NUMERICAL ALGORITHM FOR OPTIMAL MULTI-VARIABLE CONTROL OF AERO ENGINES

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Abstract

This paper is focused on practical problems of gas turbine engine optimal multi-variable control design and implementation. An algorithm for real-time resolution of optimisation problem is proposed for the optimal control design of the engine. The example of the turbo-jet engine multi-variable control presented shows high efficiency of the developed method.

1 Introduction

High performance requirements for modern aircraft power plants result in a greater degree of integrated control of aero engines. This requires complex control systems, which are multi-variable (7 to 10 control loops) and multi-functional. Operation of all control loops must be co-ordinated and where possible decoupled across all transients and steady-state conditions.

Operational requirements become greater as demand increases for accuracy and quality of transient performance. Effective control of power plants is also connected with integration of engine control with aircraft control. This requires new methods for analysis and design of control systems.

At present, control of a gas turbine engine (GTE) is usually designed using frequency domain methods of classical control theory. However, in last years, many research projects have been focused on the application of time domain methods for aero engine control (Harefors 1997). These methods are usually based on the variations of LQ (Athans 1986, Moorhouse 1994) and H-infinity (Garg 1993, Frederick 2000) methodologies of linear optimal control.

Aero engine characteristics differ over the engine fleet and can change during the engine's operation. This makes it necessary to solve the problem of real-time optimisation taking into account individual characteristics of the engine. Such an approach can provide high operational qualities for the aircraft propulsion plant.

For the solution of problems concerning optimal control of aircraft gas turbine engines, when a mathematical model of the plant is obviously inaccurate, it is preferable to use approximate methods. In real-time optimal control, the objective of optimisation in the form of integral functionals is not feasible, because it reflects average characteristics of system quality. Instead, local quality criteria are considered for optimisation, e.g. specific loss functions.

The approximate character of the mathematical model of the engine forces the designer to pass from strict optimal analytic decisions to numeric ones. Moreover, the computational speed of existing on-board computers limits the search to optimisation within a class of local-optimal control systems.

The full derivative of a Lyapunov function can be used as a local optimality criterion, which ensures asymptotic stability of the control system. Such control, along with guaranteeing stability, also becomes *optimal*, when additional constraints are imposed to control variables.

Multi-variable local-optimal control systems of gas turbine engines possess the following advantages:

- produce numerical solutions of the optimisation problem, which extends their field of application;
- the systems asymptotically converge to strict optimal solutions, i.e. they do not differ substantially from optimal systems under small perturbations;
- the synthesis procedure is based on systems analysis and enables design of multi-functional systems.

The objective of the work presented in this paper is to develop and investigate numerical algorithms of multi-variable GTE control with real-time optimisation of control laws and algorithms, along with accounting for individual characteristics of the engine.

2 Problem formulation

The problem of multi-variable GTE control can be formulated in terms of the definition of local optimal control. Suppose that the controller has m independent control actions (outputs of the controller). The objective of the control system is to maintain m control laws (e.g. maintenance of constant values of m engine parameters).

The sampling frequency f of the digital control system is known. The sampling time period is the following: $\tau = 1/f$.

The system of differential equations describes the plant dynamics in the following form:

$$\begin{aligned} \dot{x}(t) &= A(t)\,x(t) + B(t)\,u(t)\,, \\ y(t) &= C(t)\,x(t) + D(t)\,u(t)\,, \end{aligned} \qquad (1)$$

where $x(t)$ is the n-dimensioned state vector; $u(t)$ is the m-dimensioned control vector; $y(t)$ is the k-dimensioned observation (output) vector; A, B, C, and D are the matrices of the dimensions $(n\times n)$, $(n\times m)$, $(k\times n)$, and $(k\times m)$ respectively. All elements of vectors $x(t)$ and $y(t)$ are measured by corresponding sensors. The following constraints are applied to the first derivative of the control vector $u(t)$:

$$\dot{u}_1^{\lim} \le \dot{u}(t) \le \dot{u}_2^{\lim} \qquad (2)$$

where $\dot{u}_1^{\lim}$ and $\dot{u}_2^{\lim}$ are movement speed limits for the actuators.

The control computer stores in its memory matrices A, B, C, and D, which depend on the operating and flight conditions of the engine. The computer operates with the period $\Delta t = \tau = 1/f$. At each i-th cycle of its operation, current values of elements of the state vector $x(t)$ and observation vector $y(t)$ are measured, and elements of the control vector $u(t)$ are passed to actuators.

