# PR-SLIDING SECTORS FOR CONTINUOUS AND DISCRETE TIME SLIDING MODE CONTROLLERS FOR AN INDUCTION MOTOR

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## Abstract :

Induction motors, thanks to their numerous advantages, are widely used in the industry. In this study a sliding mode controller is designed to control the position of the actuator. This controller uses sliding sectors, inside which a norm of the state decreases .The control is designed to transfer the state from the outside to the inside of this sector, and when inside it, the control action becomes zero.

## 1 Introduction

Induction motors are relatively cheap, reliable and do not need maintenance, but were not used in robotics and manipulators drives due to the complexity of their model which is non linear, coupled and of high order.

Therefore these difficulties do not simplify the motor control and positioning, but the introduction of the field-oriented control technique allows the induction motor to obtain static and dynamic performances as those of the direct current motor [6]. This technique which is caracterised by the decoupling between the flux and the torque, simplifies greatly the control of the system.
As some parameters change with the heating of the motor, and as the load is usually unknown, the classical techniques of control such as PID is revealed insufficient and the use of robust control is necessary..
Variable structure control is known to be robust to parameter uncertainty and external disturbances because of the sliding motion on a predefined hyperplane. The sliding mode control is then used for the position control of the motor
Various approaches of sliding mode controllers have been proposed for continuous and discrete systems [1,4, 9].
Furuta [1] proposed the use of the non linear control law depending on subsets of the state space which are obtained by partitioning the space into sectors.
Furuta and Pan in [7] proposed the use of sliding sectors with a variable structure controller which drives the system state to an appropriately determined sector in the space inside which a norm of the system state decreases without any control action.
Variable structure control based on PR sliding sector is active only when the system state is outside of the PR-sliding sector. Such control is said “lazy” because the control input is zero as long as the state remains in the sector [2, 3,5,8].

In this paper we design a sliding sector which is a subset of the state space and inside which the norm of the system state is decreasing while the control input is zero. Such sliding sector exists for any given system including continuous or discrete time systems.
The resolution of the Riccati equation is used for the synthesis of both the continuous and the discrete time PR-sliding sectors.
The organization of the paper is as follows: In section 2, the modelisation of the motor is described. Section 3 defines the vectorial control of induction motors.
Section 4, defines the sliding sector and designs VSS controllers corresponding to these sliding sectors for continuous time systems, while section 5 presents the design of the discrete time controller based on PR-sliding sectors.

## 2 Modelisation of an asynchronous motor

After the application of the Park transformation, the mechanical and electrical equations expressed in a d-q synchronously rotating reference frame are written as follows [6]:

$$
\begin{cases}
\dfrac{d\omega_m}{dt} = \dfrac{n_p L_m}{J L_r}(\psi_{dr} i_{qs} - \psi_{qr} i_{ds}) - \dfrac{T_L}{J} \\
\dfrac{d\psi_{dr}}{dt} = \dfrac{L_m}{T_r} i_{ds} - \dfrac{R_r}{L_r}\psi_{dr} + (\omega_s - n_p\omega_m)\psi_{qr} \\
\dfrac{d\psi_{qr}}{dt} = \dfrac{L_m}{T_r} i_{qs} - (\omega_s - n_p\omega_m)\psi_{dr} - \dfrac{1}{T_r}\psi_{qr} \\
\dfrac{di_{ds}}{dt} = \Big[ -\left(\dfrac{1}{\sigma T_s} + \dfrac{1-\sigma}{\sigma T_r}\right) i_{ds} + \omega_s i_{qs} + \dfrac{1-\sigma}{\sigma L_m T_r}\psi_{dr} + \\
\qquad + \dfrac{1-\sigma}{\sigma L_m} n_p \omega_m \psi_{qr} + \dfrac{1}{\sigma L_s} v_{ds} \Big] \\
\dfrac{di_{qs}}{dt} = \Big[ -\omega_s i_{ds} - \left(\dfrac{1}{\sigma T_s} + \dfrac{1-\sigma}{\sigma T_r}\right) i_{qs} + \dfrac{1-\sigma}{\sigma L_m T_r}\psi_{qr} + \\
\qquad - \dfrac{1-\sigma}{\sigma L_m} n_p \omega_m \psi_{dr} + \dfrac{1}{\sigma L_s} v_{qs} \Big]
\end{cases}
\qquad (1)
$$

