FORCE CONTROL WITH A VELOCITY OBSERVER

J. Gudiño–Lau, M. A. Arteaga

Sección de Eléctrica,
División de Estudios de Posgrado de la Facultad de Ingeniería
Universidad Nacional Autónoma de México
Apdo. Postal 70–256, México, D. F., 04510, México,
Tel.: + 52–55–56–22–30–13, Fax: + 52–55–56–16–10–73,
e-mail: jglau@correo.unam.mx, arteaga@verona.fi-p.unam.mx

Keywords: Force control, observer design.

Abstract

The aim of this paper is to study the force/position tracking problem of robot manipulators using the well–known hybrid/position control approach. It is shown that the robot system is closed-loop stable and that the desired force can always be reached. A systematic procedure of control synthesis is presented and stability conditions for the control parameters are derived. No velocity measurements are assumed to be available, so that a nonlinear observer is proposed.

1 INTRODUCTION

In many industrial applications, robot manipulators are required to make contact with the environment, for example when deburring, grinding or in assembling tasks. In these cases, it is necessary to control not only position but also the force exerted at the contact. Several approaches to control force and position have been proposed in the literature (see [11, 12] for an overview). Depending on the adopted model of contact force, these schemes can be classified as compliant [10], impedance [4] and constrained motion control [7]. On the other hand, control techniques that apply directly position and force feedback such as hybrid control [9], and parallel control [3] have also been developed. All these schemes require an environment model, either in controller designing or for stability proofs. Also, most control schemes make use of velocity measurements, which may not be available. For example, the approaches shown in [1, 5] developed a hybrid position/force control by dividing the problem in a force controller where the motion is constrained and by a tracking controller otherwise. In this note, we proposed a similar method but using only joint measurements.

The paper is organized as follows: Section 2 describes the robot manipulator–environment system. Section 3 presents the proposed controller and the stability analysis. Section 4 illustrates the performance of the proposed controller via experimental work. Finally, Section 5 presents some conclusions.

2 System model and properties

Consider a n degrees of freedom rigid robot manipulator in contact with a smooth surface. In this case, the dynamics of the system is given by [8]

$$\begin{aligned} \boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} &+ \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{D}\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \\ &= \boldsymbol{\tau} + \boldsymbol{J}_{\varphi}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{\lambda}, \end{aligned} \tag{1}$$

where $\boldsymbol{q} \in \mathbb{R}^n$ is the vector of generalized joint coordinates, $\boldsymbol{H}(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ is the symmetric positive definite inertia matrix, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} \in \mathbb{R}^n$ is the vector of Coriolis and centrifugal torques, $\boldsymbol{g}(\boldsymbol{q}) \in \mathbb{R}^n$ is the vector of gravitational torques, $\boldsymbol{D} \in \mathbb{R}^{n\times n}$ is the positive semidefinite diagonal matrix accounting for joint viscous friction coefficients, $\boldsymbol{\tau} \in \mathbb{R}^n$ is the vector of torques acting at the joints, and $\boldsymbol{\lambda} \in \mathbb{R}^m$ is the vector of Lagrange multipliers (physically represents the force applied at the contact point). $\boldsymbol{J}_{\varphi}(\boldsymbol{q}) = \boldsymbol{\nabla}\varphi(\boldsymbol{q}) \in \mathbb{R}^{m\times n}$ is assumed to be of full rank in this paper. $\boldsymbol{\nabla}\boldsymbol{\varphi}(\boldsymbol{q})$ denotes the gradient of the object surface vector $\boldsymbol{\varphi} \in \mathbb{R}^m$ which maps a vector onto the normal plane at the tangent plane that arises at the contact point described by

$$\boldsymbol{\varphi}(\boldsymbol{q}) = \boldsymbol{0}, \tag{2}$$

while m is the number of constraints given by the surface.

Let us denote the largest (smallest) eigenvalue of a matrix by $\lambda_{\max}(\cdot)$ ($\lambda_{\min}(\cdot)$). The norm of an $n \times 1$ vector $\boldsymbol{x}$ is defined by $\|\boldsymbol{x}\| \triangleq \sqrt{\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}}$, while the norm of an $m \times n$ matrix $\boldsymbol{A}$ is the corresponding induced norm $\|\boldsymbol{A}\| \triangleq \sqrt{\lambda_{\max}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})}$. By recalling that revolute joints are considered, the following properties can be established [2, 6, 8]:

Property 2.1 The $\boldsymbol{H}(\boldsymbol{q})$ satisfies $\lambda_{\mathrm{h}}\|\boldsymbol{x}\|^2 \le \boldsymbol{x}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{x} \le \lambda_{\mathrm{H}}\|\boldsymbol{x}\|^2 \quad \forall\ \boldsymbol{q},\boldsymbol{x} \in \mathbb{R}^n$, where $\lambda_{\mathrm{h}} \triangleq \min_{\forall \boldsymbol{q}\in\mathbb{R}^n} \lambda_{\min}(\boldsymbol{H})$, $\lambda_{\mathrm{H}} \triangleq \max_{\forall \boldsymbol{q}\in\mathbb{R}^n} \lambda_{\max}(\boldsymbol{H})$, and $0 < \lambda_{\mathrm{h}} \le \lambda_{\mathrm{H}} < \infty$. △

Property 2.2 With a proper definition of $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$, $\dot{\boldsymbol{H}}(\boldsymbol{q}) - 2\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is skew–symmetric. △

Property 2.3 The vector $\boldsymbol{C}(\boldsymbol{q},\boldsymbol{x})\boldsymbol{y}$ satisfies $\boldsymbol{C}(\boldsymbol{q},\boldsymbol{x})\boldsymbol{y} = \boldsymbol{C}(\boldsymbol{q},\boldsymbol{y})\boldsymbol{x}\ \forall\ \boldsymbol{x},\boldsymbol{y} \in \mathbb{R}^n$ △

Property 2.4 It is satisfied $\|\boldsymbol{C}(\boldsymbol{q},\boldsymbol{x})\| \leq k_{\mathrm{c}}\|\boldsymbol{x}\|$ with $0 < k_{\mathrm{c}} < \infty$, and $\forall\, \boldsymbol{x} \in \mathbb{R}^n$ △

