EVOLUTIONARY DOMINANCE-BASED DESIGN OF LINEAR MULTIVARIABLE CONTROLLERS

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Abstract

Adaptive evolution strategies without crossover are used to design complete dominance-based controllers for linear multivariable plants. Such complete dominance-based controllers comprise a pre-compensator and a set of single-loop controllers, which can both be readily designed using the proposed evolutionary approach. The effectiveness of this two-stage evolutionary design technique is illustrated by synthesising a complete dominance-based multivariable controller for the Rolls-Royce Spey jet engine. The performance of this controller is compared with that of a multivariable controller for the Spey engine designed using a one-stage evolutionary design technique, which does not involve the prior achievement of diagonal dominance.

Keywords: Linear Multivariable Control, Transfer Function Matrix, Diagonal Dominance, Genetic Algorithms, Adaptive Evolution Strategies

1 Introduction

The use of diagonal dominance by Rosenbrock [1] was one of the first frequency-domain techniques to be developed for the design of linear multivariable controllers. The primary objective of this technique is to reduce plant interactions by the introduction of a multivariable static pre-compensator, so that the control system design can then be completed by using classical techniques to design a set of single-loop controllers for the compensated plant. The principal techniques developed for the achievement of diagonal dominance by the use of such static pre-compensators are the pseudo-diagonalisation method using least-squares optimisation (Hawkins [2]), the function-minimisation method using conjugate-direction optimisation (Leininger [3]), and the ALIGN algorithm developed initially in conjunction with characteristic-locus methods (Kouvaritakis [4]).

However, none of these techniques for the achievement of diagonal dominance has been accepted universally for design purposes. Indeed, the underlying optimisation techniques are often not powerful enough - even when diagonal dominance is sought only at a single frequency for plants having relatively few inputs and outputs. The situation rapidly becomes worse when diagonal dominance is sought over an entire bandwidth of frequencies for plants with large numbers of inputs and outputs. Furthermore, once a pre-compensator is found to achieve the desirable levels of diagonal dominance, it is then necessary to design a set of single-loop controllers for the compensated plant. In the literature, this second part of the design is typically described as being the ‘trivial’ part, whereas in fact poor design at this stage can severely degrade the performance of the complete closed-loop system. In both the initial achievement of diagonal dominance and the subsequent design of the single loop-controllers for the compensated plant, there is a dearth of systematic optimisation techniques sufficiently powerful for the routine design of complete diagonal dominance-based linear multivariable controllers.

In recent years evolutionary algorithms (i.e. genetic algorithms and evolution strategies) (Goldberg [5], Back [6]) have been used to solve various challenging optimisation problems in several fields of engineering. In the particular field of control engineering, genetic algorithms have been successfully used to design control systems of various kinds (see, for example, Porter [7]). It has also been shown that both non-adaptive and adaptive evolution strategies are often even more effective than genetic algorithms in solving such problems in control engineering [8].

In this paper, adaptive evolutionary strategies without crossover are accordingly used to design complete diagonal dominance-based linear multivariable controllers (i.e. pre-compensators and sets of single-loop controllers). The Rolls-Royce Spey gas-turbine engine is used as an example: by comparing these results with those obtained using previous techniques, the effectiveness of the evolutionary design technique is demonstrated. In addition, some further advantages of the evolutionary dominance-based design technique are also highlighted. Thus, for example, it is shown that the single-loop controllers can be readily optimised with respect to the overall closed-loop step-response errors, which was not previously possible using non-evolutionary dominance-based design techniques.

