DESIGN OF INFINITE IMPULSE RESPONSE (IIR) FILTERS WITH ALMOST LINEAR PHASE CHARACTERISTICS

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Abstract

We propose a method for designing Infinite Impulse Response (IIR) filters with specified magnitude frequency-response tolerances and approximately linear phase characteristics. The design method consists of two steps. First, a Finite Impulse Response (FIR) filter with linear phase is designed using standard optimisation techniques (e.g. linear programming). In the second stage, the designed FIR filter is approximated by a stable IIR filter using a Hankel-norm approximation method. This is based on a recent technique for approximating discrete-time descriptor systems and requires only standard linear algebraic routines, while avoiding altogether the solution of two matrix Lyapunov equations which is computationally expensive. A-priori bounds on the magnitude and phase errors are obtained which may be used to select the lowest-order IIR filter order which satisfies the specified design tolerances. The efectiveness of the method is illustrated with a numerical example.

1 Introduction

Finite Impulse Response (FIR) filters are widely used in many digital signal processing applications, especially in the field of communications, because they can exhibit *linear* phase characteristics. Unfortunately, in many cases the order of such filters is prohibitively high for practical implementation. In general, the number of delay elements and multipliers needed for an FIR design tends to be much higher compared to similar Infinite Impulse Response (IIR) implementations. This is especially true for filters with sharp cut-off characteristics which have a long impulse response [6, 9].

It is therefore natural to ask whether a linear-phase FIR filter $H(z)$ can be approximated by a low-order (IIR) filter, without degrading significantly its magnitude and phase characteristics. In this paper we apply Hankel-norm approximation techniques to (matrix) FIR filters. Our technique is based on a recent result involving approximations of discrete-time descriptor systems [1, 2]. This allows us to treat systems with poles at the origin and has been applied successfully in the context of mixed H_∞/H_2 optimisation problems [5]. Our results apply both to the γ-suboptimal and the strictly optimal problems, although in the later case the resulting state-space realisation is non-minimal. Using an all-pass matrix dilation technique, a state-space parametrisation of the complete family of solutions can be obtained, in the form of a linear fractional map of the ball of unstable contractions [1, 2, 4]. This can be subsequently reformulated using only the matrix Markov parameters of $H(z)$. When the results are specialised to the scalar case, magnitude and phase error bounds of the approximation error can be obtained in terms of the Hankel singular values of $H(z)$, which allows the a-priori determination of the IIR filter order satisfying specified magnitude ("ripple") specifications and an acceptable phase deviation from linearity. The effectiveness of the method is illustrated via a numerical example, involving the approximation of a high-order linear-phase FIR filter, designed using linear programming techniques [8]. Finally, possible extensions of the method are briefly discussed.

2 Notation

Most of the notation used is standard and is reproduced here for convenience. $\mathcal{D}$ denotes the open unit disc $\mathcal{D} = \{\zeta \in \mathcal{C} : |\zeta| < 1\}$, with $\bar{\mathcal{D}}$ and $\partial\mathcal{D}$ its closure and boundary respectively. $\mathcal{L}_2(\partial\mathcal{D})$ denotes the Hilbert space of all matrix-valued functions F defined on the unit circle such that

$$\int_0^{2\pi} \mathrm{trace}[F^*(e^{j\theta})F(e^{j\theta})]d\theta < \infty$$

where $(\cdot)^*$ denotes the complex conjugate transpose of a matrix. The corresponding inner product of two $\mathcal{L}_2(\partial\mathcal{D})$ functions F and G of compatible dimensions is given as:

$$< F, G >= \frac{1}{2\pi}\int_0^{2\pi} \mathrm{trace}[F^*(e^{j\theta})G(e^{j\theta})]d\theta$$