The vector $z^*(t)$ is given, defining the program trajectory. The vector $z(t)$ consists of m elements of the vectors $x(t)$ and $y(t)$, where m is equal to the number of control variables. This means that the control system has to maintain m control laws simultaneously. A simple example is maintenance of constant values of m engine parameters.

Consider a vector of trajectory deviation (or vector of errors) $e(t)$:

$$e(t)=z^*(t) - z(t). \qquad (3)$$

The objective of control is to make the current vector $z(t)$ trace the demanded $z^*(t)$ with maximum possible accuracy and quality:

$$\|e(t)\| = \|z^*(t) - z(t)\| \to \min\,, \qquad (4)$$

where $\|\cdot\|$ is the Euclidean norm of a vector. Along with delivering $\|e(t)\|_{\min}$, elements of the control vector should satisfy the constraints of Eq.(2).

In the synthesis of multi-variable GTE control, the differential equations describing the plant are to be written in the form of a system of differential matrix equations. Discrete equations are usually obtained from continuous ones. A well-known method for digitisation of continuous differential equations consists of substitution of the differentiation operation by a simple difference. This method does not provide high accuracy of difference matrix equations at low quantisation frequencies.

Here a possible method is considered for obtaining difference equations by digitisation of continuous linear differential equations of the plant. It is a digitisation method using the transitional matrix of the plant, which is generalised to a non-stationary case and explicit dependence of the observation vector on control.

The source data for a discrete model of the plant is the system of vector-matrix differential and algebraic equations represented in the normal form:

$$\begin{aligned} \Delta\dot{x}(t) &= A(t)\,\Delta x(t) + B(t)\,\Delta u(t), \\ \Delta y(t) &= C(t)\,\Delta x(t) + D(t)\,\Delta u(t), \\ \Delta x(t) &= x(t) - x^*(t), \\ \Delta y(t) &= y(t) - y^*(t), \\ \Delta u(t) &= u(t) - u^*(t), \\ x(t) &= [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}, \\ y(t) &= [y_1(t), y_2(t), \ldots, y_k(t)]^{\mathrm{T}}, \\ u(t) &= [u_1(t), u_2(t), \ldots, u_m(t)]^{\mathrm{T}}, \end{aligned} \qquad (5)$$

where $x(t)$, $y(t)$, and $u(t)$ are the state, observation and control vectors respectively; and x^*, y^*, and u^* are the vectors defining the demanded program trajectory of the plant. Matrices A, B, C and D have the dimensions $(n\times n)$, $(n\times m)$, $(k\times n)$, and $(k\times m)$ respectively. Their elements are functions of the engine state and atmospheric conditions. The system Eq.(5) is given in physical co-ordinates.

The solution of the system Eq.(5) with variable coefficients can be represented in the form:

$$\Delta x(t) = \Phi(t,t_0)\,\Delta x(t_0) + \int_{t_0}^{t} \Phi(t,\alpha)\,B(\alpha)\,\Delta u(\alpha)\;d\alpha, \qquad (6)$$

where $\Phi(t, t_0)$ is the transitional matrix and t_0 is the initial time.

Digitising time in the transitional equation of a vector-matrix type Eq.(6) with $t = i\tau$ $(i = 1, 2, \ldots)$, the solution of the system Eq.(5) at two moments of time $t_1 = i\tau$ and $t_2 = i\tau + \tau$ can be written as:

$$\Delta x(i\tau) = \Phi(i\tau,t_0)\,\Delta x(t_0) + \int_{t_0}^{i\tau} \Phi(i\tau,\alpha)\,B(\alpha)\,\Delta u(\alpha)\;d\alpha, \qquad (7)$$

$$\Delta x(i\tau+\tau) = \Phi(i\tau+\tau,t_0)\,\Delta x(t_0) + \int_{t_0}^{i\tau+\tau} \Phi(i\tau+\tau,\alpha)\,B(\alpha)\,\Delta u(\alpha)\;d\alpha. \qquad (8)$$

Using the property of the transitional matrix

$$\Phi(t_2,t_0) = \Phi(t_2,t_1)\Phi(t_1,t_0) \qquad (9)$$

and supposing that the matrices A, B, C, and D are constant over the sampling interval, one can obtain:

$$\Phi(t_2,t_0)=e^{A(t_2-t_1)}\Phi(t_1,t_0) \quad (10)$$

or

$$\Phi(i\tau+\tau,t_0)=e^{A\tau}\Phi(i\tau,t_0)\,. \quad (11)$$

Then Eq.(8) can be rewritten in the following form:

$$\Delta x(i\tau+\tau)=e^{A\tau}\Phi(i\tau,t_0)\,\Delta x(t_0)+ \\ +e^{A\tau}\int_{t_0}^{i\tau+\tau}\Phi(i\tau,t_0)\,B(\alpha)\,\Delta u(\alpha)\,d\alpha\,. \quad (12)$$