2 Evolutionary design procedure

2.1 Design of pre-compensator

Diagonal dominance is a design technique that converts a linear multivariable design problem into several single-loop design problems which can then be solved using any number of available single-loop design techniques. In the case of column dominance for a plant with transfer function matrix $G(s) = [g_{ij}(s)] \in \mathbf{C}^{m\times m}$, this involves finding a pre-compensator matrix $K = [k_{ij}] \in \mathbf{R}^{m\times m}$, such that the resulting open-loop system with transfer function matrix $Q(s) = G(s)K$ satisfies the inequality

$$| q_{ii}(s) | \geq\geq \sum_{\substack{j=1 \\ j\neq i}}^{m} | q_{ij}(s) | \quad (i = 1, \ldots, m) \tag{1}$$

where '$\geq\geq$' denotes 'at least equal to, but as much greater than as possible'. If such a K can be found, $Q(s)$ may be replaced by $\tilde{Q}(s) = diag\{Q(s)\} = [q_{ii}(s)] \in \mathbf{C}^{m\times m}$. Next, a diagonal controller matrix $D(s) = [d_{ii}(s)] \in \mathbf{C}^{m\times m}$ can be found such that $\tilde{q}_{ii}(s)d_{ii}(s)$ is as close as possible to $m_i(s)(1 - m_i(s))$, where $M(s) = diag\{m_i(s)\}$ is the desired transfer function matrix of the closed-loop system whose actual overall transfer function matrix is $T(s)$.

Note that, since $\tilde{q}_{ij}(s)d_{ij}(s) = 0 \quad (\forall\ i \neq j)$, the design of $D(s)$ can be broken down into m single-loop design problems: the transfer function matrix of the corresponding multivariable controller is $C(s) = KD(s)$. If the pre-compensator matrix K satisfies the inequality (1), then this inequality is also satisfied for $Q(s)D(s)$ since $D(s)$ post-multiplies each column of $Q(s)$ by the same gain at each frequency.

The diagonal dominance design technique has been traditionally recognised for its elegantly simple approach to the design of complex controllers. However, virtually from the earliest stages of its development, this technique has been subject to much criticism - mainly due to the non-existence of a systematic approach to the achievement of dominance. However, although recent efforts (Nobakhti et al [9, 10]) have gone some way to addressing this problem, these approaches still require significant involvement from the designer for the achievement of successful designs. Evolution strategies overcome this difficulty by offering a powerful optimisation technique which makes the routine design of dominance-based controllers significantly easier.

The evolutionary design methodology is used to find , for the plant transfer function matrix $G(s) = [g_{ij}(s)] \in \mathbf{C}^{m\times m}$, a real pre-compensator matrix $K = [k_{ij}] \in \mathbf{R}^{m\times m}$ such that $Q(i\omega) = G(i\omega)K$ is dominant over a set of frequencies $\Omega = \{\omega_f : f = 1, \ldots, n\}$ [11]. This design problem is solved by determining [12] the real pre-compensator matrix, K, such that the cost function

$$\Gamma_K(K, \Omega) = \sum_{f=1}^{n} \sum_{j=1}^{m} \frac{\sum_{\substack{i=1 \\ i\neq j}}^{m} | q_{ij}(i\omega_f) |}{| q_{ii}(i\omega_f) |} \tag{2}$$

is minimised, where

$$q_{ij}(i\omega_f) = \sum_{l=1}^{m} g_{il}(i\omega_f)k_{lj}. \tag{3}$$

Each candidate pre-compensator matrix is encoded in a chromosome comprising m^2 concatenated sub-chromosomes that each represents an element of the $m \times m$ compensator matrix. Entire populations of chromosomes of such candidate pre-compensator matrices are caused to evolve, subject - in the general case - to the actions of mutation, crossover, and selection: the measure of fitness used in such evolutionary algorithms in the present context is simply the reciprocal of the cost function defined in equation (2).