Property 2.5 $\dot{\boldsymbol{q}}$ can be written as

$$\begin{aligned} \dot{\boldsymbol{q}} &= \dot{\boldsymbol{q}} + \left(\boldsymbol{J}_\varphi^+\boldsymbol{J}_\varphi\dot{\boldsymbol{q}} - \boldsymbol{J}_\varphi^+\boldsymbol{J}_\varphi\dot{\boldsymbol{q}}\right) \\ &= \left(\boldsymbol{I}_{n\times n} - \boldsymbol{J}_\varphi^+\boldsymbol{J}_\varphi\right)\dot{\boldsymbol{q}} + \boldsymbol{J}_\varphi^+\boldsymbol{J}_\varphi\dot{\boldsymbol{q}} \\ &\triangleq \boldsymbol{Q}(\boldsymbol{q})\dot{\boldsymbol{q}} + \boldsymbol{J}_\varphi^+(\boldsymbol{q})\dot{\boldsymbol{p}}, \end{aligned} \tag{3}$$

where $\boldsymbol{J}_\varphi^+ = \boldsymbol{J}_\varphi^{\mathrm{T}}\left(\boldsymbol{J}_\varphi\boldsymbol{J}_\varphi^{\mathrm{T}}\right)^{-1} \in \mathbb{R}^{n\times m}$ stands for the Penrose's pseudoinverse and $\boldsymbol{Q} \in \mathbb{R}^{n\times n}$ satisfies $\operatorname{rank}(\boldsymbol{Q}) = n-m$. These two matrices are orthogonal, i.e. $\boldsymbol{Q}\boldsymbol{J}_\varphi^+ = \boldsymbol{O}$ (and $\boldsymbol{Q}\boldsymbol{J}_\varphi^{\mathrm{T}} = \boldsymbol{O}$). $\dot{\boldsymbol{p}} = \boldsymbol{J}_\varphi\dot{\boldsymbol{q}} \in \mathbb{R}^m$ is the so called constrained velocity. Note that

$$\dot{\boldsymbol{p}} = \boldsymbol{0} \quad \text{and} \quad \boldsymbol{p} = \boldsymbol{0}. \tag{4}$$

$\boldsymbol{p}$ is called the constrained position. △

3 Control with velocity estimation

Control Law

In this section, the tracking control problem of a cooperative system of rigid robots is studied. Consider model (1) and define the tracking and observation errors as

$$\tilde{\boldsymbol{q}} \triangleq \boldsymbol{q} - \boldsymbol{q}_{\mathrm{d}} \tag{5}$$

$$\boldsymbol{z} \triangleq \boldsymbol{q} - \hat{\boldsymbol{q}} \tag{6}$$

where $\boldsymbol{q}_{\mathrm{d}}$ is a desired smooth bounded trajectory satisfying constraint (2), and $(\hat{\cdot})$ represents the estimated value of $(\cdot)$. Other error definitions are

$$\Delta\boldsymbol{p} \triangleq \boldsymbol{p} - \boldsymbol{p}_{\mathrm{d}} \tag{7}$$

$$\Delta\boldsymbol{\lambda} \triangleq \boldsymbol{\lambda} - \boldsymbol{\lambda}_{\mathrm{d}}, \tag{8}$$

where $\boldsymbol{p}_{\mathrm{d}}$ is the desired constrained position which satisfies equation (4). $\boldsymbol{\lambda}_{\mathrm{d}}$ is the desired force to be applied by each finger on the constrained surface. Other useful definitions are

$$\begin{aligned} \dot{\boldsymbol{q}}_{\mathrm{r}} &\triangleq \boldsymbol{Q}(\boldsymbol{q})\left(\dot{\boldsymbol{q}}_{\mathrm{d}} - \boldsymbol{\Lambda}\left(\hat{\boldsymbol{q}} - \boldsymbol{q}_{\mathrm{d}}\right)\right) \\ &+ \boldsymbol{J}_\varphi^+(\boldsymbol{q})\left(\dot{\boldsymbol{p}}_{\mathrm{d}} - \beta\Delta\boldsymbol{p} + \boldsymbol{\xi}\Delta\boldsymbol{F}\right) \end{aligned} \tag{9}$$

$$\begin{aligned} \boldsymbol{s} &\triangleq \dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_{\mathrm{r}} \\ &= \boldsymbol{Q}(\boldsymbol{q})\left(\dot{\hat{\boldsymbol{q}}} + \boldsymbol{\Lambda}\left(\hat{\boldsymbol{q}} - \boldsymbol{q}_{\mathrm{d}}\right)\right) \\ &+ \boldsymbol{J}_\varphi^+(\boldsymbol{q})\left(\Delta\dot{\boldsymbol{p}} + \beta\Delta\boldsymbol{p} - \boldsymbol{\xi}\Delta\boldsymbol{F}\right) \\ &\triangleq \boldsymbol{s}_{\mathrm{p}} + \boldsymbol{s}_{\mathrm{f}} \end{aligned} \tag{10}$$

$$\Delta\boldsymbol{F} \triangleq \int_0^t \Delta\boldsymbol{\lambda}(\vartheta)\mathrm{d}\vartheta, \tag{11}$$

where $\boldsymbol{\Lambda} \in \mathbb{R}^{n\times n}$, $\boldsymbol{\xi} \in \mathbb{R}^{m\times m}$ are diagonal positive definite matrices, and β is a positive constant. Note that $\boldsymbol{s}_{\mathrm{p}}$ and $\boldsymbol{s}_{\mathrm{f}}$ are orthogonal vectors, and that $\boldsymbol{s}$ can also be written as

$$\begin{aligned} \boldsymbol{s} &= \boldsymbol{Q}(\boldsymbol{q})\left(\dot{\hat{\boldsymbol{q}}} + \boldsymbol{\Lambda}\tilde{\boldsymbol{q}} - \boldsymbol{\Lambda}\boldsymbol{z}\right) \\ &+ \boldsymbol{J}_\varphi^+(\boldsymbol{q})\left(\Delta\dot{\boldsymbol{p}} + \beta\Delta\boldsymbol{p} - \boldsymbol{\xi}\Delta\boldsymbol{F}\right). \end{aligned} \tag{12}$$

Let us analyze $\ddot{\boldsymbol{q}}_{\mathrm{r}}$. This quantity is given by

$$\begin{aligned} \ddot{\boldsymbol{q}}_{\mathrm{r}} &\triangleq \dot{\boldsymbol{Q}}(\boldsymbol{q})\left(\dot{\boldsymbol{q}}_{\mathrm{d}} - \boldsymbol{\Lambda}\left(\hat{\boldsymbol{q}} - \boldsymbol{q}_{\mathrm{d}}\right)\right) \\ &+ \dot{\boldsymbol{J}}_\varphi^+(\boldsymbol{q})\left(\dot{\boldsymbol{p}}_{\mathrm{d}} - \beta\Delta\boldsymbol{p} + \boldsymbol{\xi}\Delta\boldsymbol{F}\right) \\ &+ \boldsymbol{Q}(\boldsymbol{q})\left(\ddot{\boldsymbol{q}}_{\mathrm{d}} - \boldsymbol{\Lambda}\left(\dot{\hat{\boldsymbol{q}}} - \dot{\boldsymbol{q}}_{\mathrm{d}}\right)\right) \\ &+ \boldsymbol{J}_\varphi^+(\boldsymbol{q})\left(\ddot{\boldsymbol{p}}_{\mathrm{d}} - \beta\left(\dot{\boldsymbol{p}} - \dot{\boldsymbol{p}}_{\mathrm{d}}\right) + \boldsymbol{\xi}\Delta\boldsymbol{\lambda}\right). \end{aligned} \tag{13}$$