Comparison of Eq.(12) and Eq.(7) gives

$$\Delta x(i\tau+\tau)=e^{A\tau}\,\Delta x(i\tau)+ \\ +e^{A\tau}\int_{i\tau}^{i\tau+\tau}\Phi(i\tau,\alpha)\,B(\alpha)\,\Delta u(\alpha)\,d\alpha. \quad (13)$$

The matrix A is constant over the digitisation interval, hence the matrix $\Phi(i\tau,\alpha)$ is transformed to the exponential matrix $\Phi(i\tau\text{-}\alpha)$:

$$\Phi(i\tau-\alpha)=e^{A(i\tau-\alpha)}\,, \quad (14)$$

$$\Delta x(i\tau+\tau)=e^{A\tau}\Delta x(i\tau)+ \\ +\int_{i\tau}^{i\tau+\tau}e^{A(i\tau+\tau-\alpha)}B\Delta u(\alpha)\,d\alpha. \quad (15)$$

Denote exponent of the transitional matrix as follows:

$$F=e^{A\tau}=E+A\tau+\frac{A^2\tau^2}{2!}+\cdots\quad. \quad (16)$$

The control vector changes stepwise, being constant over the sampling period. Therefore, the integral in Eq.(15) is simplified down to the following form:

$$\int_0^{\tau}e^{A\alpha}d\alpha\;\;B\Delta u(i\tau), \quad (17)$$

where $u(i\tau)$ is the value of control u at the interval ($i\tau$, $i\tau+\tau$). Digitising time results in

$$\Delta x(i+1)=F\Delta x(i)+T\Delta u(i)\,, \quad (18)$$

where

$$T=(\int_0^{\tau}e^{A\alpha}d\alpha)\,B=\tau(E+\frac{A\tau}{2!}+\frac{A^2\tau^2}{3!}+\ldots)\,B\,. \quad (19)$$

Digitising the equation for the observation y, the system Eq.(5) can be written as follows:

$$\Delta y(i+1)\;=\;C\Delta x(i+1)\;+\;D\Delta u(i)\,. \quad (20)$$

Combining Eq.(18) and Eq.(20) gives discrete vector-matrix equations of the plant. Extrapolated plant motion parameters are estimated from their current values:

$$\Delta x(i+1)=F\Delta x(i)+T\Delta u(i)\,, \\ \Delta y(i+1)=CF\Delta x(i)+(CT+D)\Delta u(i)\,, \quad (21)$$

or

$$\Delta x(i+1)=F\Delta x(i)+T\Delta u(i)\,, \\ \Delta y(i+1)=S\Delta x(i)+P\Delta u(i)\,, \quad (22)$$

where the matrices F and T can be defined by Eq.(16) and Eq.(19). The matrices S and P are determined by

$$S=CF\,, \\ P=CT+D\,. \quad (23)$$

The system Eq.(22) consists of difference vector-matrix equations written *in deviations* from steady-state values, determining extrapolated parameters of the plant from their current values.

Consider the system Eq.(22) for the time moments $t=(i+1)\tau$ and $t=i\tau$ and subtract the second system from the first one:

$$x(i+1)-x(i)=F(x(i)-x(i-1))+T(u(i)-u(i-1))\,, \\ y(i+1)-y(i)=S(x(i)-x(i-1))+P(u(i)-u(i-1))\,, \quad (24)$$

or in another way:

$$x(i+1)=x(i)+F\delta x(i)+T\delta u(i)\,, \\ y(i+1)=y(i)+S\delta x(i)+P\delta u(i)\,, \quad (25)$$

where

$$\delta x(i)=x(i)-x(i-1), \\ \delta u(i)=u(i)-u(i-1). \quad (26)$$

Form the vector $z(i+1)$, which describes the current state:

$$z(i+1)=\begin{bmatrix}x'(i+1)\\ y'(i+1)\end{bmatrix}= \\ =\begin{bmatrix}x(i)+H_x(F\delta\,x(i)+T\delta\,u(i))\\ y(i)+H_y(S\delta\,x(i)+P\delta\,u(i))\end{bmatrix}, \quad (27)$$

where H_x and H_y are vectors of weighting coefficients, taking into account the approximate character of the mathematical model of the plant. Their values reflect the level of "uncertainty" of knowledge concerning GTE dynamics and depend on demanded engineering quality criteria.