The compensator design problem for various industrial plants has been solved using genetic algorithms, non-adaptive evolution strategies, and adaptive evolution strategies. However, the best results were usually obtained using adaptive evolution strategies without crossover. Thus, for example, the direct Nyquist array for the uncompensated open-loop Rolls-Royce Spey jet engine is shown in Figure 1: the model of this engine contains 21 states, 3 inputs (fuel flow, inlet guide vanes, nozzle area), 3 outputs (low-pressure spool speed, high-pressure spool speed, surge margin). The Spey engine is a highly interacting and non-linear system, and the linearised model used for the compensator design corresponds to 74% sea-level thrust. Indeed, Figure 1 indicates the presence of significant interactions in this engine , since the open-loop transfer function matrix is clearly not dominant. However, after using a $(\mu + \lambda)$ adaptive evolution strategy [6] to find the required pre-compensator matrix, the direct Nyquist array of the compensated engine shown in Figure 2 was obtained (where the pre-compensator matrix has been normalised such that $q_{ii}(0) = 1 \ (i = 1, 2, 3)$ in the compensated transfer function matrix,$Q(s)$). These results were obtained with population sizes of $\mu = 20$ parents and $\lambda = 20$ children for 1000 generations over a set of 50 frequency points from 0 to 75 (rads/s).

It is evident from Figure 2 that most of the interactions in the engine have been significantly reduced, since the compensated transfer function matrix is now clearly dominant. The normalised pre-compensator matrix thus found for the Rolls-Royce Spey jet engine is

$$K = \begin{pmatrix} 0.0098 & 0.002 & 0.3309 \\ -1.3304 & 1.4712 & -30.2710 \\ -0.0023 & -0.0031 & 0.7498 \end{pmatrix}. \qquad (4)$$

2.2 Design of single-loop controllers

The same evolutionary design methodology can be used again for the design of the single-loop controllers. First, Engine Handling Qualities (EHQ) guidelines were used to determine suitable response functions. These guidelines indicate [10] that an appropriate choice for the desired closed-loop transfer function matrix is

$$M(s) = diag\left\{\frac{1}{0.35s+1}, \frac{1}{1.2s+1}, \frac{1}{0.18s+1}\right\}. \qquad (5)$$

In this stage of the evolutionary design procedure, each chromosome when decoded describes a diagonal PI controller with a transfer function matrix of the form

$$\begin{aligned} D(s) &= D_P + D_I\frac{I}{s} \\ &= diag\left\{\frac{dp_{11}s + di_{11}}{s}, \frac{dp_{22}s + di_{22}}{s}, \frac{dp_{33}s + di_{33}}{s}\right\} \end{aligned} \qquad (6)$$

The fitness of each such chromosome (i.e. each candidate controller) is directly related to the closeness of the desired and actual closed-loop responses. Thus, a natural cost function is the Integral Absolute Error (IAE) between the desired closed-loop step responses (characterised by the array $[S_{ij}(t)]$) and those of the closed-loop system with the candidate controller (characterised by the array $[R_{ij}(t)]$). This cost function is given by

$$\Gamma_D(D_I, D_P) = \sum_{i,j}^{m} \int_0^{t_o} \mid R_{ij}(t) - S_{ij}(t) \mid dt \qquad (7)$$

and represents an improvement over that traditionally used in the design of single-loop controllers to be used with dominance-based pre-compensators. The cost function used in such traditional designs is clearly a special case of equation (7) with $i = j$, indicating that no attention is paid to the effects of the single-loop controllers on the off-diagonal behaviour. However, when evolution strategies are used for optimisation, cost functions like that defined in equation (7) facilitate the minimisation of the closed-loop step-response errors in *all* channels.

In order to design the single-loop PI controllers for the Spey engine, a (μ+λ) adaptive evolution strategy was employed with population sizes of $\mu = 10$ parents and $\lambda = 10$ children for 1000 generations in which the cost function (7) was evaluated over 20 sample-time points between 0 and 2.4 seconds. The resulting diagonal controller thus found is

$$D(s) = diag\left\{\frac{1.092s+4.109}{s}, \frac{1.147s+1.321}{s}, \frac{0.1962s+5.239}{s}\right\}. \qquad (8)$$

The open-loop step responses of the Spey engine are shown in Figure 3; the open-loop step responses of the engine with the pre-compensator are shown in Figure 4; and finally the closed-loop step responses of the engine with the pre-compensator and the single-loop controllers are shown in Figure 5. These results, when compared to previous dominance-based controller designs for the Spey engine, show improvements and are very encouraging. Indeed, considering that only a static pre-compensator and a set of single-loop PI controllers are being used, the results are very good and show that the adaptive evolution strategies can be readily used to design complete dominance-based linear multivariable controllers.

