

PREDICTIVE CONTROL OF NONLINEAR HAMMERSTEIN SYSTEMS AND APPLICATION TO PH PROCESSES

Z.Y. Zou^{*+}, G.P. Liu^{*†} and N. Guo⁺

^{*}School of Mechanical, Materials, Manufacturing Engineering and Management, University of Nottingham, Nottingham NG7 2RD, U.K., E-mail: guoping.liu@nottingham.ac.uk Fax: +44 (0)115 9513800

⁺Beijing Research Institute of Pharmaceutical Chemistry, P.O.Box 1043, Beijing 102205, P.R.China, E-mail: zouzhiyun@sohu.com

[†]Institute of Automation, Chinese Academy of Sciences, Beijing 100080, P.R.China

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Abstract

In this paper, a nonlinear predictive control algorithm based on discrete-time model is developed for nonlinear Hammerstein systems, which is known as NonLinear Hammerstein Predictive Control (NLHPC) algorithm. Following the predictive control strategy, this algorithm uses a Hammerstein model for control prediction. Analysis on the algorithm shows that it not only has good stability and strong robustness, but also possesses integral action itself. The NLHPC, which is implemented on an industrial computer, is applied to the PH control of a pilot plant neutralization process with strong non-linearity. The experimental results illustrate that the NLHPC gives better control performance than the commonly used industrial nonlinear PID (NL-PID) control.

1 Introduction

All practical industrial processes possess a certain degree of non-linearity. However, up to now, there are very few useful and effective nonlinear control methods. The nonlinear PID controller (NL-PID) [14] has been used to control some nonlinear systems, but it does not give optimal control and is also quite complex with regard to online tuning. A simple linear deterministic predictive control algorithm based on discrete-time model was developed by [3], and it was applied to an industrial distillation column with much better performance than PID controller. But it can't be used to control systems with strong non-linearity. So, it is desirable to use a more advanced control method to control nonlinear systems.

Predictive control is now widely used in industry and a large number of implementation algorithms, including generalised predictive control [1], dynamic matrix control [2], extended prediction self-adaptive control [6], predictive function control [13], extended horizon adaptive control [16] and unified predictive control [15], have appeared in the literature. Most predictive control algorithms are based on a linear model of the process. However, industrial processes usually contain complex nonlinearities and a linear model may be

acceptable only when the process is operating around an equilibrium point.

Recently, neural networks have been used in some predictive control algorithms that utilise nonlinear process models [5, 7]. Alternative design of nonlinear predictive control algorithms has also been studied [8, 9, 10, and 12]. However, in most algorithms for nonlinear predictive control their performance functions are minimised using nonlinear programming techniques to compute the future manipulated variables in on-line optimisation. This can make the realisation of the algorithms very difficult for real-time control.

The Hammerstein model [11] contains a nonlinear static gain and a linear dynamic section. It can be used to describe a large number of nonlinear systems in industry. In this paper, a nonlinear predictive control algorithm proposed for the nonlinear systems by combining the Hammerstein model for the control prediction model and the predictive control strategy. The developed algorithm is named as nonlinear Hammerstein predictive control (NLHPC) algorithm. In order to demonstrate the performance of the NLHPC algorithm, a comprehensive analysis of the algorithm is studied and a pilot plant neutralization process PH control experiment is carried out.

2 NonLinear Hammerstein Predictive Control

The structure of the NLHPC is presented in Figure 1. It is mainly composed of four parts: the Hammerstein model, output prediction, reference trajectory, and calculation of control action. Based on a predictive control performance function, the NLHPC is to make the prediction as close to the reference trajectory as possible, and to reach the set point quickly and smoothly at the same time.

2.1 Nonlinear Hammerstein Model

The Hammerstein model consists of a nonlinear static gain

$$x(k) = \sum_{i=1}^l r_i u^i(k) \quad (1)$$

and a linear dynamic sector

$$y_M(k) = \sum_{i=1}^{n_a} a_i y_M(k-i) + \sum_{j=1}^{n_b} b_j x(k-j) \quad (2)$$

where l , n_a , n_b are the model orders; r_i ($i=1,2,\dots,l$), a_i ($i=1,2,\dots,n_a$), b_j ($j=1,2,\dots,n_b$) are the model parameters; $x(k)$ is an intermediate variable, $y_M(k)$ is the model output. It is assumed that the model orders and parameters are known. If this is not the case, they can be estimated by the application of system identification methods [4].