Consider the Euclidean norm as a norm of the vector $e(t)$ in Eq.(4):

$$J(\delta u(i))=\sum_{j=1}^{m}\left[z_j^*(i+1)-z_j(i+1)\right]^2. \quad (28)$$

The expression for constraints Eq.(2) in a discrete form is the following:

$$\dot{u}_1^{\lim}\;\le\;\frac{u(i)-u(i-1)}{\tau}\;\le\;\dot{u}_2^{\lim}\,, \quad (29)$$

or

$$h_1(\delta u(i)) = -u(i) + u(i-1) + \tau \dot{u}_1^{\lim} =$$
$$= -\delta u(i) + \tau \dot{u}_1^{\lim} \le 0,$$
$$h_2(\delta u(i)) = u(i) - u(i-1) - \tau \dot{u}_2^{\lim} =$$
$$= \delta u(i) - \tau \dot{u}_2^{\lim} \le 0. \quad (30)$$

At each i–th step, the control vector is changed to provide the minimum of the objective function Eq.(28) under the constraints Eq.(30). In this case, optimal control represents a series of solutions of a quadratic programming problem within the control computer at discrete time instants t_i ($i = 0$, 1, …) along with execution of corresponding controls $u(t_i)$. This problem can be solved using non-linear programming methods applicable for the control computer.

Dynamic properties of the plant in a wide range of operation can be described by a set of linear models with varying coefficients. This enables engine optimal control to be built as a sequence of solutions of a quadratic programming problem in real time:

$$J = \sum_{j=1}^{m} e_j^2(\delta u(i)) \to \min, \quad (31)$$

where the error vector e includes elements of both vectors e_x and e_y :

$$e_x = x^* - x(i) - H_x[F\delta x(i) + T\delta u(i)],$$
$$e_y = y^* - y(i) - H_y[S\delta x(i) + P\delta u(i)],$$
$$\delta x(i) = x(i) - x(i-1),$$
$$\delta u(i) = u(i) - u(i-1). \quad (32)$$

with the following constraints for controls:

$$|\,\delta u(i)\,| \le \tau \dot{u}^{\lim}(i). \quad (33)$$

3 Real-time optimisation algorithm

The algorithm proposed for the solution of the mathematical programming problem (Canon 1970) in real-time is based on analysis of GTE characteristics as a multi-variable plant, and generalisation of requirements to digital multi-variable control systems.

Consider an algorithm to search for the minimum of the objective function Eq.(31). The choice of a method for solution of the problem Eq.(31), (32) and (33) depends on the following factors:

- the computational power necessary for solving the problem;
- the required accuracy of the solution;
- the demanded accuracy of constraints fulfilment;
- stable operation under short-time malfunctions.

Transforming Eq.(1), (2) and (3), the problem is represented in a standard form:

$$J = Q(x) = p^T x + x^T Cx, \quad (34)$$

$$Ax - B \le 0, \quad (35)$$

where $x = \delta u(i)$ is speed of change of the control vector at the i–th step of control; C is a symmetric positive defined m–dimensioned matrix; p^T and x^T are transposed column vectors.

The following solution represents the minimum point of the objective function Eq.(34) if the constraints Eq.(35) are not considered:

$$x^0 = -\tfrac{1}{2} C^{-1} p \quad (36)$$

If the constraints Eq.(35) are taken into account, the problem consists in searching for the point of the allowed polyhedron Eq.(35) lying at the level line

$$Q(x) = p^T x + x^T Cx$$

with the minimal value of $Q(x)$:

$$\min(Q(x) \mid Ax \le B). \quad (37)$$

The standard problem statement with the objective function Eq.(31) will lead to a search for the minimum of the function at the boundary of the allowed area. This will determine a point where reduction of control errors of some parameters is obtained only by increasing control errors in others. Therefore, in minimisation of Eq.(34), a mechanism should be developed to exclude possible increase in absolute value of any component of the vector e.

A possible method for this involves introducing into Eq.(31) additional weighting coefficients, or “penalties”, in the following form:

$$J = \sum_{j=1}^{m} \gamma_j e_j^2 \to \min, \quad (38)$$

where γ_j are weighting coefficients calculated using special formulae.