3 Factorisation and loss of performance

Diagonal dominance as a multivariable design technique was celebrated because it 'broke down' a complex design problem into a set of easily manageable problems. It did this by factoring the multivariable controller matrix $C(s) \in \mathbf{C}^{m\times m}$ into a pre-compensator matrix $K \in \mathbf{R}^{m\times m}$ and a diagonal controller matrix $D(s) \in \mathbf{C}^{m\times m}$, where the task of each part was decoupled from the other. Since $C(s) = KD(s)$, the controllers with transfer functions contained in each column of $C(s)$ have the same poles and zeros: in this sense, $C(s)$ describes a controller with a constrained multivariable structure. However, if $C(s)$ is not required to be always amenable to such factorisation, then its poles and zeros will be released from such constraints, thus constituting an unconstrained multivariable controller.

The constraint imposed by the factorisation implicit in the dominance-based approach is likely to cause loss of performance, but, hitherto, it was not possible to determine exactly the extent of such loss. However, using adaptive evolution strategies of the type already used in this paper to design the dominance-based PI controller, it is possible readily to design an unconstrained PI controller for the Spey engine, thus facilitating a quantitative assessment of such loss of performance. Thus, the parameters of each unconstrained multivariable PI controller are coded as one chromosome. This constitutes a 'one-step' design process, since each chromosome contains the data for a complete controller. The relevant cost function is similar to that used for the second stage of the dominance-based controller, namely, the IAE between the desired responses and those of the closed-loop system resulting from the candidate PI controller as given by

$$\Gamma_C(C_I, C_P) = \sum_{i,j}^{n} \int_0^{t_o} \mid R_{ij}(t) - S_{ij}(t) \mid dt. \qquad (9)$$

However, unlike the previous cost function defined in equation (7) where K_P and K_I where constrained to be diagonal, in equation (9) C_P and C_I may assume any structure. This approach was used to design an unconstrained PI controller for the Spey engine. Thus, a $(\mu + \lambda)$ adaptive evolution strategy with a population of $\mu = 20$ parents and $\lambda = 20$ children was used to optimise the parameters of this unconstrained PI controller over 20 intervals in the time range of 0 to 2.4 second. The same time-sampling points were used to evaluate the cost function in equation (9) as in the previous section, thus allowing a direct comparison between the performance of the two controllers to be made. In the present case, after running for 5000 generations, the algorithm produced the unconstrained multivariable PI controller with the transfer function matrix given by

$$C(s) = \begin{bmatrix} \frac{0.007356s+0.04189}{s} & \frac{0.002639s+0.003213}{s} & \frac{0.06148s+1.636}{s} \\ \frac{-1.455s-5.691}{s} & \frac{1.743s+1.928}{s} & \frac{-6.226s-149.6}{s} \\ \frac{-0.001713s-0.006708}{s} & \frac{-0.01335s-0.003588}{s} & \frac{0.1267s+3.711}{s} \end{bmatrix} \qquad (10)$$

The closed-loop step responses of the engine with this controller are shown in Figure 6. Notice that, as expected, the interactions have been reduced. Indeed, the unconstrained PI controller provides an amazing 65% reduction in interactions relative to those produced by the constrained PI controller. This improvement is shown even more clearly in Figures 7 and 8, which show the direct Nyquist arrays of the closed-loop system with the constrained and unconstrained sets of PI controllers. It is clear from these figures that the performance of the unconstrained PI controller is far better than that of the corresponding constrained PI controller.