$$\dot{\omega}_m = \frac{n_p}{J}\left(T_e - T_l\right)$$

The stator voltages are considered as control variables, the load torque as a disturbance and the states variables can be chosen in different ways. The state vector can be defined as follows : $X = [\psi_{dr} \quad \psi_{qr} \quad i_{ds} \quad i_{qs} \quad \omega_m]^T$

the control vector $U = [v_{ds} \quad v_{qs}]^T$

where $\sigma = 1 - \frac{L_m^2}{L_s L_r}$ $T_r = \frac{L_r}{R_r}$ $T_s = \frac{L_s}{R_s}$

## 3 Vectorial control by rotor flux orientation

The field oriented control [6] is a technique introduced by Blashke. The motor's dynamical equations can be written in a frame fixed to the rotor flux. In this new frame , by maintaining the rotor flux constant , we have a linear relation between the speed and the control variable.

Let consider the rotor flux, $\psi_r^*$ and the torque $T^*_e$ as control references and let inverse the model by rotor flux orientation , we obtain the following equations :

$$i_{ds} = \frac{1}{L_m}\psi_r^* \tag{2}$$

$$i_{qs} = \frac{L_r}{pL_m}\frac{T_e^*}{\psi_r^*} \tag{3}$$

$$\omega_{sg} = \frac{L_m}{T_r}\frac{i_{qs}}{\psi_r^*} \tag{4}$$

where $\psi_{dr} = \psi_r^*$ and $\psi_{qr} = 0$

The application of the field oriented control simplifies considerably the control structure represented on figure 1, and reduces the problem to that of a linear system of second order given by equations (5) .The field-weakening is used for the rotor flux control. This last is held constant and equal to its nominal value for speeds slower than the nominal value of the speed, and it decreases for speeds higher.

The mechanical equation of the system is given by:

$$\dot{\omega}_m(t) = \left[ -\frac{f}{J}\omega_m(t) - \frac{T_L}{J}(t) \right] \tag{5}$$

The state representation is given by :

$$\dot{X} = AX + BU + DT_L$$

The state variables are given by:

$x_1 = \theta_m - \theta_m^*$ $x_2 = \dot{\theta}_m = \omega_m$ and the control is $U = i_{qs}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{f}{J} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_t}{J} \end{bmatrix} U + \begin{bmatrix} 0 \\ -\frac{1}{J} \end{bmatrix} T_L \tag{6}$$

## 4 Sliding mode controller with PR-sliding sector

In this paper sliding mode control with PR-sliding sector is proposed.

This controller is designed such that a lyapunov function which represents a P-norm decreases with a derivative less than a specified negative value. Inside the PR-sliding sector, this norm decreases for zero input and specified velocity, and outside it the variable structure control law is used.

Let consider a linear time invariant continuous-time system:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{7}$$

where $x \in \Re^n$ and $u(t) \in \Re$ are state and input vectors. The pair (A,B) is controllable.

### 4.1 PR-sliding sector [2]

A norm is defined on $\Re^n$ and is used in the following.

The P-Norm $\|.\|_p$ of the system state is defined as :

$$\|x\|_p = \left(x^T P x\right)^{\frac{1}{2}}, \; x \in \Re^n \tag{8}$$

where $P \in \Re^{nxn}$ is a positive definite symmetric matrix.

The square of the P-norm is denoted as:

$$L = \|x\|_p^2 = x^T P x > 0, \quad \forall x \in \Re^n, x \neq 0 \tag{9}$$

If the autonomous system (7) is stable then :

$$\dot{L} = x^T\left(A^T P + PA^T\right)x \leq 0, \quad \forall x \in \Re^n \tag{10}$$

If the system (7) is instable, the inequality (10) does not hold. And we can have $\dot{L} > 0$ for some $x \in \Re^n$ and $\dot{L} \leq 0,$ for some other $x \in \Re^n$.