Another approach consists in developing additional constraints, excluding increase in control errors e_j while searching for the minimum of the objective function Eq.(31). This extends the system of limiting inequalities Eq.(33) and makes the search procedure more complicated.

A reasonable way of solving the problem of quadratic programming is the use of a method accounting for the physical sense of the objective function Eq.(31). As is seen from the formula Eq.(31), the point x^* delivering the unconditional minimum of the objective function is determined by solving the system of linear equations:

$$e_j(x) = 0, \quad j = 1,\ldots,m. \quad (39)$$

The search for the minimum of the objective function Eq.(31) within the allowed polyhedron Eq.(33) starts with an acceptable point x^0=0 and then proceeds along the beam:

$$S = x^0 + \lambda x^* \quad (40)$$

toward the point x^* until the minimum or the boundary of the allowed area. The value of λ' , where the beam crosses the boundary of the allowed area, is defined as the smallest of λ_j:

$$\lambda' = \min\{\lambda_j \mid \lambda_j > 0\}\,, \qquad (41)$$

where

$$\lambda_j = \frac{\delta \dot{u}_j^{\lim}}{|S_j|}\,. \qquad (42)$$

If the minimum of the objective function is inside of the allowed area, then obviously $\lambda=1$. The step length, defining vector x, is selected as

$$\lambda^0 = \min\{1, \lambda'\}. \qquad (43)$$

The advantage of the suggested method is that minimisation of a quadratic function takes only one iteration. Calculation of the function gradient and inverse matrices is not necessary for finding the unconditional minimum. The search for the minimum of a function along a straight line connecting the beginning of co-ordinates with the point of the unconditional minimum provides proportional reduction of control errors by all output parameters.

An easy-to-program Gaussian algorithm can be used for solving the system Eq.(39) to define the co-ordinates of the point of unconditional minimum.

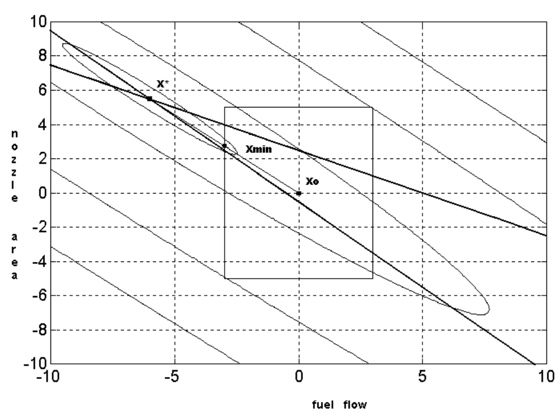

Figure 1: Trajectory of search for minimum of objective function

Figure 1 illustrates the suggested method using an example of minimisation of the following objective function:

$$J = (5 - x_W - 2x_A)^2 + (-2 - 4x_W - 4x_A)^2, \qquad (44)$$

where $x_W = \delta u_W$ and $x_A = \delta u_A$. The constraints Eq.(33) give the allowed area for search in the form of a rectangle with the initial point X_0 in the centre. The unconditional minimum of the objective function $J=0$ is in the point X^* at the intersection of the straight lines $5–X_W–X_A=0$ and $–2–4X_W–X_A=0$. These are shown by thick lines. The search for the minimum proceeds along the straight line connecting the points X_0 and X^* until the intersection with the boundary of the search area (point X_{min}). The final point determines optimal control at the current step.

The sequence of calculations according to the considered method consists of the following stages:

- the point of unconditional minimum δu^* is calculated as a solution of the system:

$$e_j(\delta u(i)) = 0, \quad j=1,\ldots,m; \qquad (45)$$

- the parameter λ' is determined at the intersection of the beam $S = \delta u^0(i) + \lambda\delta u^*(i)$ with the boundary of the allowed area:

$$\lambda_j' = \frac{\tau \dot{u}_j^{\lim}}{|S_j|}; \quad j = 1,\cdots,m; \qquad (46)$$

- parameter λ^0 is determined according to the formula:

$$\lambda^0 = \min\{1, \lambda_j'\}\,; \qquad (47)$$

- the minimum point is calculated:

$$\delta u(i)=\delta u^0(i)+\lambda^0 S. \qquad (48)$$

The developed algorithm combines high speed of calculation with low computational needs. It also provides the demanded accuracy to allow fulfilment of constraints. An additional check of whether the point found is within the allowed area Eq.(3) introduces a degree of robustness to faults resulting from transient upsets in the computational process. This enables the algorithm to be recommended for use in on-board digital control systems.