Although the structure of the Hammerstein model is simple, it can be used to represent a large number of industrial processes with strong non-linearity, such as power function, dead zone, switch and etc. Here, the Hammerstein model is used to develop nonlinear predictive control algorithm.

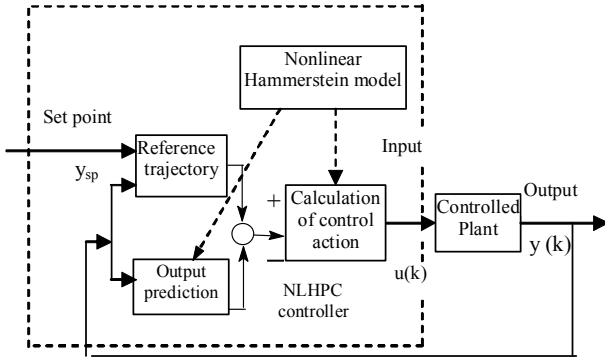


Figure 1: The NLHPC system

2.2 Output Prediction

The NLHPC algorithm predicts the future output of the nonlinear system with its Hammerstein model and past input-output data.

Suppose that at time k , a prediction of the output at time $k+1$ to $k+p$ is needed. In order to avoid the violent fluctuation of a system response, the modification of the control action is only limited in μ steps ($\mu \leq p$). So

$$u(k+\mu-1) = u(k+\mu) = \dots = u(k+p-1) \quad (3)$$

From Equations (1), (2) and (3), the p -step ahead prediction of the output at time k can be obtained by

$$Y_M(K) = Y_M^{Zi}(K) + CX(K) \quad (4)$$

where

$$Y_M(K) = [y_M(k+1|k), y_M(k+2|k), \dots, y_M(k+p|k)]^T$$

$$Y_M^{Zi}(K) = [y_M^{Zi}(k+1|k), y_M^{Zi}(k+2|k), \dots, y_M^{Zi}(k+p|k)]^T$$

$$X(K) = [x(k), x(k+1), \dots, x(k+\mu-1)]^T$$

$$C = \begin{bmatrix} c_1 & 0 & 0 & 0 \\ c_2 & c_1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ c_\mu & c_{\mu-1} & \dots & c_1 \\ \vdots & \vdots & \vdots & \vdots \\ c_p & c_{p-1} & \dots & \sum_{i=1}^{p-\mu+1} c_i \end{bmatrix}$$

Here $y_M(k+i|k)$ represents the i -step ahead prediction of the model output at time k ; $y_M^{Zi}(k+i|k)$ expresses the same value as $y_M(k+i|k)$ but with zero future control action (zero input, z_i), it can be calculated by

$$y_M^{Zi}(k+i|k) = \sum_{j=1}^{n_a} a_j y'(k+i-j) + \sum_{j=1}^{n_b} b_j x'(k+i-j), \quad i=1, 2, \dots, P \quad (5)$$

where

$$y'(k+i-j) = \begin{cases} y_M(k+i-j), & i < j+1 \\ y_M^{Zi}(k+i-j), & i \geq j+1 \end{cases} \quad (6)$$

$$x'(k+i-j) = \begin{cases} x(k+i-j), & i < j \\ 0, & i \geq j \end{cases} \quad (7)$$

Obviously, the vector $Y_M^{Zi}(K)$ denotes the future output caused by the inputs before time k , so it is a definitive term; $CX(K)$ represents the output variation caused by future control action, and it consists of the intermediate variable vector $X(K)$, so it is an unknown term. The parameters c_i ($i=1,2,\dots,p$) in matrix C can be calculated by

$$\begin{aligned} c_1 &= b_1 \\ c_i &= \sum_{j=1}^{i-1} a_j c_{i-j} + b_i, \quad 2 \leq i \leq n_b \\ c_i &= \sum_{j=1}^{n_a} a_j c_{i-j}, \quad n_b < i \leq p \end{aligned} \quad (8)$$

