}B_1 + C_2(sN - I)^{-1}B_2 \end{aligned} \tag{3}$$

and

$$\begin{aligned} C_2(sN - I)^{-1}B_2 &= -C_2B_2 - sC_2NB_2 - \cdots \\ &- s^{h-1}C_2N^{h-1}B_2 \end{aligned}$$

with $C_2N^{h-1}B_2 \neq 0$.

The aim of this paper is to investigate the $\mathcal{H}_\infty$ suboptimal model reduction for singular systems. Suppose the reduced-order singular system is

$$\begin{aligned} E_r\dot{x}_r(t) &= A_rx_r(t) + B_ru(t) \\ y(t) &= C_rx_r(t) \end{aligned} \tag{4}$$

which is assumed to be regular. Then there also exist two matrices Q_r and P_r such that

$$\begin{aligned} \dot{x}_{1r}(t) &= A_{1r}x_{1r}(t) + B_{1r}u(t) \\ y_{1r}(t) &= C_{1r}x_{1r}(t) \\ N_r\dot{x}_{2r}(t) &= x_{2r}(t) + B_{2r}u(t) \\ y_{2r}(t) &= C_{2r}x_{2r}(t) \end{aligned} \tag{5}$$

where $x_{1r}(t) \in \mathcal{R}^{n_{1r}}$, $x_{2r}(t) \in \mathcal{R}^{n_{2r}}$, $n_{1r} + n_{2r} = n_r$, $N_r \in \mathcal{R}^{n_{2r}\times n_{2r}}$ is nilpotent, and

$$\begin{aligned} &Q_rE_rP_r = diag(I, N_r), \quad Q_rA_rP_r = diag(A_{1r}, I) \\ &C_rP_r = [C_{1r} \;\; C_{2r}] \quad P_r^{-1}x_r(t) = [x_{1r}^T(t) \;\; x_{2r}^T(t)]^T \\ &Q_rB_r = [B_{1r}^T \;\; B_{2r}^T]^T, \quad y(t) = y_{1r}(t) + y_{2r}(t) \end{aligned}$$

Then the error system between the original system and the reduced order system will be

$$\begin{aligned} E_e\dot{x}_e(t) &= A_ex_e(t) + B_eu(t) \\ y_e(t) &= C_ex_e(t) \end{aligned} \tag{6}$$

where $x_e^T(t) = [x^T(t) \;\; x_r^T(t)]^T$, $y_e \in \mathcal{R}^m$, and

$$\begin{aligned} &E_e = diag(E, \;\; E_r), \quad A_e = diag(A, \;\; A_r) \\ &B_e^T = [B^T \;\; B_r^T]^T, \quad C_e = [C \;\; -C_r] \end{aligned}$$

Let

$$Q_e = diag(Q, \;\; Q_r), \quad P_e = diag(P, \;\; P_r)$$

Then the $\mathcal{H}_\infty$ norm of the transfer matrix $G_e(s)$ for the error system will be

$$\begin{aligned} \|G_e(s)\|_\infty &= \|C_eP_eP_e^{-1}(sE_e - A_e)^{-1}Q_e^{-1}Q_eB_e\|_\infty \\ &= \|C_1(sI - A_1)^{-1}B_1 - C_{1r}(sI - A_{1r})^{-1}B_{1r} \\ &+ C_2(sN - I)^{-1}B_2 - C_{2r}(sN_r - I)^{-1}B_{2r}\|_\infty \end{aligned}$$

Then, the problem of the $\mathcal{H}_\infty$ suboptimal model reduction for singular system (1) is to find a reduced-order singular system (E_r, A_r, B_r, C_r) with $\dim(E_r) < \dim(E)$ such that for a given positive number γ, the following holds

$$\|G_e(s)\|_\infty < \gamma$$

First, It is known from [9] that $||G_e(s)||_\infty$ is finite if and only if

$$C_2(sN-I)^{-1}B_2 - C_{2r}(sN_r-I)^{-1}B_{2r} = C_2B_2 - C_{2r}B_{2r}$$

i. e.,

$$C_2N^iB_2 = C_{2r}N_r^iB_{2r}, \quad i = 1, 2, \cdots, h-1 \tag{7}$$

$$C_{2r}N_r^iB_{2r} = 0, \quad i \geq h \tag{8}$$

In this case, one has

$$\|G_e(s)\|_\infty = \|C_1(sI - A_1)^{-1}B_1 - C_{1r}(sI - A_{1r})^{-1}B_{1r} + C_2B_2 - C_{2r}B_{2r}\|_\infty$$

Therefore, the $\mathcal{H}_\infty$ suboptimal model reduction problem can be solved via using the conventional approaches if (7) and (8) are met. As indicated by results in [9], the main difficulty for the model reduction problem is to find suitable (C_{2r}, N_r, B_{2r}) such that equaions (7) and (8) are satisfied.

