

MIXED μ -ANALYSIS APPLIED TO THE ATTITUDE CONTROL OF THE METOP SPACECRAFT

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Abstract

Mixed- μ analysis (hereafter also called worst-case analysis) techniques have become a powerful practice for supporting the control system design process on complex, multivariable, uncertain, time-varying and even non-linear systems. The motivation for the adaptation of such tools to space problems originated from the fact that large spacecraft or space structures exhibit low-frequency (quasi) undamped flexible modes leading to risks of interaction with high-accuracy attitude control. Although these problems have been routinely solved by control engineers for years, the large parametric uncertainty in the plant sometimes leads to destabilizing worst-case combinations that are not intuitive at all and detected late in the development process.

Astrium has successfully made use of these techniques on a variety of space applications, providing the development teams with valuable insights into the intricacies of complex systems. The article more specifically addresses the case of the polar-orbiting weather spacecraft MetOp scheduled for launch in 2005. The analysis campaign successfully detected all worst-case combinations of parameters that could destabilize the system: most were already known, but the process also allowed to discover some non-intuitive combinations involving antenna flexibilities.

Notations

ACS	Attitude Control System
CoM	Centre of Mass
DoF	Degree of Freedom
LFT	Linear Fractional Transformation

LTI	Linear Time-Invariant
MIMO	Multi Input Multi Output
OPM	OPERational Mode
SA	Solar Array
SADM	Solar Array Drive Mechanism
M_i	Generalized mass matrix of body # i ($i = 0$ - central body, $i = 1$ - SA, $i = 2, 3$ - Antennas)
L_i	Modal participation matrix of the SA flexible modes
m_ϕ	Equivalent generalized mass matrix for the SA rotation mode (state: ϕ)
i_a	Inertia modal participation of the SADM rotation axis ($\mathbf{Y}_{\text{spacecraft}}$)
n_ϕ	Inertia of the SA around the SADM rotation axis
x_G	3 DoF position vector of the centre of mass
θ_G	3 DoF rotation vector of the centre of mass
η_i	Flexible modes vector of the SA
ϕ_i	SADM rotation angle (1 DoF)
ω_i	Cantilever frequencies of the SA flexible modes
ξ	Flexible modes damping factor
K_f	SADM damping factor
K_m	SADM stiffness (model)
$T_{i, G, \text{Rsat}}$	External Force & Torque applied at the CoM

1 Worst-case analysis for space systems

1.1 Introduction: specificities of space systems

A specificity of space systems is the large range of parametric uncertainty. In addition to deterministic time-variations (motion of the solar arrays over the orbital motion, depletion of fuel over the lifetime, incidence of Sun pressure over the time of the year), the plant exhibits

large uncertainties for numerous parameters. They result from the fact that little ground testing or characterization can be done prior to flight, and also from the lack of in-flight characterization. It is indeed impossible to faithfully reproduce the free-falling vacuum conditions on the ground, and it is generally too costly to embark the sensors that would provide full observability on the plant response in-flight. The space engineer has to rely on fully modelled plants, where all small uncertainties on elementary parameters for individual components (mass, position, inertia, stiffness, damping) cumulate into large variations domains for the global plant. For instance, the uncertainty on the frequencies of the flexible modes of the solar arrays typically reaches 20 to 30%.

Robust control design is the natural approach in such situations, and worst-case analysis its vital companion to avoid over-conservative design. The latter also provides valuable insights into the intricacies of complex systems, by supplying the control engineer with the combinations of parametric variations that would lead to instability or to unacceptable performance degradation. These combinations are to be analysed to determine whether they correspond to real-life cases or are just modelling artefacts.

1.2 Worst-case analysis techniques for space systems

Worst-case analysis investigations began at Astrium in the mid 90's with tentative explorations of commercially available software in the field of μ -analysis. This exploration showed severe limitations in the applicability of such tools for space structures and space systems engineering. In particular, the very sharp and highly resonant peaks that result from large flexible appendages in vacuum conditions (solar arrays, antennas, solar sails, etc.) bring the classical frequency-gridding algorithms to their limits. In order to guarantee that the worst-case configuration is not missed, the grid has to be excessively fine, leading to unreasonable computational burden.

Research work conducted jointly with Onera [1 & 2] was then oriented towards the suppression of the frequency search, leading to the so-called *mixed- μ analysis* approach [1, 2 & 4]. The new approach could efficiently operate in the presence of narrow resonances, while allowing the same representation of system uncertainties as for traditional approaches, thus retaining full compatibility with standard μ -analysis or μ -synthesis techniques.

Promising results allowed to rapidly implement these techniques on full-scale 'real-life' systems, starting with two central domains of application: the Eurostar family of telecommunication spacecraft and the SPOT family of Earth-observation spacecraft. These two control designs are complementary in nature: in the case of telecom spacecraft, the flexible modes of the very large solar arrays are generally left outside the control bandwidth, with appropriate stiff filtering techniques; while the high-accuracy pointing control of the SPOT spacecraft makes it compulsory to control the first flexible modes and to

maintain the phase while rolling-off the gain. The results obtained on the SPOT ACS were even more interesting and promising than expected: a complex multi-parameter worst-case of a multi-loop control system (partly H_∞ controller) was found [1,2].