In order to reduce the effects of model error and unmeasurable disturbances on the accuracy of prediction, the current prediction error, $y(k) - y_M(k|k)$, is used to modify the prediction of the model output. Here $y(k)$ is the actual output, and $y_M(k|k)$ can be calculated by

$$y_M(k|k) = \sum_{i=1}^{n_a} a_i y(k-i) + \sum_{j=1}^{n_b} b_j x(k-j) \quad (9)$$

So, the modified output prediction is

$$Y_C(K) = Y_M(K) + D(K) \quad (10)$$

where

$$Y_c(K) = [y_c(k+1|k), y_c(k+2|k), \dots, y_c(k+p|k)]^T$$

$D(K) = [y(k) - y_M(k), y(k) - y_M(k), \dots, y(k) - y_M(k)]^T$
 $y_c(k+i|k)$ ($i=1, 2, \dots, p$) is the modified output prediction which is also called the closed-loop output prediction.

2.3 Reference Trajectory

The goal of industrial process control generally requires that the system output reach the set point quickly and smoothly. So, the system output should follow a reference trajectory [3].

Based on the first order approximation, the calculation algorithm of the trajectory is given by

$$y_r(k+i|k) = \alpha^i y(k) + (1 - \alpha^i) y_{sp} \quad (11)$$

$i=1, 2, \dots, p$, where $y_r(k+i|k)$ is the output reference trajectory at time $k+i$; y_{sp} is the set point of the output; and α is a parameter ($0 < \alpha < 1$). When α is smaller, the output trajectory reaches the set point with less time.

2.4 Calculation of Control Action

The performance function of the NLHPC is

$$J = \sum_{i=1}^p [y_c(k+i|k) - y_r(k+i|k)]^2 + \lambda \sum_{i=1}^{\mu} [x(k+i-1) - x(k+i-2)]^2 \quad (12)$$

where λ is the weighting factor. Equation (12) can be written as

$$J = \|Y_c(K) - Y_r(K)\|^2 + \|X(K)\|_{\lambda}^2 + \lambda x^2(k-1) - 2\lambda x(k)x(k-1) \quad (13)$$

where

$$\Lambda = \begin{bmatrix} 2\lambda & -\lambda & 0 & 0 & 0 \\ -\lambda & 2\lambda & -\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 2\lambda & -\lambda \\ 0 & 0 & 0 & -\lambda & \lambda \end{bmatrix}$$

$$Y_r(K) = [y_r(k+1|k), y_r(k+2|k), \dots, y_r(k+p|k)]^T$$

The calculation of control action is then performed in two steps.

Step1: Substituting Equations (4) and (10) into Equation (13), then taking the derivative of J with respect to $x(k)$ and finally setting the derivative to be equal to zero gives

$$x^*(k) = T \{ C^T [Y_r(K) - Y_M^Z(K) - D(K)] + \Lambda' x(k-1) \} \quad (14)$$

where $\Lambda' = [\lambda, 0, \dots, 0]^T$; $T = (C^T C + \Lambda)^{-1}$ is a matrix for control action calculation. From Equation (14), we can obtain

the optimal sequence $x^*(k), x^*(k+1), \dots, x^*(k+\mu-1)$.

Step 2: Using Equation (1) with $x^*(k+i-1)$ ($i=1, 2, \dots, \mu$), the optimal control action sequence $u^*(k+i-1)$ ($i=1, 2, \dots, \mu$) can be derived by calculating the roots of the nonlinear algebraic equation (1).

If the controlled nonlinear system has a time delay d , by modifying the performance function to be

$$J = \sum_{i=1}^p [Y_c(k+d+i|k) - Y_r(k+d+i|k)]^2 + \sum_{i=1}^{\mu} [x(k+i-1) - x(k+i-2)]^2 \quad (15)$$

The optimal control action can be obtained using the similar algorithm as the system without time delay.

