

# ADAPTIVE CONTROL OF SEPARATED FLOWS

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## Abstract

Two adaptive feedback control methods, a new adaptive integral controller used to control plants with output saturation and dead-time and the well-known extremum seeking feedback are employed to control detached flows. For the experimental validation, conducted in a wind tunnel, two different flow configurations, a backward-facing step and a diffuser, are used. These experiments confirm that the presented control strategies can be used successfully to manipulate a detached flow and thus to reduce negative concomitant effects.

## 1 Introduction

Flow separation accounts for important problems in the area of science and engineering. Technical devices characterised by a detached flow are for instance diffusers, airfoils, or air-conditioning plants. Effects of flow separation are commonly not desired. The lift of airfoils breaks down if the flow detaches. The pressure recovery in diffusers is reduced and noise and vibrations are produced in air-conditioning plants by flow separation.

To investigate different kinds of excitation to manipulate a detached flow, simple flow configurations were considered experimentally in the seventies. See [4] for a detailed list of excitation methods. In the eighties research concentrated more on understanding the physics of separation so that this knowledge may help to design technical devices in which effects of separation will be minimised. In all these investigations open-loop but not feedback control strategies were applied to affect the detached flow. A comprehensive study of flow separation control is given in [5]. Numerical studies and first experiments with successful applications of feedback control were carried out in the nineties. Sound in a diffuser [8] and drag [9] induced by a detached flow were reduced effectively by means of feedback methods. Since then the area of flow control is expanding at a fast rate but is still in a premature state. The objective of the work presented here is to regulate detached flows by using adaptive feedback control. Two standard configurations characterised by flow

separation are considered, namely a backward-facing step and a diffuser. Experiments are carried out in a wind tunnel to show the practicability of the proposed methods. This investigation has to be seen as part of a wider effort to control flows by means of various control methods [1,2,12].

The paper is organised as follows: in the next section a new adaptive integral controller and the known extremum seeking feedback will be presented. The flow configurations considered as a benchmark will be explained in section 3. Results of feedback control are discussed in section 4 followed by a conclusion.

## 2 Adaptive control

Flows are characterised by a highly non-linear behaviour of infinite dimension. Rigorous mathematical modelling leads to the well-known Navier-Stokes equations (NSE), i.e. a set of non-linear PDEs. Attempts are made to build up controllers based on NSE [3] but the obtained control laws are far from being applicable in a real-time environment in the near future. Therefore, other means of controller synthesis are needed. Examples are controllers derived from low-dimensional approximations of NSE [12] or controllers based on simple, experimentally obtained models [1,2]. Adaptive concepts are more promising candidates for the latter category.

In this section a new integral controller will be presented. Its gain will be tuned adaptively. The adaptation is not based on system identification or parameter estimation methods. The second control method used for flow control in the present study, the extremum seeking feedback, will also be explained briefly.

### 2.1 Adaptive integral controller

Consider a general non-linear plant with output saturation. The process dynamics of it can be approximated by a family of first order plus dead-time (FOPDT) models for any operating points. The static gain of the models reflects the output saturation and, thus, the gain can be related to the input variable. These kinds of plants can be controlled successfully by the following integral controller

$$u(t) = k_i(t) \cdot \int e(\tau) d\tau \quad (1)$$

based on the time-dependent gain  $k_i(t)$  and the error

$e(t)$ , i.e. the difference between reference and output variable. The gain will be tuned by the error signal according to

$$k_i(t) = e^{p_i(t) \cdot \left( \int e(\tau) d\tau \right)^2 - 1} \quad (2)$$

$$p_i(t) = p_{i0} \cdot e^{-\int p_i(\tau) \cdot e^4(\tau) d\tau}, \quad p_{i0} > 0. \quad (3)$$

This controller is an extension of the one presented in [10]. To explain its working assume the parameter  $p_i$  being constant. Then the gain  $k_i$  will increase as long as the error  $e(t)$  exists. However, to avoid closed-loop instability the parameter  $p_i$  tends towards zero according to Eq. (3) starting from a positive initial value  $p_{i0}$ . It can be shown that the time derivative of the parameter  $p_i(t)$  is negative for any values of the error  $e(t)$

$$\dot{p}_i(t) = -p_i^2(t) \cdot e^4(t). \quad (4)$$

As a result of this feature the adaptation of the gain  $k_i$  decreases with time. Furthermore, it can also be shown that the gain tends towards zero, if the error  $e(t)$  increases to infinity, thereby effectively opening the loop

$$\lim_{e(t) \rightarrow \infty} k_i(t) = 0. \quad (5)$$

As a result no instability will occur with stable plants, however a large overshoot would result in such a situation. This means that the controller possesses the feature of a so-called low-gain controller [10].