As it will be shown later, $\ddot{\boldsymbol{q}}_{\mathrm{r}}$ is necessary to implement the controller and the observer. However, this quantity is not available since $\dot{\boldsymbol{q}}$ is not measurable. In order to overcome this drawback, let us consider $\boldsymbol{Q}(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$. Then you have

$$\dot{\boldsymbol{Q}}(\boldsymbol{q}) = \begin{bmatrix} \dfrac{\partial a_{11}(\boldsymbol{q})}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}} & \cdots & \dfrac{\partial a_{1n}(\boldsymbol{q})}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}} \\ \vdots & & \vdots \\ \dfrac{\partial a_{n1}(\boldsymbol{q})}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}} & \cdots & \dfrac{\partial a_{nn}(\boldsymbol{q})}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}} \end{bmatrix}, \tag{14}$$

where $a_{\alpha\beta}$ is the $\alpha\beta$ element of $\boldsymbol{Q}(\boldsymbol{q})$. Based on equation (14), considered the following definition

$$\dot{\hat{\boldsymbol{Q}}}(\boldsymbol{q}) \triangleq \begin{bmatrix} \dfrac{\partial a_{11}(\boldsymbol{q})}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}}_{\mathrm{o}} & \cdots & \dfrac{\partial a_{1n}(\boldsymbol{q})}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}}_{\mathrm{o}} \\ \vdots & & \vdots \\ \dfrac{\partial a_{n1}(\boldsymbol{q})}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}}_{\mathrm{o}} & \cdots & \dfrac{\partial a_{nn}(\boldsymbol{q})}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}}_{\mathrm{o}} \end{bmatrix}, \tag{15}$$

with

$$\dot{\boldsymbol{q}}_{\mathrm{o}} \triangleq \dot{\hat{\boldsymbol{q}}} - \boldsymbol{\Lambda}\boldsymbol{z}. \tag{16}$$

Then, one can compute

$$\begin{aligned} \dot{\tilde{\boldsymbol{Q}}}(\boldsymbol{r}) &\triangleq \dot{\boldsymbol{Q}}(\boldsymbol{q}) - \dot{\hat{\boldsymbol{Q}}}(\dot{\boldsymbol{q}}_{\mathrm{o}}) \\ &= \begin{bmatrix} \dfrac{\partial a_{11}(\boldsymbol{q})}{\partial \boldsymbol{q}}\boldsymbol{r} & \cdots & \dfrac{\partial a_{1n}(\boldsymbol{q})}{\partial \boldsymbol{q}}\boldsymbol{r} \\ \vdots & & \vdots \\ \dfrac{\partial a_{n1}(\boldsymbol{q})}{\partial \boldsymbol{q}}\boldsymbol{r} & \cdots & \dfrac{\partial a_{nn}(\boldsymbol{q})}{\partial \boldsymbol{q}}\boldsymbol{r} \end{bmatrix}, \end{aligned} \tag{17}$$

where

$$\boldsymbol{r} \triangleq \dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_{\mathrm{o}} = \dot{\boldsymbol{z}} + \boldsymbol{\Lambda}\boldsymbol{z}. \tag{18}$$

In view of equation (15), we proposed the following substitution for $\ddot{\boldsymbol{q}}_{\mathrm{r}}$

$$\begin{aligned} \hat{\ddot{\boldsymbol{q}}}_{\mathrm{r}} &\triangleq \dot{\hat{\boldsymbol{Q}}}(\boldsymbol{q})\left(\dot{\boldsymbol{q}}_{\mathrm{d}} - \boldsymbol{\Lambda}\left(\hat{\boldsymbol{q}} - \boldsymbol{q}_{\mathrm{d}}\right)\right) \\ &+ \dot{\hat{\boldsymbol{J}}}_\varphi^+(\boldsymbol{q})\left(\dot{\boldsymbol{p}}_{\mathrm{d}} - \beta\Delta\boldsymbol{p} + \boldsymbol{\xi}\Delta\boldsymbol{F}\right) \\ &+ \boldsymbol{Q}(\boldsymbol{q})\left(\ddot{\boldsymbol{q}}_{\mathrm{d}} - \boldsymbol{\Lambda}\left(\dot{\hat{\boldsymbol{q}}} - \dot{\boldsymbol{q}}_{\mathrm{d}}\right)\right) \\ &+ \boldsymbol{J}_\varphi^+(\boldsymbol{q})\left(\ddot{\boldsymbol{p}}_{\mathrm{d}} - \beta\left(\boldsymbol{J}_\varphi(\boldsymbol{q})\dot{\boldsymbol{q}}_{\mathrm{o}} - \dot{\boldsymbol{p}}_{\mathrm{d}}\right) + \boldsymbol{\xi}\Delta\boldsymbol{\lambda}\right), \end{aligned} \tag{19}$$

where $\dot{\hat{\boldsymbol{J}}}_{\varphi}^{+}(\boldsymbol{q})$ is defined in the same fashion as $\dot{\hat{\boldsymbol{Q}}}(\boldsymbol{q})$ in equation (15). After some manipulation, it is possible to get

$$\dot{\ddot{\boldsymbol{q}}}_{\mathrm{r}} = \ddot{\boldsymbol{q}}_{\mathrm{r}} + \boldsymbol{e}(\boldsymbol{r}), \tag{20}$$

where

$$\begin{aligned} \boldsymbol{e}(\boldsymbol{r}) &\triangleq -\dot{\hat{\boldsymbol{Q}}}(\boldsymbol{q})\left(\dot{\boldsymbol{q}}_{\mathrm{d}} - \boldsymbol{\Lambda}\left(\hat{\boldsymbol{q}} - \boldsymbol{q}_{\mathrm{d}}\right)\right) \\ &- \dot{\hat{\boldsymbol{J}}}_{\varphi}^{+}(\boldsymbol{q})\left(\dot{\boldsymbol{p}}_{\mathrm{d}} - \beta\Delta\boldsymbol{p} + \boldsymbol{\xi}\Delta\boldsymbol{F}\right) \\ &+ \beta\boldsymbol{J}_{\varphi}^{+}(\boldsymbol{q})\boldsymbol{J}_{\varphi}(\boldsymbol{q})\boldsymbol{r}. \end{aligned} \tag{21}$$