4 Example of turbo jet engine control

The developed technique was used for synthesis of a two-variable control system for a twin-shaft turbo jet with a variable jet nozzle. Two independent control variables (fuel flow W_f and nozzle area A_n) allow design of multi-variable control.

A multi-variable system was developed to control maximum values of the following parameters: low pressure shaft speed n_{LP}, high pressure shaft speed n_{HP} and gas temperature T_g* behind the low pressure turbine at maximum dry thrust operation.

The source data for control design are mathematical models of the plant and requirements on dynamic and static accuracy of control. Step responses of the output parameters should be aperiodic without overshoot. The rise time must be less than one second. Static error of shaft speed control is less than 0.3 % and of temperature control is less than 5 K.

Investigation of the developed control system was performed via simulation of transients using mathematical models of the plant and controller. First, an investigation was performed using linear models of the system, then with a performance-based detailed non-linear thermo dynamic model of the engine.

The program of investigation included the following points:

- the use of various objective functions;
- response to perturbations applied to demanded values and control variables;
- influence of pure time delay;
- degree of compensation of thermocouple inertia;
- parametric perturbations;
- positional drift in hydraulic integrator actuators.

The mathematical model describes the plant Eq.(5), actuators, the thermocouple and the developed controller. Actuators of W_f and A_n are modelled with integrators with maximum speeds of 3000 (kg/hr)/s and 2000 cm^2/s respectively. The sampling frequency for transfer of control outputs to the actuators is 40 Hz. The sampling period is τ=0.025 s.

Figure 2 shows step response of the closed-loop control system to a step of 10 rpm applied to LP shaft speed demand. The n_{LP} transient is monotonic without a peak of "over-control" and lasts about 0.3 s. The T_g* transient is flat.

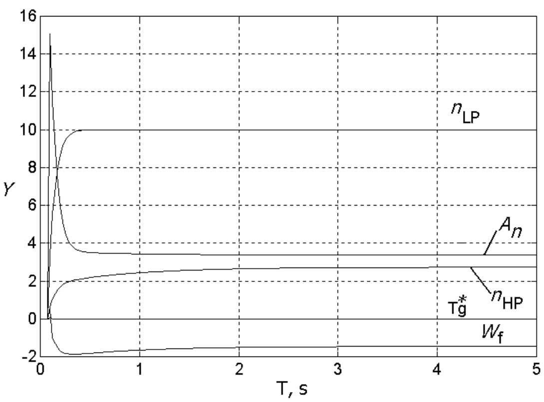

Figure 2: Step response of closed-loop system to demand of Δn_{LP} = +10 rpm

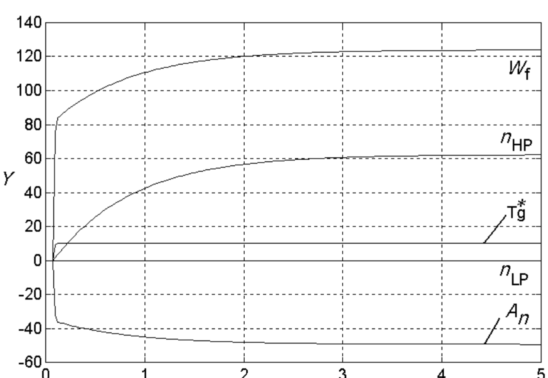

Figure 3: Step response of closed-loop system to demand of ΔT_g* = +10^0 K

Reaction caused by a step change of 10^0 K in the gas temperature demand is presented in Figure 3. The T_g* transitional process is a stepwise function lasting around 0.125 s. The LP shaft speed has no change. The controls are changed with maximum possible speed until the gas temperature achieves its new value.

The plots show that a high degree of decoupling between n_{LP} and T_g* control loops is achieved. This example proves the efficiency of the method developed for design of control systems optimised by the engine's response speed.

5 Concluding remarks

This paper presented an approach for optimal multi-variable control of aero engines. Most digital on-board engine control systems today represent mere digital realisation of analogue control laws used in hydro-mechanical control systems. However, existing control computers possess enough computing power to be used in optimisation of engine performance through a new type of control laws and algorithms. Implementation of optimal engine control leads to more efficient operation of the aircraft power plant via targeting global optimisation criteria and by accounting for individual engine characteristics. In the example discussed, a real-time algorithm was presented for optimisation of control laws and their optimal realisation. It was shown that the method developed allows design of multi-variable control of the engine with a high degree of decoupling between control loops.

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