The initial value  $p_{i0}$ , which can be viewed as a performance parameter, has to be chosen by the user. Small values  $p_{i0}$  cause a slowly increasing gain  $k_i$ . For an appropriate choice, knowledge about process dynamics is advantageous which can be obtained, for example, from step responses.

Further simulation studies with non-linear plants taken from [6] confirm that the closed-loop based on the proposed controller is stable and is able to track any constant reference signals. It should be noted that the adaptive integral controller cannot be used to control unstable and stable undamped plants or plants with right-half plane (RHP) zeros.

## 2.2 Extremum seeking feedback

A known but for a long time not considered control method used to achieve a maximal/minimal output of general non-linear plants with a static map is shown in Fig. 1. The extremum seeking controller consists of two filters, a low- and a high-pass filter, an integrator and a signal generator which supplies the controller with a sine signal.

If the actual input tends towards the optimal one, the output variable  $y(t)$  increases, assuming that the static map has a maximum and the process dynamics can be neglected. The output passes through a high-pass filter which removes the mean value but not the frequency  $\omega$ .

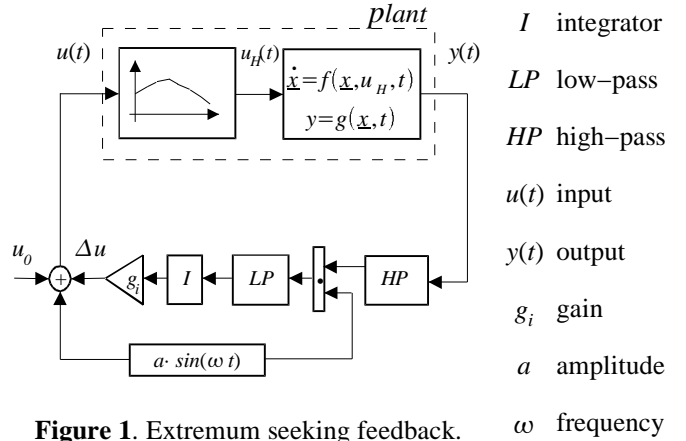


Figure 1. Extremum seeking feedback.

The product of filtered output and the zero-mean sine leads to a non zero-mean signal as long as the maximum is not obtained. This signal then passes through a low-pass filter to extract the new mean value. Change of  $\Delta u(t)$  due to integration, is the result until the actual input  $u(t)$  converges towards the optimal one.

The choice of the gain  $g_i$ , cut-off frequencies of the filters, amplitude and frequency of the sine signal determines the speed of convergence. The reader is referred to [7] for more details about extremum seeking feedback.

## 3 Flow configurations

Two flow configurations, a backward-facing step flow, where the flow detaches at the edge of the step and then reattaches in the wake of the flow, and a flow within a diffuser, where separation and reattachment locations are not fixed, are considered. These configurations possess all significant features of flow separation and serve as generic examples for more complex ones.

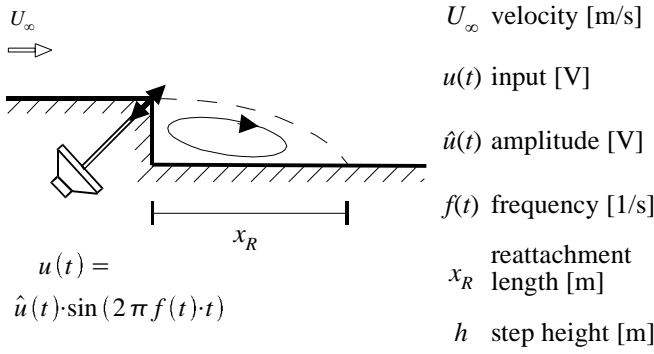
### 3.1 Backward-facing step flow

A simple flow configuration with fixed separation point is the flow over a backward-facing step, see Fig. 2. This kind of flow can be viewed, for example, as a prototype of a flow inside a burner.