3 Stability and Robustness of NLHPC

Assuming the actual model of the controlled nonlinear system as

$$y(k) = \sum_{i=1}^{n_{ao}} a_{io} y(k-i) + \sum_{j=1}^{n_{bo}} b_{jo} x(k-i) \quad (16)$$

$$x(k) = \sum_{i=1}^{l_o} r_{io} u^i(k) \quad (17)$$

where n_{ao}, n_{bo}, l_o are the actual model orders, a_{io} ($i=1, 2, \dots, n_{ao}$), b_{jo} ($j=1, 2, \dots, n_{bo}$), r_{io} ($i=1, 2, \dots, l_o$) are the model parameters.

Taking z transformation on the both sides of Equations (2) and (16), the transfer function of the model is given by

$$G(z^{-1}) = \frac{y_M(k)}{x(k)} = \frac{B(z^{-1})}{A(z^{-1})} \quad (18)$$

and the transfer function of the real system is

$$G_o(z^{-1}) = \frac{y(k)}{x(k)} = \frac{B_o(z^{-1})}{A_o(z^{-1})} \quad (19)$$

where

$$A(z^{-1}) = \sum_{i=1}^{n_a} a_i z^{-i}, \quad B(z^{-1}) = \sum_{i=1}^{n_b} b_i z^{-i}$$

$$A_o(z^{-1}) = \sum_{i=1}^{n_{ao}} a_{io} z^{-i}, \quad B_o(z^{-1}) = \sum_{i=1}^{n_{bo}} b_{io} z^{-i}$$

Making the following assumptions on the NLHPC system:

- 1) The controlled nonlinear system is stable, which means that the roots of $A_o(z^{-1})$ are all inside the unit circle in the z-plane.
- 2) $\lambda = 0$, namely, no weighting factor is added onto the intermediate variables.
- 3) $\mu = p$, namely, it does not limit the changing steps of control action.

Then, the optimal solution of minimizing the performance function J in Equation (12) is given by

$$y_c(k+1|k) = y_r(k+1|k) \quad (20) \quad G_o(z^{-1}) - G(z^{-1}) \text{ is } (\beta - 1)G(z^{-1}).$$

Based on the above assumptions, the analysis of the stability and robustness of the NLHPC system is discussed below.

3.1 Stability of the Algorithm

Assume that the model of the nonlinear system is accurate, namely $n_a = n_{ao}$, $n_b = n_{bo}$, $G(z^{-1}) = G_o(z^{-1})$, $l = l_o$, $r_i = r_{io}$ ($i = 1, 2, \dots, l$). Substituting Equation (11) and Equation (10) ($p=1$) into Equation (20) results in

$$y_M(k+1|k) + (y(k) - y_M(k|k)) = \alpha y(k) + (1 - \alpha)y_{SP} \quad (21)$$

Considering $y_M(k|k) = y_M(k)$, $y_M(k+1|k) = zy_M(k)$ and taking z transformation on the both sides of Equation (21), and also substituting Equations (18) and (19) into Equation (21), then we can obtain the following transfer function of the NLHPC system.

$$\frac{x(k)}{y_{SP}} = \frac{(1 - \alpha)A(z^{-1})A_o(z^{-1})}{(z - 1)B(z^{-1})A_o(z^{-1}) + (1 - \alpha)B_o(z^{-1})A(z^{-1})} \quad (22)$$

$$= \frac{(1 - \alpha)A_o(z^{-1})}{(z - \alpha)B_o(z^{-1})}$$

Combining Equations (19) and (22), and when all the zeros of $B_o(z^{-1})$ are inside the unit circle in the z -plane,

$$\frac{y(k)}{y_{SP}} = \frac{1 - \alpha}{z - \alpha} \quad (23)$$

Because $0 < \alpha < 1$, from Equations (22) and (23), we can conclude that the close-loop NLHPC system is stable under the present assumptions if all the roots of $B_o(z^{-1})$ are not outside the unit circle. That is to say, the NLHPC system is stable, only requires that the controlled nonlinear system is minimum phase system.