The sector where $\dot{L} \leq 0,$ is a subset in which the P-norm decreases.

The PR-sliding sector is a subset of $\Re^n$ defined by:

$$S = \left\{x \mid x^T\left(A^T P + PA^T\right)x \leq -x^T R x, \quad x \in \Re^n\right\} \tag{11}$$

where $P \in \Re^{nxn}$ is a positive definite symmetric matrix, and $R \in \Re^{nxn}$ is a positive semi-definite symmetric matrix $R = C^T C$ . $R \in \Re^{lxn}$ $l \geq 1$ and (C,A) is an observable pair. Such PR- sliding sector exists because at least the zero state satisfies this condition. Inside the PR-sliding sector, the P-Norm $\|x\|_p$ of the plant (7) without any control action decreases because: $\dot{L} \leq -x^T R x \leq 0, \quad \forall x \in S$ (12)

Furuta shows that for any plant (7) ,the PR-sliding sector defined by (11) can be rewritten as:

$$S = \left\{x \mid s^2(x) \leq \delta^2(x)\right\} \tag{13}$$

where $s^2(x) = \left(x^T P_1 x\right) \geq 0$ (14)

and $\delta^2(x) = \left(x^T P_2 x\right) \geq 0$ (15)

$P_1$ and $P_2$ are nxn a positive semi-definite symmetric matrices.

Denote $\Delta = \left(A^T P + PA^T + R\right)$ (16)

Then the PR-sliding defined by (11) is determined by :

$$x^T \Delta x \leq 0 \tag{17}$$

$$U^T \Omega U = diag(r, r_2, ..., r_{n1}) \tag{18}$$

where : $r_i$ (i=1,2,….,n) are the characteristics roots of $\Delta$ which are all real because $\Delta$ is symmetric.

Assume:

$$\overline{P}_1 = diag\left(\frac{|r_1| + r_1}{2}, \frac{|r_2| + r_2}{2}, ...., \frac{|r_n| + r_{n1}}{2}\right) \tag{19}$$

$$\overline{P}_2 = diag\left(\frac{|r_1| - r_1}{2}, \frac{|r_2| - r_2}{2}, ...., \frac{|r_n| - r_{n1}}{2}\right) \tag{20}$$

$\overline{P}_1\ \overline{P}_2$ are composed of the positive eigenvalues and the negatives eigenvalues of $\Delta$, respectively.

$$U^T \Omega U = \overline{P}_1 - \overline{P}_2 \qquad (21)$$

$$\overline{P}_i \geq 0 \ \text{(i=1,2)} \qquad (22)$$

$$P_i = U^T \overline{P}_i U \qquad (23)$$

$$\Omega = P_1 - P_2 \qquad P_i \geq 0 \ \text{(i=1,2 )} \qquad (24)$$

A simplified PR-sliding sector is a subset of $\Re^n$ defined as [ 2]:

$$S = \left\{ s \ \middle| \ \text{s(x)} \leq \delta(\text{x}), \ \ \text{x} \in \Re^{\text{n}} \right\}, \qquad (26)$$

where the linear functional s(x) ,and the square root $\delta$ (x) of the quadratic function $\delta^2(x)$ are respectively determined by:

$$s(x) = Sx, S \in \Re^{1xn} \qquad (27)$$

$$\delta(x) = \sqrt{x^T \Delta x}\ , \Delta \in \Re^{nxn} \text{ and } \Delta \geq 0 \ (\Delta \neq 0) \qquad (28)$$

Inside the simplified PR-sliding sector S (26), the P-norm decreases with zero input and the derivative of the candidate Lyapunov function L(t) satisfies the condition

$$\dot{L} = \frac{d}{dt} x^T P x = s^2(x) - \delta^2(x) - x^T R x$$

$$\dot{L} \leq -x^T R x, \ \ \forall \ \text{x(t)} \in \text{S} \qquad (29)$$

A simplified PR-sliding sector can be designed by using the following RICCATI equation:

$$A^T P + PA^T - PBB^T P = -Q \qquad (30)$$

where Q$\in \Re^{nxn}$ is a positive definite-symmetric matrix.