In a highly simplified explanation the flow looks as follows. Because of its inertia the oncoming flow with a free-stream velocity  $U_\infty$  is not able to follow the sudden expansion at the step. Therefore, the flow detaches at the edge of the step. The free-stream moves on above the recirculation zone bounded by the dashed curve and reattaches at the bottom wall at a location  $x_R$  downstream of the step.

Within the recirculation zone a secondary flow develops. In a first approximation this flow can be imagined as a rotating vortex. Near the main flow, the secondary flow moves in the same direction and reverse close to the bottom wall. The energy required for the rotation is extracted from the main flow itself.

A widely accepted method to control a separated flow is to introduce disturbances at the separation location, i.e. at the edge of the step.



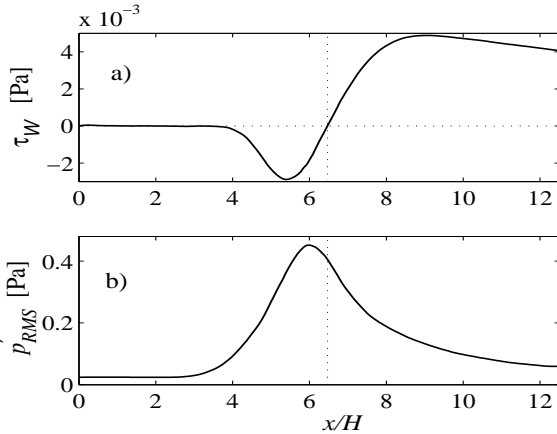
**Figure 2.** Backward-facing step flow with actuation.

Low cost sources to generate such disturbances are loudspeakers supplied with a periodic voltage signal. This kind of excitation method counts to the active ones, because an outer energy source is required.

The time-dependent amplitude  $\hat{u}(t)$  and excitation frequency  $f(t)$  are the input variables of the plant to be controlled. The plant output which characterises the detached flow in the wake shall be the reattachment length  $x_R$ . This length is defined by a zero-mean wall-shear stress  $\tau_w$

$$\tau_w = \eta \cdot \left. \frac{\partial \bar{u}}{\partial y} \right|_{y=0} \quad (7)$$

with  $\eta$  being the dynamic viscosity.  $y$  is the co-ordinate vertical to the bottom wall and  $\bar{u}$  denotes in fluid mechanics the component of the time-averaged velocity in the  $x$ -direction, i.e. the direction of mean flow. It should not be mistaken for the input variable  $u(t)$ . The time-averaged wall-shear stress in the wake of the step for a free-stream velocity  $U_\infty=3\text{m/s}$  obtained from a numerical solution of the NSE is given in Fig. 3.



**Figure 3.** Wall-shear stress (a) and wall pressure fluctuations in the wake of the step. The  $x$ -direction is normalised by the step height  $H$ .

Due to the reverse near-wall flow the wall-shear stress in the wake becomes negative over a wide range ( $x=2\dots6.4H$ ). The wall-shear stress vanishes at the location  $x=6.4H$ . Hence, the main flow reattaches at this position (Fig. 3a).

There are several techniques to measure the wall-shear stress, i.e. to determine the reattachment length indirectly.

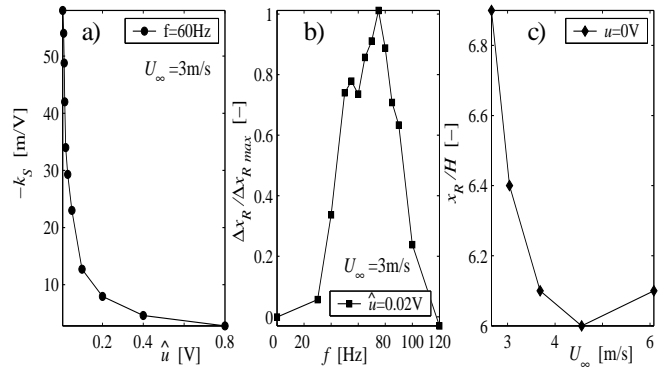
However, these techniques are very expensive, unpractical for our purpose and a large time-averaging is required for an accurate measurement of the wall-shear stress. An appropriate choice to determine the reattachment length is to measure the wall-pressure fluctuations in the wake by microphones. Several authors have confirmed that the root mean square of near-wall fluctuations rise to a maximum in the reattachment region [11] (Fig. 3b).