The proposed controller is then given by

$$\begin{aligned} \boldsymbol{\tau} &\triangleq \boldsymbol{H}(\boldsymbol{q})\dot{\ddot{\boldsymbol{q}}}_{\mathrm{r}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}_{\mathrm{r}})\dot{\boldsymbol{q}}_{\mathrm{r}} \\ &+ \boldsymbol{D}\dot{\boldsymbol{q}}_{\mathrm{R}} + \boldsymbol{g}(\boldsymbol{q}) \\ &- \boldsymbol{K}_{\mathrm{R}}\left(\dot{\boldsymbol{q}}_{\mathrm{o}} - \dot{\boldsymbol{q}}_{\mathrm{r}}\right) - \boldsymbol{J}_{\varphi}^{\mathrm{T}}(\boldsymbol{q})\left(\boldsymbol{\lambda}_{\mathrm{d}} - k_{\mathrm{f}}\Delta\boldsymbol{F}\right), \end{aligned} \tag{22}$$

where $\boldsymbol{K}_{\mathrm{r}} \in \mathbb{R}^{n\times n}$ is a diagonal positive definite matrix and k_{f} is a positive constant. Note that from equations (10) and (18) it is $\dot{\boldsymbol{q}}_{\mathrm{o}} - \dot{\boldsymbol{q}}_{\mathrm{r}} = \boldsymbol{s} - \boldsymbol{r}$. Thus, from equation (20) one gets

$$\begin{aligned} \boldsymbol{\tau} &= \boldsymbol{H}(\boldsymbol{q})\left(\ddot{\boldsymbol{q}}_{\mathrm{r}} + \boldsymbol{e}(\boldsymbol{r})\right) + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}_{\mathrm{r}})\dot{\boldsymbol{q}}_{\mathrm{r}} \\ &+ \boldsymbol{D}\dot{\boldsymbol{q}}_{\mathrm{R}} + \boldsymbol{g}(\boldsymbol{q}) \\ &- \boldsymbol{K}_{\mathrm{R}}\left(\boldsymbol{s} - \boldsymbol{r}\right) - \boldsymbol{J}_{\varphi}^{\mathrm{T}}(\boldsymbol{q})\left(\boldsymbol{\lambda}_{\mathrm{d}} - k_{\mathrm{f}}\Delta\boldsymbol{F}\right). \end{aligned} \tag{23}$$

Substituting equation (23) into (1), one can compute the closed loop dynamics after some manipulation as

$$\begin{aligned} \boldsymbol{H}(\boldsymbol{q})\dot{\boldsymbol{s}} &= -\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{s} - \boldsymbol{K}_{\mathrm{DR}}\boldsymbol{s} \\ &+ \boldsymbol{H}(\boldsymbol{q})\boldsymbol{e}(\boldsymbol{r}) - \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}_{\mathrm{r}})\boldsymbol{s} \\ &+ \boldsymbol{K}_{\mathrm{R}}\boldsymbol{r} + \boldsymbol{J}_{\varphi}^{\mathrm{T}}(\boldsymbol{q})\left(\Delta\boldsymbol{\lambda} + k_{\mathrm{f}}\Delta\boldsymbol{F}\right), \end{aligned} \tag{24}$$

where $\boldsymbol{K}_{\mathrm{DR}} \triangleq \boldsymbol{K}_{\mathrm{R}} + \boldsymbol{D}$. In order to get equation (24), Property 2.3 has been used.

Observer definition

The proposed dynamics of the observer is given by

$$\begin{aligned} \dot{\hat{\boldsymbol{q}}} &= \dot{\boldsymbol{q}}_{\mathrm{o}} + \boldsymbol{\Lambda}\boldsymbol{z} + k_{\mathrm{d}}\boldsymbol{z} & (25)\\ \ddot{\boldsymbol{q}}_{\mathrm{o}} &= \dot{\ddot{\boldsymbol{q}}}_{\mathrm{r}} + k_{\mathrm{d}}\boldsymbol{\Lambda}\boldsymbol{z} & (26)\\ &+ \boldsymbol{H}^{-1}(\boldsymbol{q})\boldsymbol{J}_{\varphi}^{\mathrm{T}}(\boldsymbol{q})\left(\Delta\boldsymbol{\lambda} + k_{\mathrm{f}}\Delta\boldsymbol{F}\right), \end{aligned}$$

where k_{d} is a positive constant. Note that equation (26) can be rewritten as

$$\begin{aligned} \ddot{\boldsymbol{q}}_{\mathrm{o}} &= \ddot{\boldsymbol{q}}_{\mathrm{r}} + \boldsymbol{e}(\boldsymbol{r}) + k_{\mathrm{d}}\boldsymbol{\Lambda}\boldsymbol{z} \\ &+ \boldsymbol{H}^{-1}(\boldsymbol{q})\boldsymbol{J}_{\varphi}^{\mathrm{T}}(\boldsymbol{q})\left(\Delta\boldsymbol{\lambda} + k_{\mathrm{f}}\Delta\boldsymbol{F}\right). \end{aligned} \tag{27}$$

Now, from equation (25) you have

$$\ddot{\boldsymbol{q}}_{\mathrm{o}} = \ddot{\hat{\boldsymbol{q}}} - \boldsymbol{\Lambda}\dot{\boldsymbol{z}} - k_{\mathrm{d}}\dot{\boldsymbol{z}}. \tag{28}$$

Thus, from equation (27) one has

$$\begin{aligned} \dot{\boldsymbol{s}} &= \dot{\boldsymbol{r}} + k_{\mathrm{d}}\boldsymbol{r} + \boldsymbol{e}(\boldsymbol{r}) \\ &+ \boldsymbol{H}^{-1}(\boldsymbol{q})\boldsymbol{J}_{\varphi}^{\mathrm{T}}(\boldsymbol{q})\left(\Delta\boldsymbol{\lambda} + k_{\mathrm{f}}\Delta\boldsymbol{F}\right). \end{aligned} \tag{29}$$

By multiplying both sides of equation (29) by $\boldsymbol{H}(\boldsymbol{q})$, and by taking into account equation (24) one gets

$$\begin{aligned} \boldsymbol{H}(\boldsymbol{q})\dot{\boldsymbol{r}} &= -\boldsymbol{H}_{\mathrm{rd}}\boldsymbol{r} - \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{s} \\ &- \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}_{\mathrm{r}})\boldsymbol{s} - \boldsymbol{K}_{\mathrm{DR}}\boldsymbol{s}, \end{aligned} \tag{30}$$

where $\boldsymbol{H}_{\mathrm{rd}} \triangleq k_{\mathrm{d}}\boldsymbol{H}(\boldsymbol{q}) - \boldsymbol{K}_{\mathrm{R}}$. Finally, by using Property 2.3 again and after some manipulation, it is

$$\begin{aligned} \boldsymbol{H}(\boldsymbol{q})\dot{\boldsymbol{r}} &= -\boldsymbol{H}_{\mathrm{rd}}\boldsymbol{r} - \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{r} \\ &- \boldsymbol{C}(\boldsymbol{q}, \boldsymbol{s} + 2\dot{\boldsymbol{q}}_{\mathrm{r}})\boldsymbol{s} \\ &+ \boldsymbol{C}(\boldsymbol{q}, \boldsymbol{s} + \dot{\boldsymbol{q}}_{\mathrm{r}})\boldsymbol{r} - \boldsymbol{K}_{\mathrm{DR}}\boldsymbol{s}, \end{aligned} \tag{31}$$