Also it is known from [2] that the transfer matrix for a system is determined only by the controllable and observable subsystem, Therefore, one problem in this paper is to discuss the model reduction of the fast subsystems (N, I, B_2, C_2). i.e., to find the fast subsystem $(N_r, I_r, B_{2r}, C_{2r})$ with $n_{2r} < n_2$ and satisfy the conditions (7) and (8).

The approach adopted in [9] is to find N_r first, then one tries to solve (7) and (8) to obtain B_{2r} and C_{2r}. The proposed approach there has two significant disadvantages. On one hand, for a given N_r, the equations (7) and (8) may not have solutions for B_{2r}, C_{2r}. On the other hand, even the solutions for these equations exist, it is still not easy to solve them due to their nonlinear nature.

In this paper, the following questions related to the model reduction problem will be addressed. The existence of $(N_r, I_r, B_{2r}, C_{2r})$ satisfying (7) and (8) will be tackled and their solutions will be investigated. Also the lowest order of N_r will be identified and a model reduction algorithm will be presented.

3 Silverman-Ho algorithm

The Silverman-Ho algorithm in [1] is about the property of a matrix polynomial. It has many applications in system analysis and design. First, it can be stated as below.

Lemma 1 *[1] For any polynomial matrix P(s), there always exist matrices N, B, and C, with N nilpotent, such that* $P(s) = C(sN - I)^{-1}B$

For a given polynomial matrix

$$P(s) = P_0 + P_1 s + \cdots + P_{h-1}s^{h-1}, P_i \in \mathcal{R}^{r\times m}$$

The above lemma guarantees the existence of B, C, and the nilpotent matrix N, such that

$$P(s) = C(sN - I)^{-1}B$$

The following process presents an approach for finding a minimal realization (C, N, B), in the sense that they are impulsive controllable and observable [2]. Let

$$\begin{aligned} M_0 &= \begin{bmatrix} -P_0 & -P_1 & \cdots & -P_{h-2} & -P_{h-1} \\ -P_1 & -P_2 & \cdots & -P_{h-1} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -P_{h-2} & -P_{h-1} & \cdots & \cdots & 0 \\ -P_{h-1} & 0 & \cdots & \cdots & 0 \end{bmatrix} \\ &\in \mathcal{R}^{hr\times hm} \end{aligned}$$

$$\begin{aligned} M_1 &\triangleq \begin{bmatrix} -P_1 & -P_2 & \cdots & -P_{h-1} & 0 \\ -P_2 & -P_3 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -P_{h-1} & 0 & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix} \\ &\in \mathcal{R}^{hr\times hm} \end{aligned}$$

and denote

$$\tilde{n} \triangleq \mathrm{r}ankM_0$$

Then one decompose M_0 into the following form

$$M_0 = L_1 L_2$$

where $L_1 \in \mathcal{R}^{hr\times\tilde{n}}$, $L_2 \in \mathcal{R}^{\tilde{n}\times hm}$ are matrices with full column and row rank, respectively. Further, Let $\tilde{B}$ and $\tilde{C}$, respectively, be the first m columns of L_2 and the first r rows of L_1. Then

$$\tilde{N} = \left(L_1^T L_1\right)^{-1} L_1^T M_1 L_2^T \left(L_2 L_2^T\right)^{-1}$$

will be nilpotent and $(\tilde{N}, \tilde{B}, \tilde{C})$ forms a minimal realization for $P(s)$ as desired. This algorithm will be used in the sequel to design a procedure for solving the model reduction problems proposed in the previous section.

4 Main Results

From the Silver-Ho algorithm, it can be seen that the order of the minimal realization for $P(s)$ is determined by the rank of the matrix M_0. For a given fast subsystem (N, B_2, C_2), let $P_i = C_2 N^i B_2,\ \ i = 0, 1, \cdots, h-1$. Then the suboptimal model reduction problem is to find a matrix $\tilde{P}_0$ such that $n_2 = \mathrm{r}ankM_0 > \mathrm{r}ank\tilde{M}_0 = n_{2r}$, where $\tilde{M}_0$ corresponds to $\tilde{P}_0, P_1, \cdots, P_{h-1}$. So the existence of such matrix $\tilde{P}_0$ will determine whether the original fast subsystem can be reduced. The following theorem will give a necessary and sufficient condition for the existence of such matrix $\tilde{P}_0$.