The location of maximal pressure fluctuations varies slightly between  $0.85\dots0.95 \cdot x_R$  and this relation holds for a wide range of free-stream velocities. To determine the root mean square value time averaging is needed. Experiments have shown that time-averaging of about three seconds is necessary to obtain this relation [2].

Recent investigations of a flow over a backward-facing step [14] confirm that the reattachment length depends in a non-linear fashion on the two input variables which are amplitude and frequency of the sine voltage signal  $u(t)$ , as well as on the free-stream velocity  $U_\infty$ , see as well Fig. 4a,b and Fig. 4c, respectively.

An output saturation characterises the process, shown in Fig. 4a. Furthermore, it was shown that the input/output behaviour can be approximated by a family of linear first or second order black-box models [2]. Because of these facts, the adaptive integral controller proposed in section 2.1 can be used to control this process. The output of the controller will be a time-dependent amplitude  $\hat{u}(t)$  required to track the reference.

Fig. 4b shows that two optimal frequencies exist by forcing with a constant amplitude, a local optimum near  $f=55\text{Hz}$  and a global one at  $f=80\text{Hz}$ . The extremum seeking feedback can be used therefore as well.

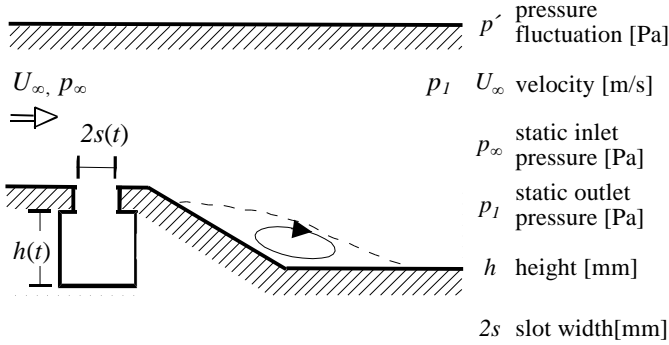


**Figure 4.** Reattachment length as a function of amplitude and frequency of inputs (a, b) and free-stream velocity (c). Note that the product of static gain  $k_S$  and amplitude yields the reattachment length.  $\Delta x_R$  represents the reduction of the reattachment length.

The relation to free-stream velocity (Fig. 4c) will be used to simulate disturbances which affect the reattachment length and also the optimal forcing frequencies. It should be noted that a higher free-stream velocity leads to a reduced reattachment length but shifts the optimal forcing frequency to higher values. See [14] for more details of these complex flow properties.

### 3.2 Diffuser flow

The second flow configuration, which is often accompanied by the occurrence of flow separation, is the flow within a diffuser (Fig. 5). A diffuser slows down a mean flow, converting its kinetic energy into a pressure rise.

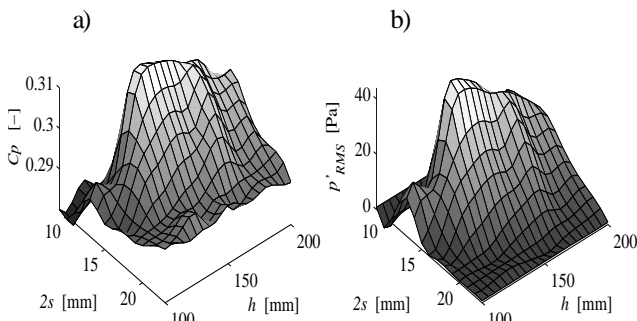


**Figure 5.** Diffuser flow with an acoustic resonator.

This pressure gradient can cause flow separation for large opening angles because the kinetic energy of near-wall fluid is not sufficient to overcome the pressure rise. Therefore, the pressure recovery in a diffuser can be less than its theoretical value. Again the recirculation zone is dominated by a large rotating vortex which is supplied with energy by the main flow itself. The efficiency of diffusers is commonly quantified by the pressure recovery coefficient