As the pair (A,B) in (7) is assumed to be controllable, the positive definite-symmetric solution $P \in \Re^{nxn}$ of the Riccati equation (30) exists.

If we take the solution P to design the PR-sliding sector, then for zero input

$$\dot{L}(t) = x^T(t)\left(A^T P + PA^T\right)x(t) \qquad (31)$$

$$\dot{L}(t) = x^T(t)\left(PBB^T P\right)x(t) - x^T Q x \qquad (32)$$

For any controllable plant , if the positive definite symmetric solution P of the Riccati equation is used to define the P-norm and the positive semi-definite symmetric matrix R is chosen so that $\Delta = Q - R \geq 0$ and $\Delta \neq 0$ where Q$\in \Re^{nxn}$ in the Riccati equation (30) is a positive definite-symmetric matrix, then the PR-sliding sector defined in (13) can be rewritten as the following simplified PR- sliding sector.

$$S = \left\{ x \mid \ |s(x)| \leq \delta(x), \qquad x \in \Re^{\text{n}} \right\} \qquad (33)$$

where $\quad s(x) = Sx(t) \ \ S = B^T P \qquad (34)$

$$\delta(x) = \sqrt{x^T(t) \Delta x(t)} \qquad (35)$$

$$\Delta = Q - R \qquad (36)$$

### 4.2 Variable structure controller design

The controller is designed using the following steps:

a-The parameter matrices Q, P,R, and Δ may be chosen as:

- Choose an nxn positive-definite symmetric matrix Q.
- Choose a positive constant r (0<r<1) and let $\Delta = rQ$ and $R = (1-r)Q$

b- Solve the Riccati equation for the positive definite symmetric matrix P

c- To avoid the chattering occurring on the boundary of the PR-sliding sector an inner sector $S_i$ and an outer sector $S_o$ are subsets of the PR-sliding sector and defined as :

$$S_i = \left\{ x \mid |s(x)| \leq \alpha \delta(x), \qquad x \in \Re^{\text{n}} \right\} \qquad (37)$$

$$S_o = \left\{ x \mid \quad \alpha\,\delta(x) < |s(x)| \leq \delta(x), \qquad x \in \Re^{\text{n}} \right\} \qquad (38)$$

where $0 < \alpha < 1$

$S = S_i \cup S_o$ and $S_i \cap S_o = \aleph$ with $\aleph$ the null set in $\Re^n$.

d- The variable structure control law will be active to move the state from the outside of the sliding sector into the inside of the inner sector, then the input will become zero until the state move to the outside of the PR-sliding sector.

Furuta in [2] shows that :

Corresponding to the inner and outer sectors of the PR-sliding sector S, with $\Delta = rQ$ and $R = (1-r)Q$ for some positive constants $0 < \alpha < 1$ and $0 < r < 1$, the variable structure control law: $u(t) = -\sigma(s(x), \delta(x))(SB)^{-1}(SAx + Ks(x)) \quad (39)$

Ensures the movement of the state from the outside of the sliding sector into the inside of the inner sector and the decreasing of the P-norm, and results in a quadratically stable variable structure control system if the positive coefficient K is large enough so that $K > \max\left\{\frac{SB}{2}, K_0\right\}$, where the positive constant $K_0$, satisfies the following quadratic inequality:

$$2K_0 \alpha^2 rQ + S^T SA + A^T S^T S > 0 \qquad (40)$$

$\sigma(s(x), \delta(x)))$ is a hysterisis dead zone function on s(x) and $\delta(x)$ and is defined as:

$$\sigma(s(x), \delta(x)) = \begin{cases} 0 & \text{x} \in \text{S}_{\text{i}} \\ \text{unchanged} & \text{x} \in \text{S}_{\text{o}} \\ 1 & \text{x} \overline{\in} \text{S} \end{cases}$$

$SB$ is equal to $B^T PB$ which is non singular.