Now, let us define

$$\boldsymbol{x} \triangleq \left[\boldsymbol{s}^{\mathrm{T}} \quad \boldsymbol{r}^{\mathrm{T}} \quad \Delta\boldsymbol{F}^{\mathrm{T}}\right]^{\mathrm{T}}, \tag{32}$$

as state for (11), (24), and (31), and consider for each $\boldsymbol{x}$ a ball given by

$$S_{\mathrm{a}} = \{\boldsymbol{x} : \|\boldsymbol{x}\| \le a\}. \tag{33}$$

For any $\boldsymbol{x}$ in the ball (33), $\boldsymbol{s}$, $\boldsymbol{r}$, and $\Delta\boldsymbol{F}$ are bounded. In view of the definition (18), one has that both $\boldsymbol{z}$, and $\dot{\boldsymbol{z}}$ are bounded. However, $\tilde{\boldsymbol{q}}$ and $\dot{\tilde{\boldsymbol{q}}}$ require more attention. Recall from equation (12) that $\boldsymbol{s}$ is given by

$$\begin{aligned} \boldsymbol{s} &= \boldsymbol{Q}(\boldsymbol{q})\left(\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{\Lambda}\tilde{\boldsymbol{q}} - \boldsymbol{\Lambda}\boldsymbol{z}\right) \\ &+ \boldsymbol{J}_{\varphi}^{+}(\boldsymbol{q})\left(\Delta\dot{\boldsymbol{p}} + \beta\Delta\boldsymbol{p} - \boldsymbol{\xi}\Delta\boldsymbol{F}\right). \end{aligned}$$

By assumption, the finger joints are revolute, thus meaning that both $\boldsymbol{Q}(\boldsymbol{q})$ and $\boldsymbol{J}_{\varphi}^{+}(\boldsymbol{q})$ are bounded. Also, $\boldsymbol{p}$, $\dot{\boldsymbol{p}}$, $\boldsymbol{p}_{\mathrm{d}}$ and $\dot{\boldsymbol{p}}_{\mathrm{d}}$ must be zero in view of Property 2.5, and constraint (4). This means that

$$\boldsymbol{Q}(\boldsymbol{q})\left(\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{\Lambda}\tilde{\boldsymbol{q}}\right) \tag{34}$$

must be bounded. Since $\boldsymbol{Q}(\boldsymbol{q})$ is not a full rank matrix, in general one cannot conclude that $\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{\Lambda}\tilde{\boldsymbol{q}}$ is bounded. However, it can be shown that, if the desired region of attraction a in (33) is small enough, then the boundedness of (34) does not only guarantee that both $\dot{\tilde{\boldsymbol{q}}}$, and $\tilde{\boldsymbol{q}}$ are bounded, but also that they will tend to zero if (34) tends to zero [6, 8]. This assumption means that $\tilde{\boldsymbol{q}}$, and $\dot{\tilde{\boldsymbol{q}}}$ are bounded if $\boldsymbol{x}$ is bounded and small enough. Finally, in view of the fact that $\dot{\boldsymbol{q}}_{\mathrm{d}}$ must be bounded, if $\boldsymbol{x}$ belongs to (33), then from definition (21) there must exist a constant

$$M_{\mathrm{e}} \triangleq M_{\mathrm{e}}(a) \tag{35}$$

such that

$$\|\boldsymbol{e}(\boldsymbol{r})\| \le M_{\mathrm{e}}\|\boldsymbol{r}\| < \infty \tag{36}$$

holds. We define the following Lyapunov function for system (11), (24), and (31)

$$V(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{x}, \tag{37}$$

where $\boldsymbol{M} \triangleq$ block diag$\{\boldsymbol{H}(\boldsymbol{q}), \quad \boldsymbol{H}(\boldsymbol{q}), \quad \boldsymbol{\xi}\}$, and $V(\boldsymbol{x})$ satisfies

$$\lambda_1 \|\boldsymbol{x}\|^2 \le V(t) \le \lambda_2 \|\boldsymbol{x}\|^2, \tag{38}$$

with

$$\begin{aligned} \lambda_1 &\triangleq \frac{1}{2} \min_{\forall \boldsymbol{q} \in \mathbb{R}^n} \lambda_{\min}(\boldsymbol{M}) \\ \lambda_2 &\triangleq \frac{1}{2} \max_{\forall \boldsymbol{q} \in \mathbb{R}^n} \lambda_{\max}(\boldsymbol{M}). \end{aligned} \tag{39}$$

Next, we analyze the stability properties of our control–observer approach.

Theorem 3.1 Consider the cooperative system dynamic given by equations (1) and (2), in closed loop with the control law (22) and the observer (25)–(26), where $\boldsymbol{q}_\mathrm{d}$, and $\boldsymbol{p}_\mathrm{d}$ are the desired bounded joint and constrained positions, whose derivatives $\dot{\boldsymbol{q}}_\mathrm{d}$, $\ddot{\boldsymbol{q}}_\mathrm{d}$, $\dot{\boldsymbol{p}}_\mathrm{d}$, and $\ddot{\boldsymbol{p}}_\mathrm{d}$ are also bounded, and they all satisfy constraint (4). Consider also equation (39), and a given region of attraction defined by equation (33), where the bound a is chosen small enough so that if

$$\|\boldsymbol{x}\| \le x_{\max}, \tag{40}$$

with

$$x_{\max} \triangleq \sqrt{\frac{\lambda_2}{\lambda_1}} a, \tag{41}$$

then the boundedness of equation (34) does not only guarantee that both $\dot{\tilde{\boldsymbol{q}}}$, and $\tilde{\boldsymbol{q}}$ are bounded, but also that they will tend to zero if equation (34) tends to zero. Then, asymptotic stability of tracking, observation and force errors arise, i.e.