$$C_p = \frac{p_1 - p_\infty}{0.5 \rho U_\infty^2} \quad (8)$$

To minimise the separation in such a diffuser and, consequently, to enhance the static outlet pressure an acoustic resonator is located upstream of the expansion (see Fig. 5). Acoustic oscillations within the cavity of the resonator are excited by the oncoming flow itself. The generated sound characterised by frequency and amplitude depends, among other parameters, on the slot width  $2s$  and the volume height  $h$  of the cavity. The static map of the resonator which possesses several local optima is presented in Fig. 6b.



**Figure 6.** Relation between slot width  $2s$  and volume height  $h$  of an acoustic resonator to the pressure recovery  $C_p$  (a) and the pressure fluctuations  $p'_{RMS}$  (b) in the cavity itself measured for an inlet velocity of  $U_\infty=10\text{m/s}$ .

To generate a maximal sound pressure level within the cavity a slot width of  $2s=13\text{mm}$  and a volume height of  $h=140\text{mm}$  are required. It is interesting to note that by using these settings the maximal pressure recovery in this diffuser can be also achieved, see the static map in Fig. 6a. The existence of optimal values allows to control this process by extremum seeking feedback aimed to maximise the pressure recovery.

Note that the proposed excitation method counts to the passive ones because the energy required for the generation of the sound is extracted from the oncoming flow itself. The reader is referred to [13] for more details about separation control by acoustic resonators.

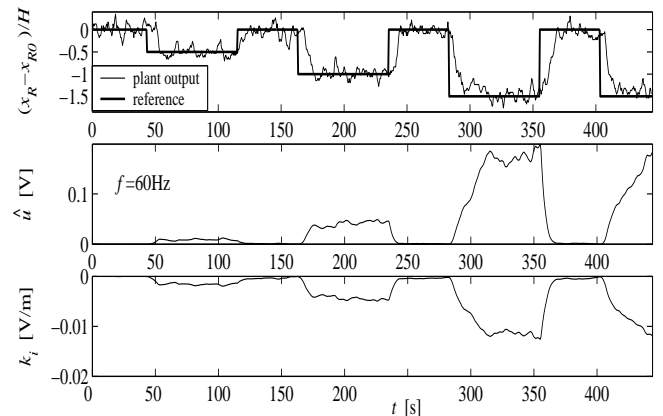
## 4 Results of feedback control

In this section results of feedback control are discussed. First, the performance of the closed-loop achieved by the new proposed adaptive integral controller will be shown (section 4.1.1). In this case the excitation frequency is kept constant close to the optimal one. Afterwards the control strategy is expanded in such a way that the excitation frequency is determined by the extremum seeking feedback (section 4.1.2). The maximising of the pressure recovery in a diffuser by using two extremum seeking feedback methods is then reported in section 4.2.

### 4.1 Separation control – backward-facing step flow

#### 4.1.1 Adaptive integral control

Fig. 7 shows an experimental result of the closed-loop performance achieved by the proposed adaptive integral controller. Simulation studies, in which the dynamics and output saturation of the process are modelled approximately based on results of step experiments, confirm to set the initial value of the tuning parameter to  $p_{i0}=5 \cdot 10^{-5}$ . The closed-loop tracks the reference signal well. But poor transient behaviour can be seen after changes in the reference signal.



**Figure 7.** Reduction of the reattachment length behind a backward-facing step by using adaptive integral feedback control for a free-stream velocity of  $U_\infty=3\text{m/s}$ .

This is caused by the averaging time (three seconds) required for an accurate correlation between root mean square of wall-pressure fluctuations and the reattachment

length (see section 3.1).

It was mentioned in section 2.1 that the tuning parameter  $p_i(t)$  decreases when large errors occur. Hence, tracking of large steps in the reference signal which causes large errors at the beginning, e.g. at time  $t=280$ s, leads to a slower adaptation of the controller gain  $k_i$  and, consequently, a larger transient-time is implied.