$\mathcal{H}_2(\partial\mathcal{D})$ and $\mathcal{H}_2^\perp(\partial\mathcal{D})$ are the closed subspaces of $\mathcal{L}_2$ consisting of all functions analytic in $\mathcal{C} \setminus \bar{\mathcal{D}}$ and $\mathcal{D}$, respectively. $\mathcal{L}_\infty(\partial\mathcal{D})$ is the space of all uniformly-bounded matrix-valued functions in $\partial\mathcal{D}$, i.e. all functions defined on the unit circle whose norm:

$$\|F\|_\infty = \sup_{\theta\in[0,2\pi)} \bar{\sigma}[F(e^{j\theta})]$$

is finite. Here $\bar{\sigma}(\cdot)$ denotes the largest singular value of a matrix. $\mathcal{H}_\infty(\partial\mathcal{D})$ and $\mathcal{H}_\infty^-(\partial\mathcal{D})$ denote the closed subspaces of $\mathcal{L}_\infty(\partial\mathcal{D})$ consisting of all functions analytic in $\mathcal{C}\setminus\bar{\mathcal{D}}$ and $\mathcal{D}$, respectively, while $\mathcal{H}_\infty^{-,k}(\partial\mathcal{D})$ is the set of all functions in $\mathcal{L}_\infty(\partial\mathcal{D})$ with no more than k poles in $\mathcal{D}$. Spaces of real-rational functions will be indicated by the suffix $\mathcal{R}$ before the corresponding space symbol. The *unit ball* of $\mathcal{H}_\infty^-(\partial\mathcal{D})$ is the set $\mathcal{BH}_\infty^-(\partial\mathcal{D}) = \{U \in \mathcal{H}_\infty^-(\partial\mathcal{D}) : \|U\|_\infty \le 1\}$. If $G \in \mathcal{L}_\infty(\partial\mathcal{D})$, then the Hankel operator with symbol G is defined as:

$$\Gamma_G : \mathcal{L}_2(\partial\mathcal{D}) \to \mathcal{H}_2(\partial\mathcal{D}), \;\; \Gamma_G = \Pi_+ M_G|_{\mathcal{H}_2^\perp(\partial\mathcal{D})}$$

in which M_G denotes the multiplication operator $M_G : \mathcal{L}_2(\partial\mathcal{D}) \to \mathcal{L}_2(\partial\mathcal{D})$, $M_G f = Gf$ and Π_+ denotes the orthogonal projection $\Pi_+ : \mathcal{L}_2(\partial\mathcal{D}) \to \mathcal{H}_2(\partial\mathcal{D})$. If $H \in \mathcal{L}_\infty(\partial\mathcal{D})$ has dimensions $(p_1+p_2)\times(m_1+m_2)$ and $U \in \mathcal{RL}_\infty(\partial\mathcal{D})$ has dimensions $m_2 \times p_2$, we define the *lower linear fractional map* of H and U as $\mathcal{F}_l(H,U) = H_{11} + H_{12}U(I - H_{22}U)^{-1}H_{21}$, provided the indicated inverse exists. If $\mathcal{U}$ is a set of $m_2 \times p_2$ matrix functions, then $\mathcal{F}_l(H,\mathcal{U})$ denotes the set $\{\mathcal{F}_l(H,U) : U \in \mathcal{U}\}$.

3 FIR filters with linear phase response

The fact that FIR filters can have a linear phase response has been used extensively in many digital signal processing applications. A filter with a nonlinear phase characteristic causes distortion to its input signal, since the various frequency components in the signal will be delayed, in general, by amounts which are not proportional to their frequency, thus altering their mutual harmonic relationships. A distortion of this form is undesirable in many practical digital signal processing applications such as music, video, data transmission and biomedicine and can be avoided by using filters with linear phase over the frequency range of interest [9].