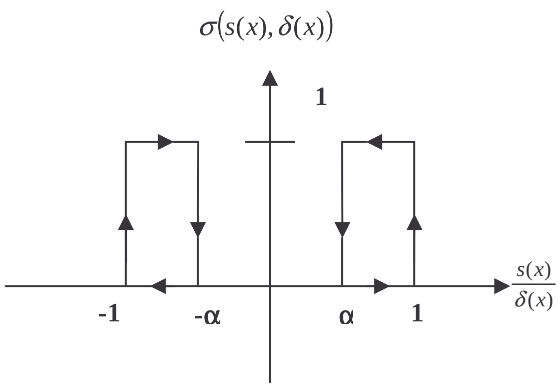
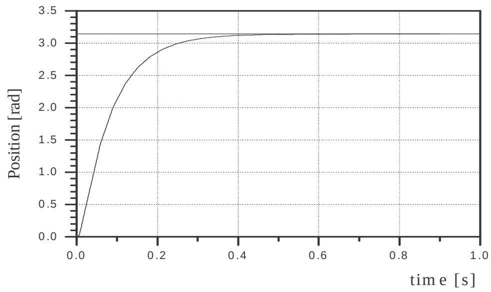



Figure 1 -Hysterisis dead zone function σ(s(x),δ(x)

$$S_o = \left\{ x \mid \quad \alpha\delta(x) < |s(x)| <, \delta(x), x \in \Re^n \right\}$$

## 4.3 Simulation results :

Simulation results shown on figure 2 were carried out for a step position of 3.14 rad and a sampling time of Ts=.01s, the coefficient of the control law is K=45 and the coefficient defining inner and outer sectors is $\alpha$=0.28 **.** We see that the position converges towards its reference value rapidly and the control input is sometimes inactive.

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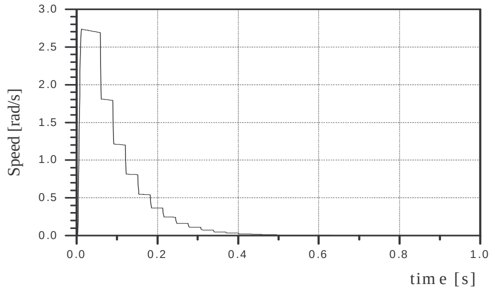


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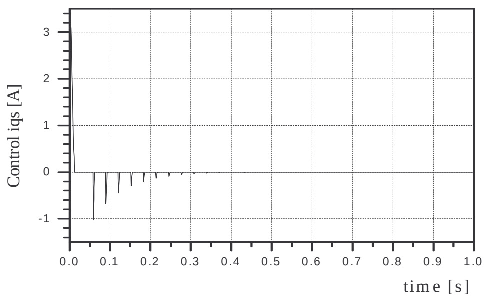


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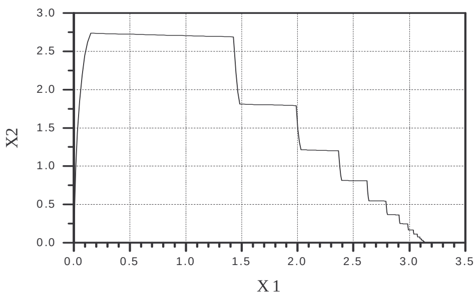


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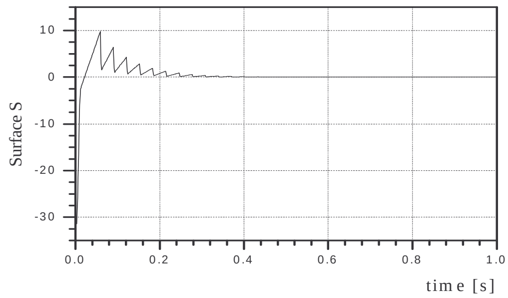