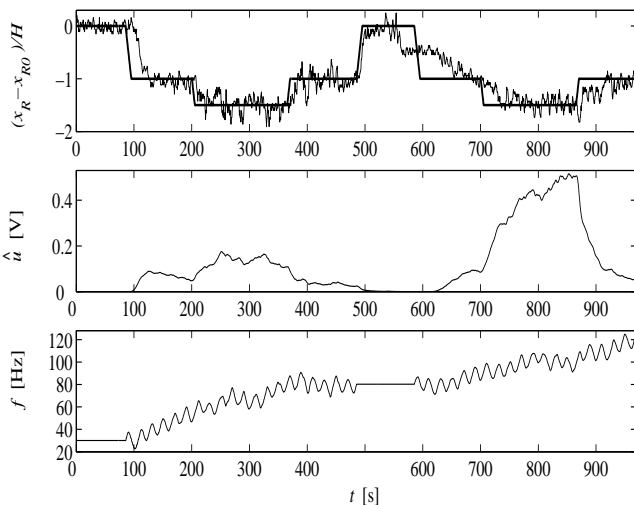
The authors suggest to implement a mechanism for supervision of the adaptation, so that a decrease of the tuning parameter  $p_i(t)$  is stopped for periods in which output and reference signals do not differ. Furthermore, the initial parameter can be reinitialised, i.e. the initial parameter  $p_{i0}$  can be increased slowly while no strong overshoot of the output variable occurs.

#### 4.1.2 Adaptive integral control and extremum seeking feedback

Next, the control strategy is expanded, so that the excitation frequency is now determined by the extremum seeking feedback, i.e. the output  $u(t)$  of the controller of Fig. 1 is the excitation frequency  $f(t)$ . Before choosing the parameters of the extremum seeking feedback it should be mentioned that these two controllers can work against each other. Thus, changing the excitation frequency, which varies the reattachment length, can be viewed as a disturbance which is rejected by the adaptive integral controller.

Therefore, the initial parameter  $p_{i0}$  and the gain  $g_i$  have to be chosen carefully. An appropriate choice for the problem considered in this work is to set  $p_{i0} = 5 \cdot 10^{-5}$  and  $g_i = -1.5$ . The sine frequency is set equal to the cut-off frequency of the high-pass filter  $\omega = \omega_h = 0.31 \text{ rad/s}$  and the cut-off frequency of the low-pass filter is set to  $\omega_l = 10\omega_h$ . At first a free-stream velocity of  $U_\infty = 3 \text{ m/s}$  and an initial excitation frequency of  $f_0 = 30 \text{ Hz}$  are chosen. Compared to Fig. 4b this initial frequency differs significantly from the optimal one  $f_{opt} = 75 \text{ Hz}$ .

Closed-loop performance achieved by the coupling of the proposed control methods can be seen in Fig. 8.



**Figure 8.** Coupling of adaptive integral feedback control and extremum seeking feedback for an energy-saving reduction of reattachment length.

Due to the inefficient initial frequency  $f_0$  high amplitudes are required to track the first setpoint change at  $t=90$ s. The fact that the optimal frequency is actually not used leads to a slow increase of the excitation frequency  $f(t)$  by the extremum seeking feedback, so that the excitation is more effective and high amplitudes are not longer required for tracking. The transient time after further reference changes, e.g. at  $t=200$ s, could then obviously be reduced.

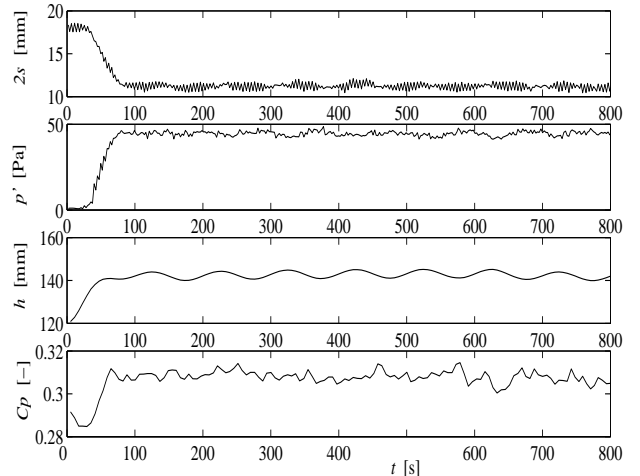
It is interesting to see what happens if disturbances affect the closed-loop. Hence, the free-stream velocity is set to  $U_\infty = 4.5 \text{ m/s}$  at  $t=550$ s. An increased inlet velocity reduces the reattachment length, compare Fig. 4c.