Suppose that the unit pulse response of an FIR filter is given by $\{h(0), h(1), h(2), \ldots, h(N)\}$. For this filter to have a *linear* phase response, say $\theta(\omega) = -\alpha\omega$ for α constant, it must have an impulse response with *positive* symmetry, i.e. $h(n) = h(N-n-1)$, where n varies as $n = 0, 1, 2, \ldots, \frac{N-1}{2}$ for N odd and $n = 0, 1, 2, \ldots, \frac{N}{2} - 1$ for N even. Such a filter has identical phase and group delay and these are independent of frequency, i.e.

$$T_p = -\frac{\theta(\omega)}{\omega} = T_g = -\frac{d\theta(\omega)}{d\omega} = \alpha$$

An FIR filter whose impulse response has *negative* symmetry, i.e. $h(n) = -h(N-n-1)$, has an *affine* phase-response characteristic of the form $\theta(\omega) = \beta - \alpha\omega$ with α and β constant and its group delay $T_g = \alpha$ is independent of ω.

The simple relationship between the impulse response of an FIR filter and its frequency response has been exploited to design filters of this type via a variety of optimisation methods [3], [7], [8], [10]. One of the earliest and simplest approaches is [8], in which a linear programming procedure is proposed for designing linear phase FIR filters with specified magnitude-response characteristics (low-pass, high-pass, etc). As an example, consider the filter

$$H(z) = h(0) + h(1)z^{-1} + \ldots + h(p-1)z^{-(p-1)} + h(p)z^{-p} + h(p+1)z^{-(p+1)} + \ldots + h(2p)z^{-2p}$$

in which positive symmetry of the impulse response is enforced around the central coefficient $h(p)$ by setting $h(0) = h(2p), h(1) = h(2p-1), \ldots, h(p-1) = h(p+1)$. The frequency response of the filter may be written as:

$$H(e^{j\omega}) = e^{j\omega p} M(\omega)$$

where $M(\omega)$ is a real linear function of the filter's coefficients:

$$M(\omega) = \begin{pmatrix} 2\cos(p\omega) & 2\cos((p-1)\omega) & \ldots & 1 \end{pmatrix} \begin{pmatrix} h(0) \\ h(1) \\ \vdots \\ h(p) \end{pmatrix}$$

Suppose now that we want to satisfy the following specifications: (a) $1-\delta \le |H(e^{j\omega})| \le 1+\delta$ for all frequencies in the pass-band $\omega \in [0, \omega_p]$, (b) $|H(e^{j\omega})| \le \delta$ for all frequencies in the stop-band $\omega \in [\omega_s, \pi]$ where $\omega_p < \omega_s$, and (c) minimise the "ripple" δ subject to constraints (a) and (b). Specifications (a) and (b) can enforced by discretising the pass-band and stop-band frequency intervals using n frequencies, say, i.e.

$$0 = \omega_1 < \omega_2 < \ldots < \omega_n = \omega_p$$

and

$$\omega_s = \omega_{n+1} < \omega_{n+2} < \ldots < \omega_{2n} = \pi$$

and enforcing the specifications via the inequalities:

$$1-\delta \le M(\omega_i) \le 1+\delta, \;\; i = 1, 2, \ldots, n \tag{1}$$

and

$$-\delta \le M(\omega_i) \le \delta, \;\; i = n+1, n+2, \ldots, 2n \tag{2}$$

The optimisation problem: $\min\delta$ subject to (1) and (2), can now be formulated as a linear programme in the standard form:

$$\min c'x \quad \text{s.t.} \quad Ax \le b$$

where $x' = (h(0)\ h(1) \ldots h(p)\ \delta)$, $c' = (0 \ldots 0\ 1)$, b is a $4n$-dimensional vector and A is a $4n \times (p+2)$-dimensional matrix. This can be solved efficiently using standard techniques, e.g. the simplex algorithm or interior point methods.

4 Hankel-norm approximation of FIR filters

In this section we propose a Hankel-norm method for approximating FIR filters via lower-order IIR filters. The method is based on a recent result involving model-reduction of descriptor discrete-time systems [1], [2].