$$\begin{aligned} \lim_{t\to\infty} \dot{\tilde{\boldsymbol{q}}} &= \boldsymbol{0} \quad \lim_{t\to\infty} \tilde{\boldsymbol{q}} = \boldsymbol{0} \quad \lim_{t\to\infty} \dot{\boldsymbol{z}} = \boldsymbol{0} \\ \lim_{t\to\infty} \boldsymbol{z} &= \boldsymbol{0} \quad \lim_{t\to\infty} \Delta\boldsymbol{\lambda} = \boldsymbol{0}, \end{aligned} \tag{42}$$

if the following conditions are satisfied

$$\lambda_{\min}(\boldsymbol{K}_\mathrm{R}) > \mu_1 + 1 \tag{43}$$

$$k_\mathrm{d} > \frac{\lambda_{\max}(\boldsymbol{K}_\mathrm{R}) + \alpha}{\lambda_\mathrm{H}}, \tag{44}$$

where $\alpha = \mu_2 + \frac{1}{4}(\lambda_\mathrm{d} + \mu_3 + \mu_4)^2$, and

$$\mu_1 \triangleq \max_{\|\boldsymbol{x}\| \le x_{\max}} \|\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}_\mathrm{r})\| \tag{45}$$

$$\mu_2 \triangleq \max_{\|\boldsymbol{x}\| \le x_{\max}} \|\boldsymbol{C}(\boldsymbol{q}, \boldsymbol{s} + \dot{\boldsymbol{q}}_\mathrm{r})\| \tag{46}$$

$$\mu_3 \triangleq \max_{\|\boldsymbol{x}\| \le x_{\max}} \|\boldsymbol{C}(\boldsymbol{q}, \boldsymbol{s} + 2\dot{\boldsymbol{q}}_\mathrm{r})\| \tag{47}$$

$$\mu_4 \triangleq M_\mathrm{e}(x_{\max})\lambda_\mathrm{H} \tag{48}$$

$$\lambda_\mathrm{d} \triangleq \lambda_{\max}(\boldsymbol{D}), \tag{49}$$

where equation (35), and Properties 2.1 and 2.4 have been used.

Proof: Rewrite equation (37) as

$$\begin{aligned} V(\boldsymbol{x}) &= \frac{1}{2}\boldsymbol{s}^\mathrm{T}\boldsymbol{H}(\boldsymbol{q})\boldsymbol{s} + \frac{1}{2}\boldsymbol{r}^\mathrm{T}\boldsymbol{H}(\boldsymbol{q})\boldsymbol{r} \\ &+ \frac{1}{2}\Delta\boldsymbol{F}^\mathrm{T}\boldsymbol{\xi}\Delta\boldsymbol{F}. \end{aligned} \tag{50}$$

By using Properties 2.2 and 2.5, the derivative of (50) along (11), (24), and (31) can be computed as

$$\begin{aligned} \dot{V}(\boldsymbol{x}) &= -k_\mathrm{f}\Delta\boldsymbol{F}^\mathrm{T}\boldsymbol{\xi}\Delta\boldsymbol{F} - \boldsymbol{s}^\mathrm{T}\boldsymbol{K}_\mathrm{DR}\boldsymbol{s} \\ &- \boldsymbol{r}^\mathrm{T}\boldsymbol{H}_\mathrm{rd}\boldsymbol{r} - \boldsymbol{s}^\mathrm{T}\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}_\mathrm{r})\boldsymbol{s} \\ &+ \boldsymbol{r}^\mathrm{T}\boldsymbol{C}(\boldsymbol{q}, \boldsymbol{s} + \dot{\boldsymbol{q}}_\mathrm{r})\boldsymbol{r} \\ &- \boldsymbol{r}^\mathrm{T}\boldsymbol{D}\boldsymbol{s} - \boldsymbol{r}^\mathrm{T}\boldsymbol{C}(\boldsymbol{q}, \boldsymbol{s} + 2\dot{\boldsymbol{q}}_\mathrm{r})\boldsymbol{s} \\ &+ \boldsymbol{s}^\mathrm{T}\boldsymbol{H}(\boldsymbol{q})\boldsymbol{e}(\boldsymbol{r}). \end{aligned} \tag{51}$$

Since $\boldsymbol{x}(0)$ is in region (33), there must exist a time greater than zero such that

$$\begin{aligned} \dot{V}(\boldsymbol{x}) &\le -k_\mathrm{f}\lambda_{\min}(\boldsymbol{\xi})\|\Delta\boldsymbol{F}\|^2 \\ &- (\lambda_{\min}(\boldsymbol{K}_\mathrm{R}) - \mu_1)\|\boldsymbol{s}\|^2 \\ &- (k_\mathrm{d}\lambda_{\mathrm{H}i} - \lambda_{\max}(\boldsymbol{K}_\mathrm{R}) - \mu_2)\|\boldsymbol{r}\|^2 \\ &+ (\mu_3 + \mu_4 + \lambda_\mathrm{d})\|\boldsymbol{s}\|\|\boldsymbol{r}\| \end{aligned}$$

is valid. By taking conditions (43)–(44) into account, one concludes that $\dot{V}(\boldsymbol{x}) \le 0 \; \forall \; t \ge 0$, and that $\dot{V}(\boldsymbol{x}) = 0$ only if $\boldsymbol{x} = \boldsymbol{0}$. Thus, $\boldsymbol{x} \to \boldsymbol{0}$. From definition (18) one has directly

$$\lim_{t\to\infty} \boldsymbol{z} = \boldsymbol{0} \qquad \lim_{t\to\infty} \dot{\boldsymbol{z}} = \boldsymbol{0}.$$

On the other hand, since $\boldsymbol{s} \to \boldsymbol{0}$, both $\boldsymbol{s}_\mathrm{p}$, and $\boldsymbol{s}_\mathrm{f}$ defined in equation (10) tend to zero as well. Since

$$\begin{aligned} \boldsymbol{J}_\varphi(\boldsymbol{q})\boldsymbol{s} &= \boldsymbol{J}_\varphi(\boldsymbol{q})\boldsymbol{Q}(\boldsymbol{q})\left(\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{\Lambda}\tilde{\boldsymbol{q}} - \boldsymbol{\Lambda}\boldsymbol{z}\right) \\ &+ \boldsymbol{J}_\varphi(\boldsymbol{q})\boldsymbol{J}_\varphi^+(\boldsymbol{q})\left(\Delta\dot{\boldsymbol{p}} + \beta\Delta\boldsymbol{p} - \boldsymbol{\xi}\Delta\boldsymbol{F}\right) \\ &= \Delta\dot{\boldsymbol{p}} + \beta\Delta\boldsymbol{p} - \boldsymbol{\xi}\Delta\boldsymbol{F}, \end{aligned}$$

one also gets $\lim_{t\to\infty} \Delta\dot{\boldsymbol{p}} = \boldsymbol{0}$ $\lim_{t\to\infty} \Delta\boldsymbol{p} = \boldsymbol{0}$. Furthermore, in view of the assumption made on $x_{\max}$, and the fact that $\boldsymbol{s}_\mathrm{p} \to \boldsymbol{0}$, it can be shown that [6, 8]

$$\lim_{t\to\infty} \dot{\tilde{\boldsymbol{q}}} = \boldsymbol{0} \qquad \lim_{t\to\infty} \tilde{\boldsymbol{q}} = \boldsymbol{0}.$$