These results clearly confirm, in a quantitative way, what was known qualitatively about the dominance-based approach to the design of linear-multivariable controllers, namely, that this approach offers simplicity at the expense of performance. However, simplicity arises in two contexts, the first of which involves the technique itself. This advantage all but disappears when the dominance problem is formulated as an optimisation problem, since the only difference between the two optimisations is that a direct optimisation of the unconstrained PI is more computationally expensive than the two stages of the dominance approach put together.

The second context in which simplicity is an issue is in the structure of the controller itself. It may seem, on the surface, that both constrained and unconstrained controllers have the same complexity; but, from an implementation point of view, this is not correct. Thus, let $C_c(s) = K \times D(s)$ represent a constrained controller and $C_u(s)$ its corresponding unconstrained controller (not necessarily in PI form). If the state-space representation of $D(s)$ is

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t), \end{aligned} \qquad (11)$$

then it is evident that the state-space representation of the complete constrained controller is

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BKu(t) \\ y(t) &= Cx(t) + DKu(t) \end{aligned} \qquad (12)$$

since K is only a static matrix. Hence, it is clear that the number of the states of the constrained controller, $C_c(s)$, equals the number of the states of the diagonal controller, $D(s)$. Specifically, the number of *distinct* poles of $D(s)$ determines the minimum number of the states. This could have been directly deduced and indeed been expected from the constraint that, as a result of the factorisation, the poles and zeros of each column of $C_c(s)$ are the same. The situation, however, is different in the case of $C_u(s)$. Here, since each element is independent of the others, the number of states may increase compared to those of a $C_c(s)$ with the same structure. Thus, for example, consider the case when the elements of both $C_c(s) \in \mathbf{C}^{m\times m}$ and $C_u(s) \in \mathbf{C}^{m\times m}$ are simple lead compensators (so that both systems are multivariable lead controllers): in this case, whilst the maximum number of the states of $C_c(s)$ is m, $C_u(s)$ may contain up to m^2 states.

4 Concluding remarks

Diagonal dominance as a design technique lay dormant for a long time due to the lack of a universal technique powerful enough for the routine design of dominance-based controllers. In this paper, evolution strategies have been proposed as one such powerful technique that can easily be adapted to solve many minimisation problems including that of diagonal dominance. Using the Rolls-Royce Spey engine as an example, it has been shown that the two-stage design of the dominance-based controller may be readily effected using evolution strategies. In the case of this engine, the evolutionary design methodology has also been used to synthesise an unconstrained multivariable PI controller which exhibited superior performance to that of the dominance-based constrained controller.

The possession of such a powerful minimisation algorithm raises the following question: what is the justification for first finding a pre-compensator and then a set of single-loop controllers, if the controller can be designed as a single, unfactorised and unconstrained structure? In answer to this question, it has been shown that a dominance-based controller will usually yield a simpler implementation than its corresponding unconstrained controller. Also, the constrained version would yield higher integrity with respect to loop failures. Therefore, it is concluded that the final decision rests with the control engineer as to which approach is suitable for the problem at hand and whether the priority lies with performance or implementation issues. However, the one fact remaining is that evolution strategies are very effective tools in tackling both formulations of the multivariable controller design problem.

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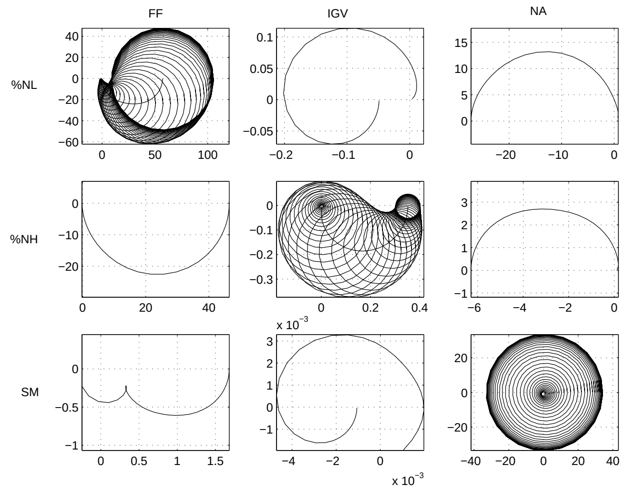