The main advantage of Hankel-norm approximation methods over other model-reduction techniques is that they offer tight bounds on the infinity-norm of the approximation error; in particular, it is shown in [4] how to construct k-th order approximations $X(z) \in \mathcal{RH}_\infty(\partial\mathcal{D})$ of $H(z) \in \mathcal{RH}_\infty(\partial\mathcal{D})$ with $\deg X(z) = k \leq \deg H(z) = n$, such that:

$$\|H(z) + X(z)\|_\infty \leq \sum_{i=k+1}^{n} \sigma_i(\Gamma_H) \tag{3}$$

where $\sigma_i(\Gamma_H)$ denotes the i-th singular value of Γ_H, indexed in non-increasing order of magnitude. This inequality can be used to determine a-priori the order k of the low-order system $X(z)$ which satisfies magnitude error specifications.

Theorem 2 below parametrises all solutions $X(z)$ to the Hankel-norm approximation problem $\|\Gamma_H + \Gamma_X\| = \gamma$, in which $H(z)$ is the matrix FIR filter

$$H(z) = H_0 + H_1 z^{-1} + \ldots + H_n z^{-n}$$

and $X(z)$ is a matrix IIR filter of degree $\deg X(z) \leq k$. The parametrisation is given in descriptor form and hence applies both in the sub-optimal case ($\sigma_{k+1}(\Gamma_H) < \gamma < \sigma_k(\Gamma_H)$) and the optimal case ($\gamma = \sigma_{k+1}(\Gamma_H)$). Before stating this theorem, however, the following preliminary result is needed:

Theorem 1: Let

$$\tilde{H}(z) = H(z) - H_0 = H_1 z^{-1} + H_2 z^{-2} + \ldots + H_n z^{-n}$$

with $H_i \in \mathcal{R}^{p\times l}$ for $i = 1, 2, \ldots, n$. Then:

1. The singular values of $\Gamma_{\tilde{H}}$, $\sigma_i(\Gamma_{\tilde{H}})$ (indexed in decreasing order of magnitude) are the singular values of the (Hankel) matrix:

$$R_1 = \begin{pmatrix} H_1 & H_2 & \ldots & H_{n-1} & H_n \\ H_2 & H_3 & \ldots & H_n & 0 \\ H_3 & H_4 & \ldots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ H_{n-1} & H_n & \ldots & 0 & 0 \\ H_n & 0 & \ldots & 0 & 0 \end{pmatrix}$$

2. A state-space realisation of $\tilde{H}(z)$ is given by $\tilde{H}(z) = C(zI - A)^{-1}B$ with:

$$A = \begin{pmatrix} 0 & 0 & 0 & \ldots & 0 & 0 \\ I_p & 0 & 0 & \ldots & 0 & 0 \\ 0 & I_p & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & I_p & 0 \end{pmatrix} \in \mathcal{R}^{np\times np}$$

$$B = \begin{pmatrix} H_n' & H_{n-1}' & \ldots & H_2' & H_1' \end{pmatrix}' \in \mathcal{R}^{np\times l}$$

$$C = \begin{pmatrix} 0 & 0 & \ldots & 0 & I_p \end{pmatrix}' \in \mathcal{R}^{p\times np}$$

   Further, the realisation is output balanced, i.e. the unique solution of the Lyapunov equation $Q = A'QA + C'C$ is given by $Q = I_{np}$.

3. The controllability grammian of the realisation in part 2, i.e. the unique solution of the Lyapunov equation $P = APA' + BB'$, can be factored as $P = R_2 R_2'$ where:

$$R_2 = \begin{pmatrix} H_n & 0 & \ldots & 0 \\ H_{n-1} & H_n & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_1 & H_2 & \ldots & H_n \end{pmatrix}$$

   Equivalently P can be block-partitioned as:

$$(P_{ij})_{i=1,2,\ldots n}^{j=1,2,\ldots n} \text{ where } P_{ij} = \sum_{k=1}^{\min(i,j)} H_{n-i+k} H_{n-j+k}'$$

   and $\sigma_i^2(\Gamma_{\tilde{H}}) = \lambda_i(P)$ for each i.