Finally, we know from equation (12) that $\Delta\boldsymbol{F} \to \boldsymbol{0}$. This, however, does not necessarily means that $\Delta\boldsymbol{\lambda}$ tends to zero as well. In order to prove it, one may use again the fact that $\boldsymbol{J}_\varphi(\boldsymbol{q})\boldsymbol{s} = \Delta\dot{\boldsymbol{p}} + \beta\Delta\boldsymbol{p} - \boldsymbol{\xi}\Delta\boldsymbol{F}$, which means that

$$\begin{aligned} \boldsymbol{J}_\varphi(\boldsymbol{q})\dot{\boldsymbol{s}} &+ \dot{\boldsymbol{J}}_\varphi(\boldsymbol{q})\boldsymbol{s} \\ &= \Delta\ddot{\boldsymbol{p}} + \beta\Delta\dot{\boldsymbol{p}} - \boldsymbol{\xi}\Delta\boldsymbol{\lambda} = -\boldsymbol{\xi}\Delta\boldsymbol{\lambda}. \end{aligned} \tag{52}$$

The result on the right hand side of the last equation is valid since constraint (4) must be satisfied. When the time t is large enough, (24) becomes

$$\begin{aligned} \boldsymbol{H}(\boldsymbol{q})\dot{\boldsymbol{s}} &= \boldsymbol{J}_\varphi^\mathrm{T}(\boldsymbol{q})\Delta\boldsymbol{\lambda} \\ \Rightarrow \dot{\boldsymbol{s}} &= \boldsymbol{H}^{-1}(\boldsymbol{q})\boldsymbol{J}_\varphi^\mathrm{T}(\boldsymbol{q})\Delta\boldsymbol{\lambda}. \end{aligned}$$

By multiplying this last equation by $\boldsymbol{J}_\varphi(\boldsymbol{q})$ and taking equation (52) into account one gets

$$-\dot{\boldsymbol{J}}_\varphi(\boldsymbol{q})\boldsymbol{s} - \boldsymbol{\xi}\Delta\boldsymbol{\lambda} = \boldsymbol{J}_\varphi(\boldsymbol{q})\boldsymbol{H}^{-1}(\boldsymbol{q})\boldsymbol{J}_\varphi^{\mathrm{T}}(\boldsymbol{q})\Delta\boldsymbol{\lambda},$$

or

$$\left(\boldsymbol{\xi} + \boldsymbol{J}_\varphi(\boldsymbol{q})\boldsymbol{H}^{-1}(\boldsymbol{q})\boldsymbol{J}_\varphi^{\mathrm{T}}(\boldsymbol{q})\right)\Delta\boldsymbol{\lambda} = -\dot{\boldsymbol{J}}_\varphi(\boldsymbol{q})\boldsymbol{s}.$$

Since $\boldsymbol{\xi} + \boldsymbol{J}_\varphi(\boldsymbol{q})\boldsymbol{H}^{-1}(\boldsymbol{q})\boldsymbol{J}_\varphi^{\mathrm{T}}(\boldsymbol{q})$ is a nonsingular matrix and $\dot{\boldsymbol{J}}_\varphi(\boldsymbol{q})$ remains bounded because of the fact that the tracking error is bounded and tends to zero for bounded desired velocities, we arrive to the conclusion that

$$\lim_{t\to\infty} \Delta\boldsymbol{\lambda} = \boldsymbol{0}.$$

$\triangle$

4 EXPERIMENTAL RESULTS

In this section, some experimental results are presented. To this end, a test bed with a A465 of CRS Robotics industrial robots is employed (see Figure 1). Even though it has six degrees of freedom, only the first three joints are used for the experiments, while the rest of them are mechanically braked. Each joint is actuated by a CD motor. Thus, in order to implement control law (22) and observer (25)–(26), the motors dynamics has to be taken into account.

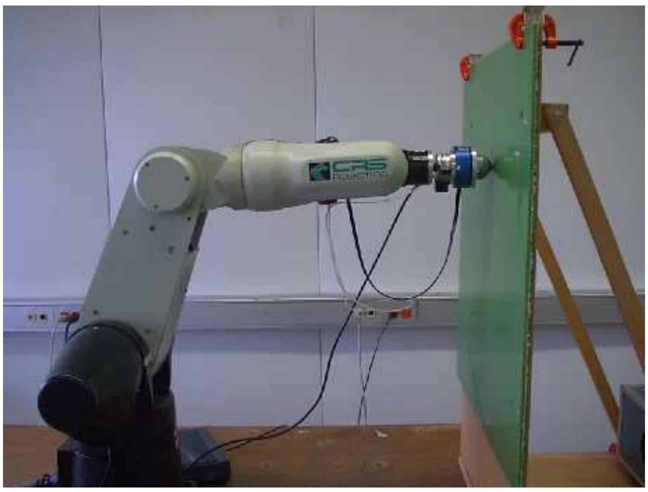

Figure 1: Robot A465 of CRS Robotics.

The constraint in Cartesian coordinates is simply given by

$$\varphi = x - b = 0, \tag{53}$$

with b a positive constant. The desired trajectories are given by

$$\begin{aligned} x_{\mathrm{d}} &= 0.626[\mathrm{m}] \qquad (54)\\ y_{\mathrm{d}} &= 0.05\sin(\omega(t-t_{\mathrm{i}}))[\mathrm{m}]\\ z_{\mathrm{d}} &= (0.585 + 0.05\cos(\omega(t-t_{\mathrm{i}})) - 0.05)[\mathrm{m}]. \end{aligned}$$

Note that the inverse kinematics of the manipulator has to be employed to compute $\boldsymbol{q}_{\mathrm{d}}$. These trajectories are valid from an initial time t_{i} to a final time t_{f}, while ω is a fifth order polynomial designed to satisfy $\omega(t_{\mathrm{i}}) = \omega(t_{\mathrm{f}}) = 0$. Also, the derivatives of ω are zero at t_{i} and t_{f}. By choosing (54), the robot will make a circle in the y–z plane. Also, no force control is carried until the manipulator is in the initial position for the circle, at $(0.626, 0, 0.585)$[m]. The desired pushing force is given by

$$f_{\mathrm{d}x} = 45 + 30\sin(2\pi(t-t_{\mathrm{i}})/40)[\mathrm{N}], \tag{55}$$

and $f_{\mathrm{d}y} = f_{\mathrm{d}z} = 0$[N]. The different control and observer parameters are $\boldsymbol{\Lambda} = 20\boldsymbol{I}$, $\boldsymbol{K}_{\mathrm{R}} = 70\boldsymbol{I}$, $k_{\mathrm{d}} = 8$, $k_{\mathrm{f}} = 10$, $\boldsymbol{\xi} = 0.001\boldsymbol{I}$.