Figure 1: Open-loop direct Nyquist array of the Spey engine

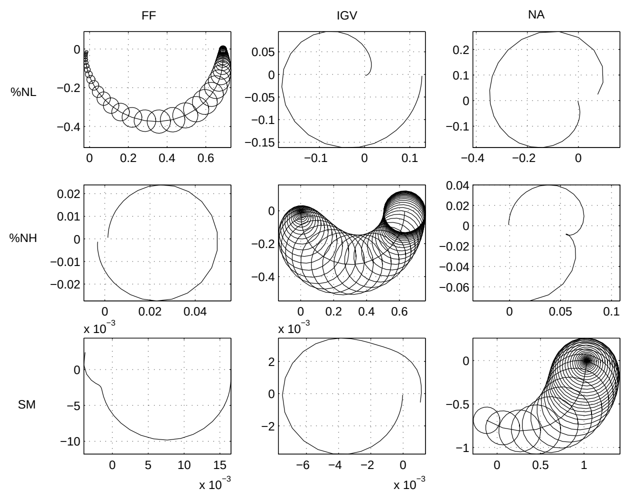

Figure 2: Open-loop direct Nyquist array of the compensated Spey engine

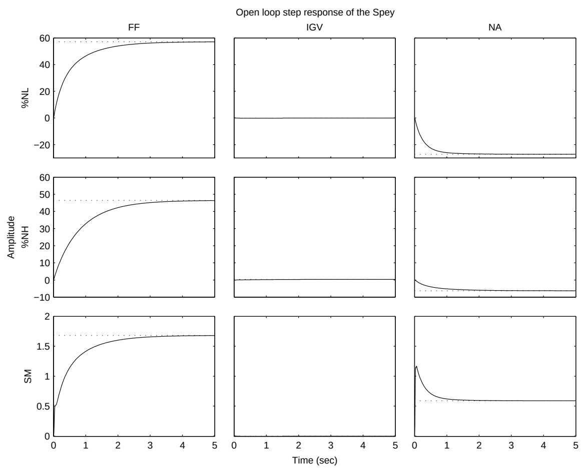

Figure 3: Open-loop step responses of the Spey engine

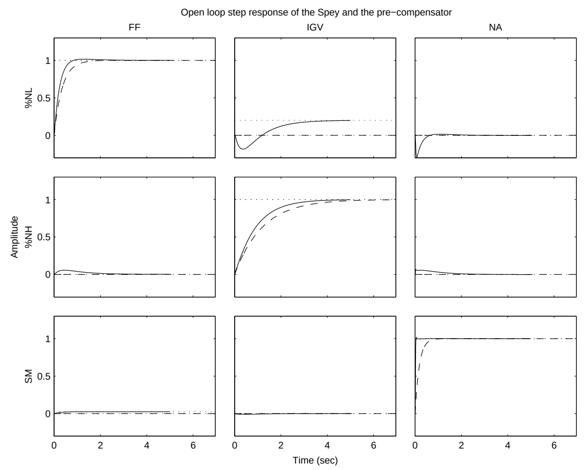

Figure 4: Open-loop step responses of the compensated Spey engine(dashed line = desired response)