Proof: Straightforward and hence omitted. □

Theorem 2: Let $\tilde{H}(z)$ be the matrix FIR filter defined in Theorem 1. Let γ satisfy $\sigma_{k+1}(\Gamma_{\tilde{H}}) \leq \gamma < \sigma_k(\Gamma_{\tilde{H}})$. Then all $X(z) \in \mathcal{H}_\infty^{-,k}(\partial\mathcal{D})$ such that $\|\tilde{H}(z) + X(z)\|_\infty \leq \gamma$ are generated via the lower linear fractional transformation:

$$\mathcal{X} = \{\mathcal{F}_l(S_a, U) : \gamma U \in \mathcal{BH}_\infty^-(\partial\mathcal{D})\}$$

where:

$$S_a = D_S + C_S(zE_S - A_S)^{-1}B_S$$

with

$$E_S = \begin{pmatrix} I_{np} & 0 \\ 0 & 0 \end{pmatrix}, \quad A_S = \begin{pmatrix} -I_{np} & \gamma^2 I_{np} - P \\ X & A_0 \end{pmatrix}$$

$$B_S = \begin{pmatrix} 0 & 0 \\ B & B_0 \end{pmatrix}, \quad C_S = \begin{pmatrix} 0 & CX^{-1}P \\ 0 & C_0 \end{pmatrix}$$

and

$$D_s = \begin{pmatrix} CX^{-1}B & \gamma I_p \\ \gamma I_l & 0 \end{pmatrix}$$

where A, B, C and P are defined in Theorem 1, and $X = I_{np} + A$, $A_0 = P - \gamma^2 XA'X^{-\prime}$, $B_0 = \gamma XX^{-\prime}C'$ and $C_0 = \gamma B'X^{-\prime}$.

Proof: Minor adaptation of result in [1], [2]. □

The proof of Theorem 2 follows readily by specialising the results of [1] and [2] to our case, using the state-space description in Theorem 1. Note that the generator of all Hankel norm approximations $S_a(z)$ is given in descriptor form; this can be converted into a state-space form, if required, using a standard procedure. In particular, it is always possible to obtain a state-space realisation of $S_a(z)$ of order $2np - \mathrm{Rank}(A_0)$; if A_0 is non-singular, we obtain a state-space description of order np. Note also, that although Theorem 2 is still valid in the optimal case $\gamma = \sigma_{k+1}(\Gamma_H)$, the resulting realisation is non-minimal. In this case, a minimal realisation can be obtained in closed form via a singular perturbation argument (see [2] for details).

It is clear from Theorem 2 that the solution of Lyapunov equations is completely avoided in our framework; in fact the generator $S_a(z)$ is completely defined by the Markov parameters of $\tilde{H}(z)$ (and γ), although we have not made this dependence explicit. Due to the special structure of the state-space model describing $\tilde{H}(z)$ in this case, it is possible to give a closed-form expression of the state-space realisation of $S_a(z)$ directly in terms of the Markov parameters. Due to space limitations, we do not pursue this direction here.

In the remaining part of this section we specialise Theorem 1 to the scalar case and obtain bounds on the phase of the approximation error. Note first, that in the scalar case, the solution to the Hankel-norm approximation problem is, in general, unique only in the optimal case. The so-called "central solution" is obtained by setting $U = 0$. Having obtained a state-space realisation of an approximation, one next needs to remove its anti-stable component; this extraction of the stable projection can be performed efficiently by transforming the system to block-Schur form using an appropriate orthogonal state-space transformation and ordering the eigenvalues of the "A" matrix in ascending order of magnitude; decoupling of the stable and anti-stable parts then requires the solution of a matrix Sylvester equation.