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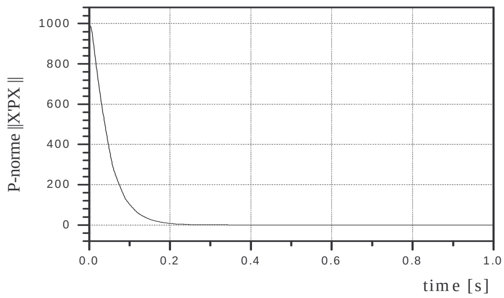


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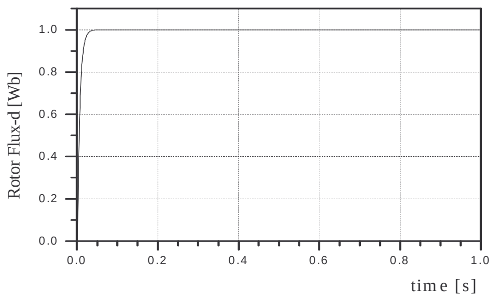


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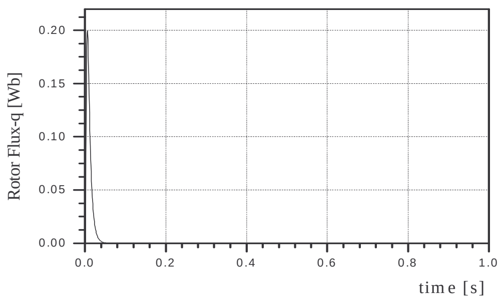



(h)

Figure 2 Responses of continuous time VS control

(a) position, (b) speed (c) control, (d) phase plane, (e)surface, (f) P-norme, (g) direct rotor flux component, (h) quadrature rotor flux component

This is due to the fact that when the state is inside the PR-sliding control, there is no control action until it moves outside of it.The system state P- norm decreases in the state space.

Furthermore the rotor fluxes are maintained to their reference values $\psi_{dr}$=1Wb and$\psi_{qr}$= 0Wb

## 5 Discrete-time sliding mode controller

The discretization of the system: $\dot{x}(t) = Ax(t) + Bu(t)$

yields to: $x_{k+1} = \phi x_k + \Gamma u_k$ (41)

The control is given at every sampling instant $kT_s$, where $T_s$ is the sampling period" The control, input is held constant between sampling : $u(t)=u_k k\Delta \le t \le (k+1)\Delta$ (42)

where $x_k = x(k\Delta)$ ; $\phi = e^{A\Delta}$ ; $\Gamma = \int_0^{\Delta} e^{A\tau} d\tau$

### 5.1 Discrete PR-sliding sector [3]

As for the continuous case a norm is defined on $\Re^n$

The $P_d$ -Norm $\|.\|_{p_d}$ of the system state is defined as :

$\|x_k\|_{p_d} = \left(x_k^T P_d x_k\right)^{\frac{1}{2}}$, $x_k \in \Re^n$ where $P_d \in \Re^{nxn}$ is a positive definite symmetric matrix.

The square of the $P_d$ -norm is denoted as:

$$\bar{L} = \|x_k\|_{p_d}^2 = x_k^T P_d x_k > 0, \quad \forall x_k \in \Re^n, x \ne 0 \quad (43)$$

The discrete PR-sliding sector is a subset of $\Re^n$ defined by:

$$S_d = \left\{ x \mid x_k^T \left(\phi^T P_d \phi - P_d\right) x_k \le -x_k^T R_d x_k, \quad x_k \in \Re^n \right\} (44)$$

where $P_d$ is a nxn positive definite symmetric matrix.

and $Rd$ is a nxn positive semi-definite symmetric matrix.