The observer–controller scheme has been programmed in a PC computer, while the sampling time is 7ms. Only the time from $t_{\mathrm{i}} = 15$s to $t_{\mathrm{f}} = 55$s is shown because otherwise no force control is being used. The results for the tracking and observation errors can be seen in Figure 2 in joint coordinates. It is evident that the observer is working pretty well, while the tracking errors are relatively large. This can also be appreciated in Figure 3 in Cartesian coordinates, where the force measurements are shown. Note that the desired force is being applied pretty well. The rather poor results in the position tracking is mainly due to the fact that an exact knowledge of the manipulator dynamics is required and the model used in the experiment is not very accurate. On the other hand, it is remarkable that the observer and the force controller work very well.

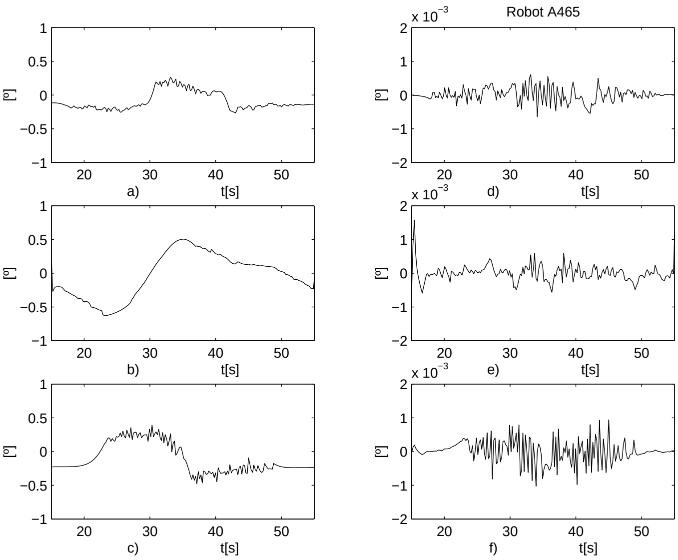

Figure 2: Tracking errors in joint coordinates. a) $\tilde{q}_{11}$. b) $\tilde{q}_{12}$. c) $\tilde{q}_{13}$. Observation errors. d) z_{11}. e) z_{12}. f) z_{13}.

5 CONCLUSIONS

The tracking control problem for position and force application of rigid robots without velocity measurements

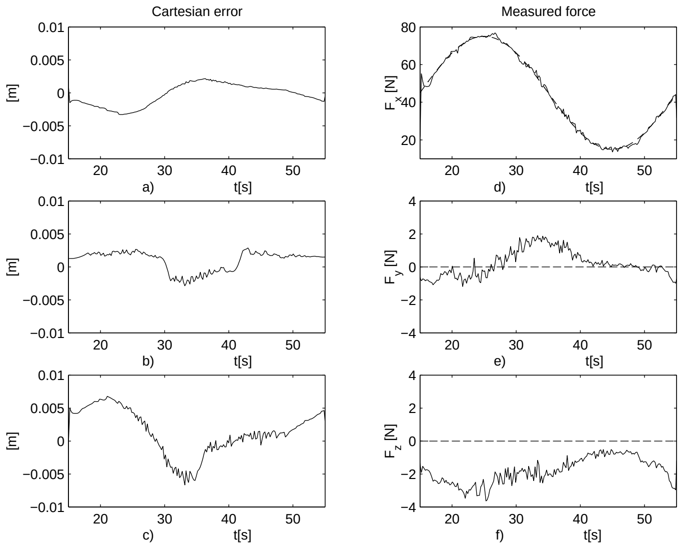

Figure 3: Tracking errors in Cartesian coordinates. a) $\tilde{x}_1$. b) $\tilde{y}_1$. c) $\tilde{z}_1$.
Force measurements. d) F_{x_1}. e) F_{y_1}. f) F_{z_1}.
—— measured value. - - - desired value.

is considered in this paper. The control law is a decentralized approach which takes into account motion constraints. By assuming that the robot dynamics is well known and that contact force measurements are available, a nonlinear observer is proposed that requires only the knowledge of the inertia matrix.

Experimental results have been carried out to test the proposed approach. The overall results can be considered good, even though it has become clear that the approach should be modified to take into account inaccuracies in the robot model and the possible lack of force sensors.

Acknowledgements

This work is based on research supported by DGAPA–UNAM under grant IN106901 and by the CUDI.

References

[1] S. Arimoto, Y. H. Liu, and T. Naniwa, "Model–based adaptive hybrid control for geometrically constrained robots," in Proceedings of the 1993 IEEE International Conference on Robotics and Automation, pp. 618–623, 1993.

[2] M. A. Arteaga Pérez, "On the properties of a dynamic model of flexible robot manipulators," ASME Journal of Dynamic Systems, Measurement, and Control, vol. 120, pp. 8–14, 1998.

[3] S. Chiaverini, B. Siciliano, and L. Villani, "Force/position regulation of compliant robot manipulators," IEEE Transactions on Automatic Control, vol. 15, no. 2, pp. 381–386, 1994.

[4] N. Hogan, "Impedance control: an approach to manipulation. Parts i–iii," ASME Journal of Dynamic Systems, Measurement, and Control, vol. 10, pp. 1–24, 1985.

[5] Y. H. Liu and S. Arimoto, "Implicit and explicit force controllers for rheo–holonomically constrained manipulators and their extension to distributed cooperation control," in IFAC 13th Triennial World Congress, (San Francisco. USA), 1996.

[6] Y.-H. Liu, S. Arimoto, V. Parra-Vega, and K. Kitagaki, "Decentralized adaptive control of multiple manipulators in cooperations," International Journal of Control, vol. 67, no. 5, pp. 649–673, 1997.

[7] H. McClamroch and D. Wang, "Feedback stabilization and tracking of constrained robots," IEEE Transactions on Automatic Control, vol. 33, pp. 419–426, 1988.

[8] V. Parra-Vega, A. Rodríguez-Ángeles, S. Arimoto, and G. Hirzinger, "High precision constrained grasping with cooperative adaptive handcontrol," Journal of Intelligent and Robotic Systems, vol. 32, pp. 235–254, 2001.

[9] M. Raibert and J. Craig, "Hybrid position/force control of manipulators," ASME Journal of Dynamic Systems, Measurement, and Control, vol. 103, pp. 126–133, 1981.

[10] M. W. Spong and M. Vidyasagar, Robot Dynamics and Control. New York: John Wiley & Sons, 1989.

[11] T. Yoshikawa, "Force control of robot manipulators," in Proc. IEEE Int. Conf. on Robotics and Automation, (San Francisco, USA), April 2000.

[12] G. Zeng and A. Hemami, "An overview of robot force control," Robotica, vol. 15, pp. 473–482, 1997.