Reference [4] shows that having solved the k-th order Hankel-norm approximation problem, it is always possible to choose a constant term x_0 so that the approximation error satisfies the bound:

$$\|h(z) + (x(z) + x_0)\|_\infty \leq \sum_{i=k+1}^{n} \sigma_i(\Gamma_h) \qquad (4)$$

Since this bound applies uniformly in frequency, it can be used to give an immediate bound on the approximation phase error:

Theorem 3: Let $\hat{x}(z) = x(z) + x_0$ be a k-th order optimal Hankel norm approximation of the scalar FIR filter $h(z)$ such that (4) holds. Then, if $\phi(\omega) = \arg(h(e^{j\omega})) + \arg(\hat{x}(e^{j\omega}))$, we have that:

$$|\sin(\phi(\omega))| \leq \frac{\sum_{i=k+1}^{n} \sigma_i(\Gamma_h)}{|h(e^{j\omega})|} \qquad (5)$$

for every $\omega \in [0, \pi)$. In particular, if the frequency interval $[\omega_1\ \omega_2]$ lies in the filter's passband in which $1-\delta \leq |h(e^{j\omega})| \leq 1+\delta$, then

$$|\sin(\phi(\omega))| \leq \frac{\sum_{i=k+1}^{n} \sigma_i(\Gamma_h)}{1-\delta} \qquad (6)$$

for every $\omega \in [\omega_1\ \omega_2]$.

Proof: Straightforward and hence omitted. □

The phase error bound given in Theorem 3 can be used to select the minimum order of approximation k consistent with a worst-case phase-error specification; If $h(z)$ is a linear-phase FIR filter, then the RHS of (5) or (6) quantifies the deviation in the phase of the IIR approximation of $h(z)$ from linearity.

5 Example

In this section some of the results presented in the paper are illustrated by means of a computer example. First, a linear phase FIR filter of order $n = 21$ was designed using the linear programming procedure outlined in section 3. The design specifications were defined as: $\omega_p = 1$ rad/sample, $\omega_s = 1.5$ rads/sample and the frequency intervals $[0, \omega_p]$ and $[\omega_s, \pi]$ were each discretised using 50 equally-spaced frequencies. The linear programme was then set up and solved using Matlab's function lp.m. The minimum ripple was obtained as $\delta = 0.0232$; the first 11 optimal impulse response coefficients of the resulting filter $h(z)$ are tabulated below (the last 10 coefficients are symmetric and are not included):

i	$h(i)$	i	$h(i)$	i	$h(i)$
0	0.0017	4	0.0358	8	0.0919
1	-0.0212	5	-0.0015	9	0.2995
2	-0.0123	6	-0.0662	10	0.3980
3	0.0178	7	-0.0561		

Table 1: Impulse response coefficients

The magnitude frequency response of the filter is shown in Figure 1 below. Note that the response in the passband and the stopband lies within the desired bounds $1 \pm \delta$.

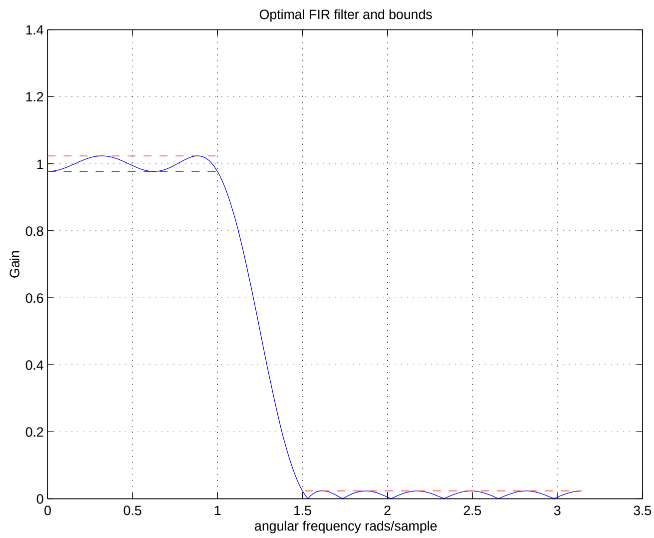

Figure 1: *Magnitude frequency response*

Next the Hankel-norm approximation method outlined in Theorem 2 was applied to the FIR filter $h(z)$. The Hankel singular values of $h(z)$ are shown in Table 2. Parameter γ was chosen as $\gamma = 0.03$ resulting in a 7-th order (sub-optimal) Hankel-norm approximation. The seven stable poles of the approximant were obtained as: 0.7467, $0.2841 \pm 0.8152j$, $0.4977 \pm 0.6347j$ and $0.6886 \pm 0.3363j$.