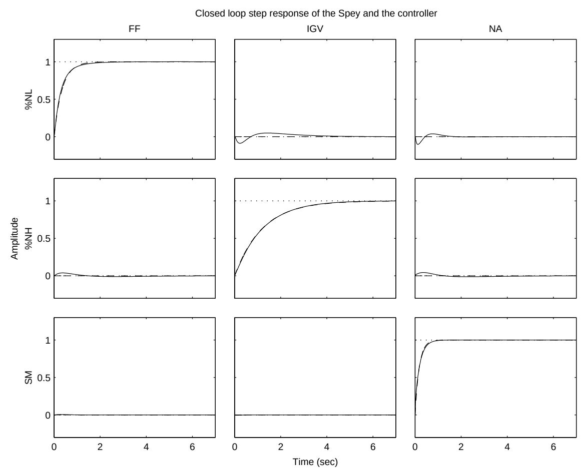

Figure 5: Closed-loop step responses of the Spey engine with complete dominance based controller (Dashed line = desired response)

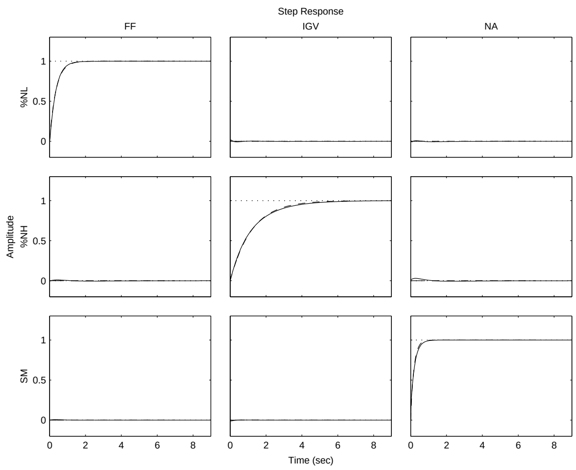

Figure 6: Closed-loop step responses of the Spey engine with the unconstrained PI controller (dashed line = desired)

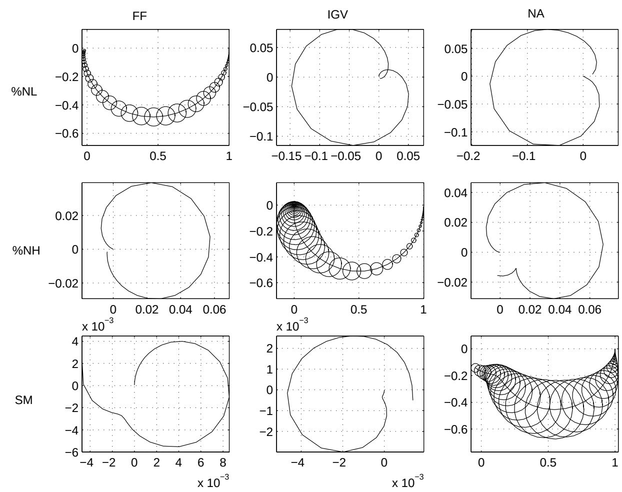

Figure 7: Closed-loop direct Nyquist array of the Spey engine with the constrained PI controller

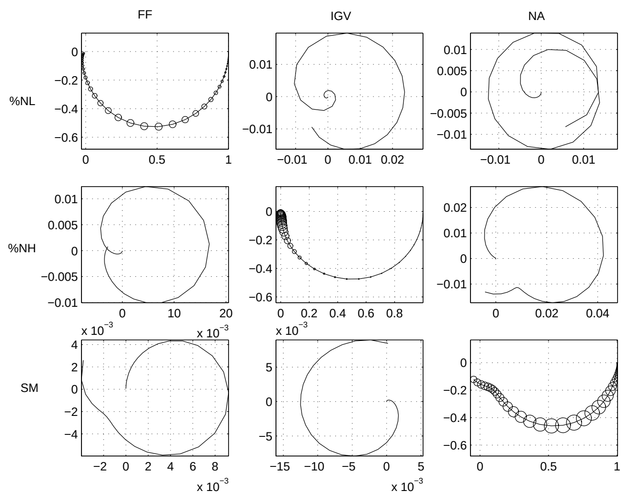

Figure 8: Closed-loop direct Nyquist array of the Spey engine with the unconstrained PI controller