3.2 Robustness of the Algorithm

It assumes that there exists an error in the system model, but the nonlinear static gain of the nonlinear model is accurate, and the linear dynamic sector of the nonlinear model has a β time difference from the real system, that is

$$G_o(z^{-1}) = \beta G(z^{-1}) \quad (24)$$

From Equation (22), the closed-loop transfer function of the NLHPC system is

$$G_c(z^{-1}) = \frac{(z - 1)B(z^{-1})A_o(z^{-1}) + (1 - \alpha)B_o(z^{-1})A(z^{-1})}{(z - 1 + (1 - \alpha)\beta)B(z^{-1})A_o(z^{-1})} \quad (25)$$

Assume that all the roots of $A_o(z^{-1})$ are inside the unit circle and the model of the nonlinear system is minimum phase.

If $z = 1 - (1 - \alpha)\beta$ is inside the unit circle, namely, β satisfies

$$0 < \beta < 2(1 - \alpha)^{-1} \quad (26)$$

Then from Equation (25), we can conclude that the nonlinear system is stable, namely, it is robust when the modeling error

3.3 Property in the Steady State

Assume the NLHPC system is stable. As the time approaches infinity, $x(k)$ will keep constant, and the increment of $x(k)$ will be zero, then Equation (12) will become

$$J = \sum_{i=1}^p [y_c(k+i|k) - y_r(k+i|k)]^2 \quad (27)$$

Because $u(k)$, $y(k)$ and $y_M(k)$ will keep constant after the system getting stabilized, Equation(3) will no longer place limit on the control action in the steady state. From Equation (27), it is clear that the optimal control will result in

$$y_c(k+1|k) = y_r(k+1|k) \quad (28)$$

which leads to

$$y_M(k+1|k) + [y(k) - y_M(k|k)] = \alpha y(k) + (1 - \alpha)y_{sp} \quad (29)$$

As $y_M(k+1|k) = y_M(k|k)$, from Equation (29) we have $y(k) = y_{sp}$. This implies that there exists no steady state error for the NLHPC system. The above result shows that the NLHPC algorithm exists integral action which eliminates the system error in the steady state, no matter whether there exists the modeling error, the weighting factor is placed on the increment of $x(k)$, or the change steps of control action is limited.

4 Application to a PH Control Process

The NLHPC algorithm was applied to a pilot plant PH process, as shown in Figure 2. Its control results were compared with the control results of a nonlinear PID (NL-PID) controller.

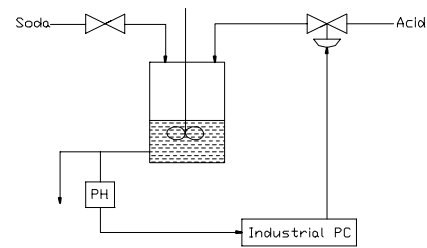


Figure 2: The control system of the PH process

The nonlinear PID controller is composed of a linear PID controller described by

$$u(k) = e(k) + k_I \sum_{i=1}^k e(i) + k_D [e(k) - e(k-1)] \quad (30)$$

and a sector of nonlinear static gain which is shown in Figure 3. In Equation (30), $e(k)$ is the error, k_I is the integral coefficient and k_D is the derivative coefficient. In Figure 3, k_1 , k_2 and k_3 are the static gains.

As the NLHPC algorithm has good robustness, the Hammerstein model developed several years ago for the PH Process is used to design the NLHPC controller at present time.

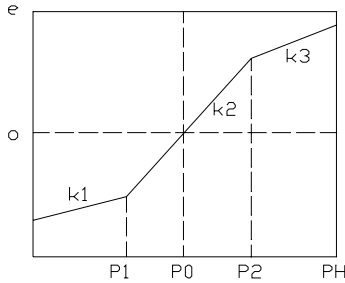


Figure3: The nonlinear static gains of the NL-PID controller

The Hammerstein model of the PH process is

$$x(k) = u(k) - 1.207u^2(k) + 1.15u^3(k) \quad (31)$$

$$y(k) - 1.558y(k-1) + 0.597y(k-2) = 0.0185x(k-2) + 0.0173x(k-3) + 0.00248x(k-4) \quad (32)$$

where $u(k)=u_m(k)-u_s$, $y(k)=y_m(k)-y_s$, $u_m(k)$ is the electric current to adjust the valve position of the acid flow, $y_m(k)$ is the PH value of the neutral liquid. $u_s=2.5mA$, $y_s=5.713PH$, and the sampling period $T_s=30$ seconds.