Without loss of generality, let

$$\begin{aligned} M_2 &\triangleq \begin{bmatrix} -P_0 & -P_1 & \cdots & -P_{h-2} & -P_{h-1} \end{bmatrix} \\ &\triangleq \begin{bmatrix} \alpha_1^T & \alpha_2^T & \cdots & \alpha_m^T \end{bmatrix}^T \end{aligned}$$

$$M_4=\left[\begin{array}{ccccc} -P_1 & -P_2 & \cdots & -P_{h-1} & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ -P_{h-2} & -P_{h-1} & \cdots & \cdots & 0\\ -P_{h-1} & 0 & \cdots & \cdots & 0 \end{array}\right]$$

Then M_0 can be partitioned as

$$M_0=\left[\begin{array}{c} M_2\\ \hline M_4 \end{array}\right]$$

Let the vector set $\left[\begin{array}{cccc}\alpha_1, & \alpha_2, & \cdots, & \alpha_d\end{array}\right]$ be the maximal linearly independent set of $\left[\begin{array}{cccc}\alpha_1, & \alpha_2, & \cdots, & \alpha_m\end{array}\right]$ such that

$$\mathrm{r}ankM_0=\mathrm{r}ank\left[\begin{array}{c} \alpha_1\\ \alpha_2\\ \vdots\\ \alpha_d\\ \hline M_4 \end{array}\right]=rank[M_4]+d, d\leq m$$

This is possible since one can choose the independent vectors from the bottom to top in M_0. Also, it can be seen that d is determined by M_4 and M_2. Further, suppose that

$$\left[\begin{array}{c} M_4\\ \alpha_1\\ \alpha_2\\ \vdots\\ \alpha_d\\ \alpha_{d+1}\\ \vdots\\ \alpha_m \end{array}\right]=\left[\begin{array}{c|c|c} M_4^1 & M_4^2 & M_4^3\\ \hline *_1 & *_2 & *_3\\ \hline p_{k+1} & \beta_{k+1} & \eta_{k+1}\\ p_{k+2} & \beta_{k+2} & \eta_{k+2}\\ \vdots & \vdots & \vdots\\ p_d & \beta_d & \eta_d\\ \hline p_{d+1} & \beta_{d+1} & \eta_{d+1}\\ p_{d+2} & \beta_{d+2} & \eta_{d+2}\\ \vdots & \vdots & \vdots\\ p_m & \beta_m & \eta_m \end{array}\right]$$

where $k=\mathrm{r}ank[P_{h-1}]$, $*_1,*_3\in\mathcal{R}^{k\times q}, *_2\in\mathcal{R}^{k\times(h-2)q}$. The matrix $*_3$ are of full row rank, p_i, $\eta_j\in\mathcal{R}^{1\times q}$ with

$$rank\left[\begin{array}{c} *_3\\ \hline \eta_j \end{array}\right]=rank[*_3]$$

$\beta_i\in\mathcal{R}^{1\times(h-2)q}$, $k+1\leq i\leq m$, $k+1\leq j\leq m$. This simplification is possible due to the fact that

$$\mathrm{r}ankP_{h-1}=\mathrm{r}ank[*_3]$$

Then among

$$\beta_{k+1},\quad \beta_{k+2},\quad \cdots,\quad \beta_d$$

there are two possible cases. (i) If

$$\mathrm{r}ank\left[\begin{array}{c|c} M_4^2 & M_4^3\\ *_2 & *_3\\ \hline \beta_i & \eta_i \end{array}\right]=\mathrm{r}ank\left[\begin{array}{c|c} M_4^2 & M_4^3\\ *_2 & *_3 \end{array}\right]$$

which indicates that p_i can affect the rank of M_0 due to the following fact,

$$\mathrm{r}ank\left[\begin{array}{c|c|c} M_4^1 & M_4^2 & M_4^3\\ \hline *_1 & *_2 & *_3\\ \hline p_i & \beta_i & \eta_i \end{array}\right]>\mathrm{r}ank\left[\begin{array}{c|c|c} M_4^1 & M_4^2 & M_4^3\\ \hline *_1 & *_2 & *_3 \end{array}\right]$$