i	σ_i	i	σ_i	i	σ_i	i	σ_i
1	1.0000	6	0.1765	11	0.0117	16	0.0116
2	0.9973	7	0.0602	12	0.0117	17	0.0002
3	0.9563	8	0.0232	13	0.0117	18	0.0002
4	0.7791	9	0.0135	14	0.0116	19	0.0000
5	0.4344	10	0.0118	15	0.0116	20	0.0000

Table 2: Hankel singular values

The magnitude response of $h(z)$ and its IIR approximation is shown in Figure 2. It can be seen that the IIR filter has a slightly larger ripple in the pass-band. Figure 3 shows the impulse responses of the two filters.

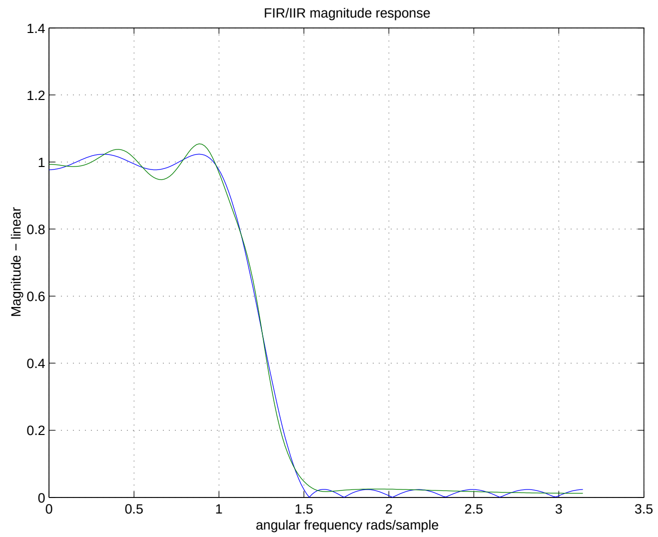

Figure 2: *FIR/IIR magnitude frequency response*

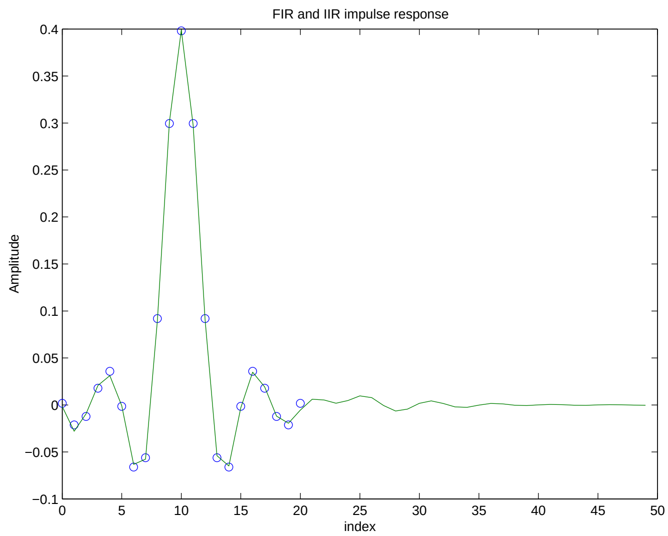

Figure 3: *FIR/IIR impulse response*

Finally, Figure 4 shows the phase error in the passband $0 \leq \omega \leq 1$ rad/sample arising from the approximation (i.e. the deviation of the phase of the IIR filter from linearity), while Figure 5 shows the pole/zero pattern of the IIR filter (there is an additional zero at $z = -15.6521$ not shown in the figure).