Inside the $P_d R_d$-sliding sector, the P-Norm $\|x\|_{p_d}$ of the plant (41) without any control action decreases because

$$\Delta L_k = L_{k+1} - L_k = x_k^T \left(\phi^T P_d \phi - P_d\right) x_k$$

$$\le -x_k^T R_d x_k \le 0, \quad \forall x_k \in S_d \ (45)$$

A simplified PR-sliding sector is defined as

$$S_d = \left\{ x_k \mid \quad |s_k| \le \delta_k, x_k \in \Re^n \right\} \quad (46)$$

where $s_k = S_d x_k$ and $\delta_k = \sqrt{x_{Tk}^T \Delta_d x_k}$

To design the discrete-time simplified $P_d Rd$-sliding sector the following discrete time Riccati equation is solved:

$$P_d = Q_d + \phi^T P_d \phi - \phi^T P_d \Gamma (1 + \Gamma P_d \Gamma)^{-1} \Gamma^T P_d \phi \quad (47)$$

where $Q_d \in \Re^{nxn}$ is a positive definite symmetric matrix.

If the positive definite symmetric solution $P_d$ of the Riccati equation is used to define the $P_d$ norm and the positive semi-definite symmetric matrix $R_d$ is chosen so that $\Delta_d = Q_d - R_d$ and $\Delta_d \ne 0$ then the $P_d R_d$-sliding sector defined in (44) can be rewritten as:

$$S_d = \left\{ x_k \mid \quad |s(x_k)| \le \delta(x_k), x_k \in \Re^n \right\} \quad (48)$$

where $s_k = S_d x_k$ $S_d = \Gamma^T P_d \phi \ / \sqrt{1 + \Gamma^T P_d \Gamma_k}$

$\Delta_d = Q_d - R_d$

$\delta_k = \sqrt{x_{Tk}^T \Delta_d x_k}$ $\Delta_d \ge 0$ and $Q_d \in \Re^{nxn}$ is a positive definite symmetric matrix.

The discrete-time $P_d R_d$-sliding sector can be defined as a subset of the continuous PR-sliding sector.The PR-Sliding sector becomes smaller after sampling.The discrete PR-Sliding sector becomes closer to the continuous-time one when the sampling frequency increases.

### 5.2 variable structure controller design

The design of the discrete sliding mode controller is done as follows:

- Choose the nxn positive definite symmetric matrix $Q_d$.
- Solve the discrete time Riccati equation (47) with $P_d$ its solution to design the discrete time $P_d R_d$-sliding sector.
- Choose the positive constant r (0<r<1) and let $\Delta_d = rQ_d$ and $R_d = (1-r)Q_d$
- The discrete time control law to move the system state from the outside to the inside of the $P_d R_d$-sliding sector is given by:

$$u_k = \begin{cases} 0 & x_k \in S_d \\ -(S_d \Gamma)^{-1} \left(S_d \phi x_k + K_d (sign(S_d \Gamma s_k) \delta_k\right) & x_k \notin S_d \end{cases}$$

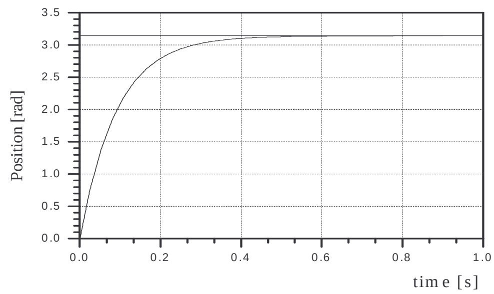
where the coefficient $K_d$ satisfies the condition $0 \le K_d < 1$.

### 5-3 Simulation results

Simulation results shown on figure 3 were carried out for a step position of 3.14 rad.

$$Q = 100 * \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \ r=0.91; \ alpha =.28; \ Kd=0.012$$

We see a fast convergence of the position to its reference value.We notice that in some time intervals, the control is zero: this is due to the fact that the state moves into the PR-sliding sector.