It needs to mention that the model described by Equations (31) and (32) exists a modelling error, which is mainly caused by the variable characteristics of equipments and instruments, and the changing concentration of acid and soda. However the good robustness of NLHPC can overcome these modeling errors and control the PH process well.

4.1 Simulation results of PH control

In the simulation, the parameters of NLHPC were $\alpha=0.85, \lambda=0.005, p=6, \mu=3$; the parameters of the NL-PID achieved by optimal tuning were $k_I=0.15, k_D=0, k_I=0.12, k_2=0.60, k_3=0.12, P_1=4.8, P_2=7.3$.

Set point tracking

When the set point of PH was raised from 5.7 to 8.0, the control result of the NLHPC and NL-PID is illustrated in Figure 4. From Figure 4 it can be found that the NLHPC controller has a short set point tracking time and a good dynamic output response, and NL-PID controller has a long tracking time and an oscillating output response.

Disturbance rejection

When the PH value of the neutralization process was stabilized at 5.7, a deterministic disturbance signal was added to the process. The control result was shown in Figure 5.

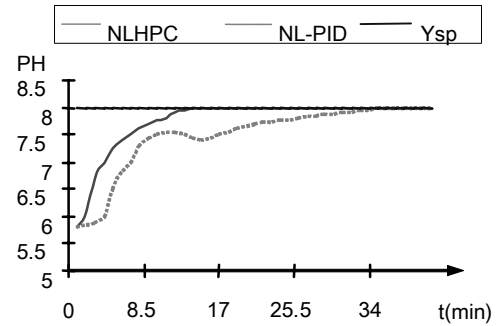


Figure 4: The output response of set point tracking

The PH output response of the NLHPC quickly returns to the set point with a short transient response and nearly no over-shooting. The over-shooting of the NL-PID controller is high, and its transient response time is much longer than the NLHPC.

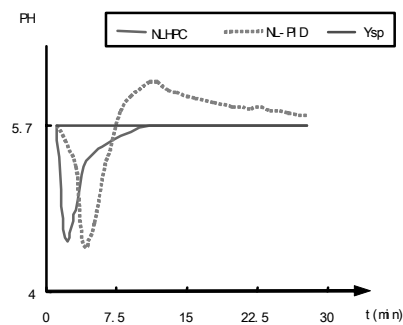


Figure 5: The output response of anti-disturbance

4.2 Experimental results of the PH control

In the PH control experiment of the neutralization process using the NLHPC, the control parameters were selected as $\alpha=0.85, \lambda=0.005, p=6, \mu=3$. At the beginning, the PH value was stabilized at about 7.3. Then the set point was suddenly changed to 6.638. The set point tracking response of the PH process controlled by the NLHPC was recorded down as shown in Figure 6. It can be seen from the results that the PH process has a very good output response. The PH value reaches the new set point quickly and smoothly without a steady state error.

It needs to mention that these PH control results of NLHPC were achieved using the Hammerstein model developed several years ago. This model now has significant difference with the present PH process. So this demonstrates that the robustness of NLHPC is very strong.

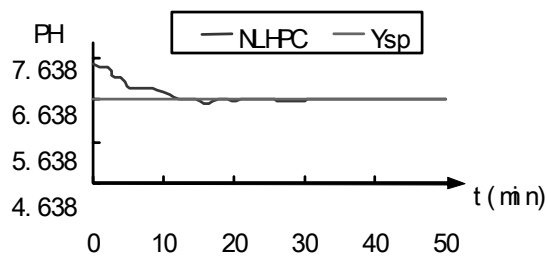


Figure 6: The set point tracking of the NLHPC system

5 Conclusions

This paper has presented a nonlinear predictive control strategy for nonlinear Hammerstein systems. Both analysis and experiments have shown that the NLHPC is an effective method for the control of nonlinear systems. It not only gives good control response, but also possesses good stability and robustness even when there exists large modeling error. Furthermore, its algorithm is simple and its implementation is convenient and flexible. So, the NLHPC algorithm is applicable to a large number of industrial systems with strong nonlinearities.

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