It should be emphasised that the optimal excitation frequency also depends on the inlet velocity. In this case the optimal frequency shifts towards higher values if the inlet velocity is increased.

For the new inlet velocity the excitation with a frequency of  $f=80 \text{ Hz}$  is inefficient. Hence, the closed-loop behaviour is characterised again by a long transient time and higher values of amplitude are required to track the reference signal. The extremum seeking feedback shifts the excitation frequency towards  $f=120 \text{ Hz}$ , close to the optimal one. Comparing these two cases of different inlet velocities, it should be noted that the considered plant will become more insensitive, i.e. higher amplitudes are needed for tracking, if the inlet velocity increases.

#### 4.2 Separation control – flow within a diffuser

A further successful application of feedback control is the enhancement of the pressure recovery in a diffuser, shown in Fig. 9. Two extremum seeking feedbacks are used to control the inputs separately.



**Figure 9.** Extremum seeking feedback for enhancement of pressure recovery and fluctuations in the cavity of an acoustic resonator for a free-stream velocity  $U_\infty = 10 \text{ m/s}$ .

The first one maximises the sound level by changing the slot width for the instantaneous volume height. This control-loop can be viewed as an inner one which is characterised by small time constants, i.e. this closed-loop works fast. The second control-loop varies the volume height in order to maximise the pressure recovery. This outer loop will work slower thus the coupling between

increased sound level in the cavity and pressure recovery enhancement in a diffuser can be neglected.

As initial values the slot width  $2s_0=18\text{mm}$  and the cavity height  $h_0=120\text{mm}$  are chosen. For these settings the effect of excitation on the detached flow can be neglected and the pressure recovery is below  $C_p=0.29$  (see also Fig. 6). Due to the sinusoidal variation of inputs the sound level in the cavity enhances slowly which leads to a reduction of the slot width. It was mentioned above that an enhanced sound level involves stronger excitation, so that flow separation will be delayed and thus a rising pressure recovery is implicated. Near optimal values for slot width and cavity height are finally found by the extremum seeking feedbacks.

It should be mentioned that the transient time of the included high-pass filter aided the seeking of an extremum. This fact will be explained briefly by the following filter equation

$$y_i = \frac{C_{pi} - C_{pi-1} + y_{i-1}}{1 + \omega_h \cdot \Delta t} \quad (9)$$

based on the filter output  $y$ , the pressure recovery coefficient  $C_p$  as the filter input, the cut-off frequency  $\omega_h$  and the sampling interval  $\Delta t$ . At the starting time ( $i=1$ ) values for output and input one time step back ( $i=0$ ) are required. These were set to zero, so that for  $i=1$  a positive steplike input exists which causes a suddenly increased output, i.e. the height of cavity will be enhanced even though the pressure recovery coefficient decreased slightly. Consequently, the extremum seeking feedback would increase the cavity height at first irrespective whether an initial value  $h_0$  above the optimal one, e.g.  $h_0=180\text{mm}$  (see also Fig. 6a), would be chosen.

## 5 Conclusion

A new adaptive integral controller and the known extremum seeking feedback are used to reduce the effects of separation in two simple flow configurations. In the first case, the flow over a backward-facing step, the excitation amplitude and frequency can be determined separately. Experiments carried out in a wind tunnel show that the closed-loop tracks the reference signal after a transient time which is used to shift the actual frequency to the optimal one. So, for the reduction of reattachment length the effort of energy can be reduced.

In the second case, the flow within a diffuser is considered. To affect the flow separation an acoustic resonator is mounted in front of the expansion. Two extremum seeking feedbacks are used to tune the geometry of this resonator, i.e. slot width and height of cavity, separately. It can be shown that optimal settings for the geometry are found and consequently the pressure recovery coefficient is maximised.

## Acknowledgement

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