Then one can decrease the rank of M_0 by changing p_i as discussed below. (ii) If

$$\mathrm{r}ank\left[\begin{array}{c|c} M_4^2 & M_4^3\\ *_2 & *_3\\ \hline \beta_i & \eta_i \end{array}\right]>\mathrm{r}ank\left[\begin{array}{c|c} M_4^2 & M_4^3\\ *_2 & *_3 \end{array}\right]$$

Then it implies that p_i does not affect the rank of M_0. In case (i), one can reduce the rank of M_0 by changing the elements p_i as described below. Since $[\beta_i,\eta_i]$ is linearly dependent on

$$\left[\begin{array}{c|c} M_4^2 & M_4^3\\ *_2 & *_3 \end{array}\right]$$

There exists vectors x_1 and x_2 such that

$$[\beta_i,\eta_i]=[x_1^TM_4^2+x_2^T*_2, x_1^TM_4^3+x_2^T*_3]$$

In this case, one replace p_i with

$$\tilde{p}_i=x_1^TM_4^1+x_2^T*_1$$

one can find that

$$\mathrm{r}ank\left[\begin{array}{c|c|c} M_4^1 & M_4^2 & M_4^3\\ \hline *_1 & *_2 & *_3\\ \hline \tilde{p}_i & \beta_i & \eta_i \end{array}\right]=\mathrm{r}ank\left[\begin{array}{c|c|c} M_4^1 & M_4^2 & M_4^3\\ \hline *_1 & *_2 & *_3 \end{array}\right]$$

which indicates that the rank can be reduced. However, the vector among $[p_j,\beta_j,\eta_j]$, $d<j<m+1$ may become independent with

$$\left[\begin{array}{c|c|c} M_4^1 & M_4^2 & M_4^3\\ \hline *_1 & *_2 & *_3\\ \hline p_{k+1} & \beta_{k+1} & \eta_{k+1}\\ p_{k+2} & \beta_{k+2} & \eta_{k+2}\\ \vdots & \vdots & \vdots\\ p_d & \beta_d & \eta_d \end{array}\right] \qquad (9)$$

after changing p_i. In order to avoid the possible rank incremental in this case, one need to change p_j accordingly as bellow. Remind that $[p_j,\beta_j,\eta_j]$, $d<j<m+1$

is linearly dependent with (9), so there exist vectors y_1, y_2, y_3 satisfying

$$[p_j, \beta_j, \eta_j] = [y_1^T M_4^1 + y_2^T *_2 + y_3^T P, *, *]$$

In order to keep the rank not increasing after P is replaced by $\tilde{P}$, one can replace p_j with

$$\tilde{p}_j = y_1^T M_4^1 + y_2^T *_2 + y_3^T \tilde{P}$$

After changing the p_i,$0 < i < d+1$ in case (i) and related p_j, $d < j < m+1$, the rank of M_0 has been reduced, which indicates that a reduced model can be obtained. Now, with notation for $k < i < d+1$,

$$S = \left\{ \eta_i \mid \mathrm{rank} \left[\begin{array}{c|c} M_4^2 & M_4^3 \\ *_2 & *_3 \\ \hline \beta_i & \eta_i \end{array} \right] = \left[\begin{array}{c|c} M_4^2 & M_4^3 \\ *_2 & *_3 \end{array} \right] \right\}$$

Clearly, this set will provide a necessary and sufficient condition for the existence of the lower order fast subsystems.

Theorem 2 *Given (N, B_2, C_2), there exists a reduced-order, controllable and observable system (N_r, B_{2r}, C_{2r}) with dimension $n_{2r} < n_2$ such that the $\mathcal{H}_\infty$ norm of the error system is finite if and only if the set S is not empty.*

With this theorem, the following results are obvious.

Corollary 3

1. *The lowest order n_r of the reduced-order system is $n_2 - N(S)$, where $N(S)$ is the number of the elements in S. Further,*

$$n_2 - N(S) \geq rank \;\; M_1 + k$$

2. *For single-input(or single-output), controllable and observable system, there does not exist reduced-order system such that the $\mathcal{H}_\infty$ norm of the error system exists.*

Prior to give a model reduction algorithm, one should note a fact that the corresponding p_i in case (ii) can choose any value freely without affecting the rank of M_0. This set can be a free parameter for optimal reduction algorithm given below.