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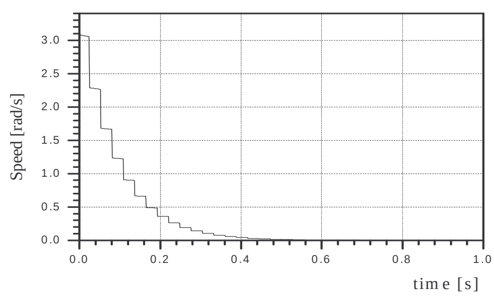


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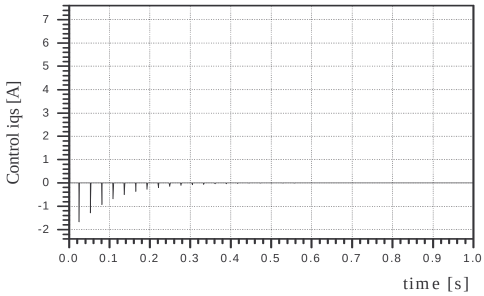


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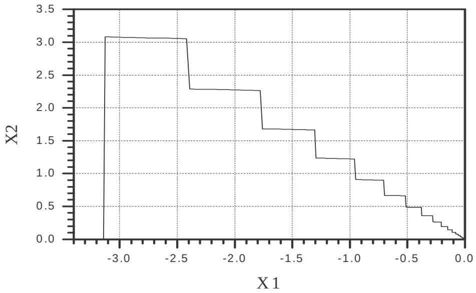


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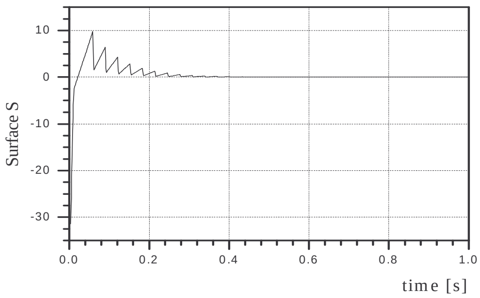
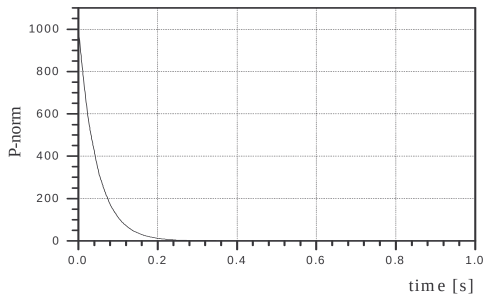



(e)




(f)

Figure 3 Responses of discrete time VS control $\tau$=.01s
(a) position, (b) speed (c) control, (d) phase plane, (e)surface, (f) P-norm

## 6- Conclusion

In this paper Sliding mode controllers based on sliding sectors have been proposed for both continuous and discrete time systems.The PR-Sliding sectors, which are subsets of the state space, have been defined such that the P-norm of the system state decreases inside them.The control law is designed such that to move the state from the outside to the inside of the PR-sector and when in it, the control becomes inactive.To avoid chattering in the continuous time case, inner and outer sectors are used. In this case the control moves the state from the outside of the PR-sliding sector to the inner sector and remains inactive until the system state leaves the PR-sliding sector.Simulation results show the effectiveness of the method and good results.

## 7 References

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## Notations

**d,q** : indexes corresponding to the reference frame
**$R_s$ $R_r$** : stator and rotor resistances
**$L_s$, $L_r$ $L_m$**: stator , rotor and mutual inductances
**$T_s$, $T_r$** : stator and rotor time
**$\sigma$** : total leakage coefficient 1-$L_m^2$ /$L_S L_r$
**$n_p$** : number of poles pairs
**J, f** : inertia, viscous coefficient
**$\omega_s$ , $\omega_r$** : stator and rotor electrical angular velocity
**$\omega_{sg}$**$\omega_m$ : electrical mechanical speed
**$\psi$** : flux
**$v_{ds}$, $v_{qs}$** : stator voltages
**$i_{ds}$, $i_{qs}$** : stator currents
**$T_e$ $T_l$** : electromagnetic torque load torque

**Machine parameters :**$P_N$ : 1.5KW $R_S$= 4.85Ω $R_r$=3.805Ω ; $f_N$=50Hz $n_p$=2 ; f=.00114 $N_N$=1420 trs/mn ; $L_s$=0.274H $L_r$=.274H, $L_m$=.258H. J=.031kg $m^2$ ; $V_N$=220V