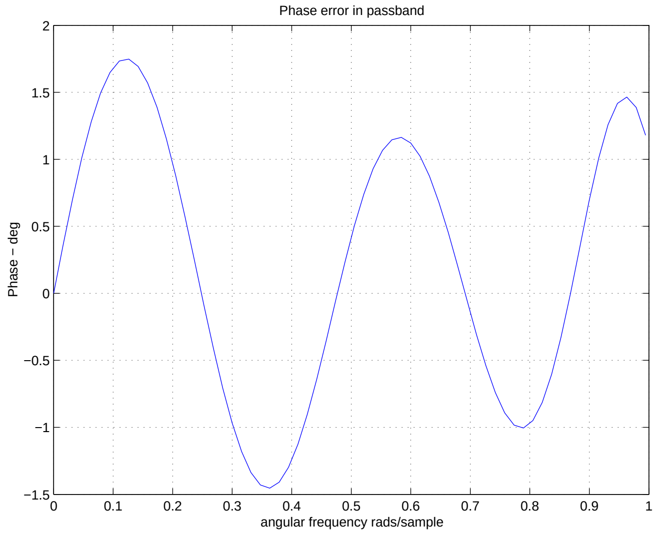

Figure 4: *Phase error in passband (deg)*

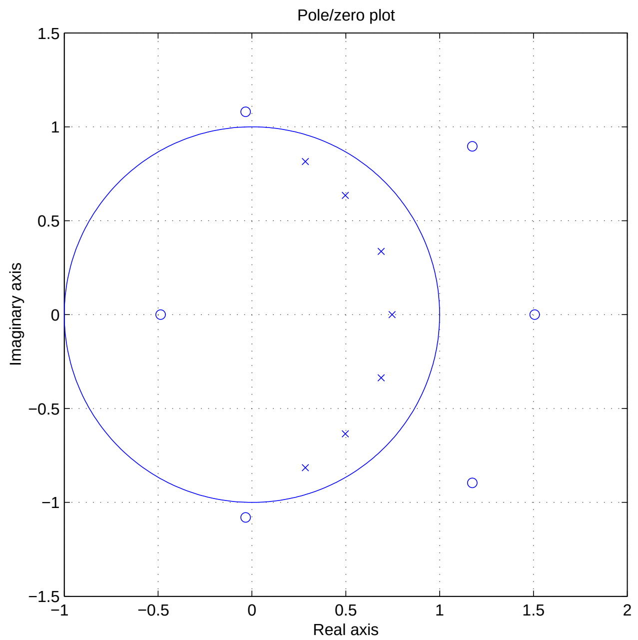

Figure 5: *Pole/zero pattern of IIR filter*

6 Conclusions

In this work we have presented a systematic way for designing low-order IIR filters with approximately linear phase response characteristics. The method relies on two steps: (a) First a linear-phase FIR filter is designed using linear programming or an alternative optimisation technique; (b) In step 2, the FIR filter is approximated by a low-order IIR filter using Hankel-norm approximation methods. The method is computationally efficient and relies on recent results for approximating discrete-time descriptor systems. A-priori bounds are obtained for the magnitude and phase approximation error which can assist the designer to choose an approximation order consistent with the design specifications. A low-order example has illustrated the effectiveness of the method.

There are a number of issues related to the proposed technique that we intend to pursue in the future. These include:

- Derivation of closed-form expressions for the IIR approximation filter, directly from the Markov parameters of the FIR filter.
- Derivation of tighter error bounds than those applying in general for the Hankel-norm approach. This seems possible given the special structure of FIR filters (all poles lie at the origin).
- Investigation of the sensitivity properties of the proposed design under finite-precision implementation.
- Generalisation of the method to other types of approximation, e.g. relative-error and general frequency weighted approximations.
- Extension of the method to more general application domains.

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