Let the order of the reduced order system be n_{2r} satisfying $n_2 - N(S) \leq n_{2r} < n_2$. Without loss of generality, suppose that the original P_0 can be partitioned as

$$P_0 = \left[\begin{array}{c} P_{01} \\ \hline P_{02} \\ \hline P_{03} \end{array} \right]$$

where P_{01} is the vector set in case (i) and P_{02} is the vector set in case (ii) and P_{03} is the vector set with index $d < j < m+1$. In this case, one can iteratively reduce the rank of M_0 by changing P_{01} and P_{03}. Recall that P_{02} will not affect the rank of M_0, therefore P_{03} will be a function of P_{02} which is a free parameter used in the optimization process below. So the suboptimal model reduction problem can be converted into an optimization problem for finding A_{r1}, B_{r1}, C_{r1}, and $P_{03}(P_{02})$ such that

$$\|C_1(sI - A_1)^{-1}B_1 - C_{1r}(sI - A_{1r})^{-1}B_{1r} + + P_{02}(P_{03})\|_\infty$$

is minimized. This can be solved by the standard optimization technique introduced in [10].

5 Algorithm and Illustrative Example

Now a model reduction algorithm can be presented based on previous discussions.

1. Decompose the given singular system into the slow and fast subsystems. If there exists controllable and observable part and denote it as (N, I, B_2, C_2). Otherwise, go to step 4.
2. Compute $P_i = -C_2 N^i B_2$ and obtain M_0.
3. Change elements in P_{01} to P_{01}^* such that the rank of matrix M_0^* is reduced as desired.
4. Solve the unconstrained optimization problem and obtain A_{r1}, B_{r1}, C_{r1}, P_{03}^* and then P_0^*.
5. Use the Silverman-Ho algorithm to obtain the minimal realization (N_r, I_r, B_r, C_r) for the reduced order system.

Now, we give one numerical example to illustrate the algorithm. Consider the fast system (N, B, C) with

$$N = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 3 & 0 & 3 & 2 \\ 1 & 0 & 2 & 1 & 0 \\ 3 & 2 & 3 & 1 & 1 \end{bmatrix}$$

It can be verified that this system is a minimal realization. Computing

$$-P_0 = CB = \begin{bmatrix} 7 & 12 \\ 5 & 2 \\ 8 & 9 \end{bmatrix}$$

$$-P_1 = CNB = \begin{bmatrix} 3 & 5 \\ 0 & 3 \\ 2 & 7 \end{bmatrix}$$

Then one can obtain that $N(S) = 1$. Implementing this algorithm, one can obtain the following parameters.

$$N_r = \begin{bmatrix} 0.3284 & 0.1584 & -0.0582 & 0.0001 \\ -0.3969 & -0.3370 & -0.0025 & 0.0663 \\ 0.7732 & -0.0241 & -0.3357 & 0.1813 \\ -0.0215 & -0.7651 & -0.3740 & 0.3443 \end{bmatrix}$$

$$B_r = \begin{bmatrix} -0.5660 & -0.7371 \\ 0.5363 & -0.6610 \\ 0.5055 & -0.0223 \\ -0.3694 & 0.1392 \end{bmatrix}$$

$$C_r = \begin{bmatrix} -14.9254 & -1.5002 & -1.3974 & -0.1695 \\ -5.3407 & 2.9334 & 0.9190 & 0.1649 \\ -16.4778 & 2.5852 & 0.1916 & 0.0886 \end{bmatrix}$$

Also the $\mathcal{H}_\infty$ norm of the error system is

$$\|G_e(s)\|_\infty^2 = \|CB - C_r B_r\|_\infty^2 = 9.8025$$

With the proposed algorithm in [9], one can obtain a reduced system with the $\mathcal{H}_\infty$ norm

$$\|G_e(s)\|_\infty^2 = \|CB - C_r B_r\|_\infty^2 = 27.5005 > 9.8025$$

This illustrates that the new proposed algorithm is much better than one reported in [9].

6 Conclusions

In this paper, we developed a new approach of $\mathcal{H}_\infty$ suboptimal reduction algorithm for singular systems. A necessary and sufficient condition is presented in which one can guarantee the existence of a reduced-order system with finite $\mathcal{H}_\infty$ norm of the error system. Also an explicit algorithm is presented for obtaining the reduced order systems. Compared to the results in the exiting literature, one can find the following the importance contributions of this paper

- A necessary and sufficient condition is obtained for the existence of the reduced order systems.
- The possible lowest order of the reduced system is obtained.
- One free parameter in the reduction process has been identified.
- The optimization process is a combination of slow and fast subsystems.

The similar technique can be applied to discrete singular systems.

The disadvantage of the proposed algorithm is the decomposition of the system matrices into slow and fast systems. This overhead may bring numerical difficulties in practice.

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