The promises of the tools were verified on other applications such as the current one: the mixed- μ analysis for the ACS of the MetOp spacecraft.

1.4 The MetOp spacecraft and its mission

Astrium develops the MetOp spacecraft under the leadership of the European Space Agency (ESA). The MetOp mission objectives jointly established by ESA and EUMETSAT are twofold:

- To ensure continuity, improvement and availability for operational purposes of polar meteorological observations.
- To provide Europe with an enhanced capability for the routine observation of Earth from space and with the possibility for European states to participate in the long term Earth climate monitoring.

The MetOp spacecraft is put into heliosynchronous circular orbit at about 800 km altitude. This satellite consists of an Earth pointing central body, connected to a flexible solar array that provides the energy to the on-board equipment (see Figure 1). The ASCAT instrument whose antennas are of interest for our study is a scatterometer (dedicated to the measure of wind velocity).

Worst-case analysis is conducted on the Attitude Control System (ACS) for the Operational Mode (OPM). This mode uses an Earth sensor for pitch/roll estimation and a Sun sensor for yaw depointing estimation. The actuation system relies on a 3-wheels assembly unit.

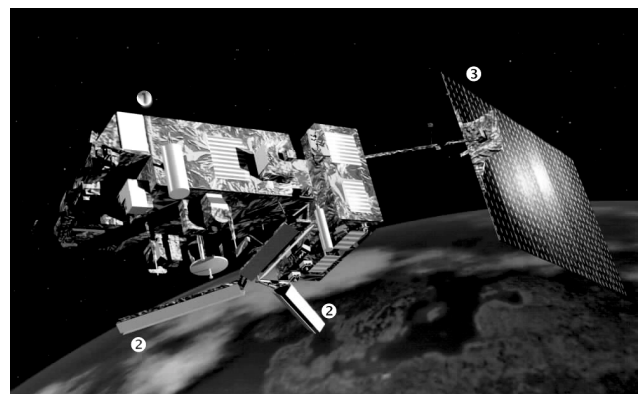


Figure 1: Artist's view of the MetOp spacecraft, showing the central body (1), the two deployable ASCAT antennas (2), and the solar array (3)

*The in-orbit dimensions reach 17.6m*6.6m*5.0m whereas the launch configuration measures 6.3m*3.4m*3.5m.*

1.3 Methodology for running μ -analysis

The methodology for the use of the mixed- μ techniques is easy to follow. First a LFT model representative of all the potential variations of the plant is modelled. This first step consisted for MetOp ACS in re-using the generic LFT elements already developed for SPOT. It is actually straightforward to create a complete LFT model using LFT description of its elementary pieces. Furthermore, the LFT toolbox created by ONERA allows manipulating the LFT models as easily as the LTI objects under the Matlab environment.

Then the combined use of the tools computing the lower and upper bounds of μ (minimal singular value) allows to derive precisely and rapidly one or several valid worst-case combination leading to the destabilisation of the system. Finally these results are carefully analysed and possibly compared to the usual Monte Carlo statistical campaign results.

Another advantage of the LFT modelling is its modularity. By using either the whole set of uncertain parameters or only a reduced set of specific parameters, one can verify the robustness of the control design or check the sensitivity of the stability margins to several parameters.

2 Mathematical modelling for MetOp

2.1 Dynamics elements

The MetOp spacecraft is very similar to the SPOT platform in terms of dynamics (same kind of bus and solar array) as well as in terms of control (recurring control algorithms structure).

The dynamics of the spacecraft used for the design of the control system and for the worst-case analyses is an aggregation of elementary dynamics for rigid and flexible bodies, namely:

- The central body of the spacecraft, considered as a rigid body.
- The solar array drive mechanism, on which the solar array is hinged, which rotates the array by 360° every 90 minutes (orbital period) to ensure the best solar energy collection.
- The solar array itself, with its first 17 flexible modes (from ~ 0.1 Hz to ~ 5 Hz).
- The deployable ASCAT antennas, with their first two flexible modes. This is where the MetOp spacecraft fundamentally differs from its cousin SPOT. The analysis later showed that these antennas were responsible for unexpected destabilizing combinations of parameter uncertainties.

The flexible modes themselves are described by their frequencies and damping coefficients. Their impact on the motion of the central body is characterized in the form of modal participation coefficients expressed at the center of

mass of the appendage. The corresponding dynamics equations then have to be transported with respect to the center of mass of the spacecraft and to its reference frame in order to obtain the influence of appendage modes on the motion of the central body.

In addition, the solar array is rotated about its axis by a motor (solar array drive mechanism). The modelling of this degree of freedom in rotation leads to a fundamental non-stationarity, as the frame in which the flexible modes of the appendage appear rotates about the reference frame of the spacecraft.

This leads to the following set of non-linear equations describing the dynamics of the spacecraft and its appendages:

$$M_i \cdot \begin{bmatrix} \ddot{x}_G \\ \ddot{\theta}_G \end{bmatrix} + L_i^T \cdot \ddot{\eta}_i + m_\varphi \cdot \ddot{\phi}_i = T_{i,G,Rsat} \quad (1)$$

$$L_i \cdot \begin{bmatrix} \ddot{x}_G \\ \ddot{\theta}_G \end{bmatrix} + \ddot{\eta}_i + C_i \dot{\eta}_i + K \eta_i + i_a^T \cdot \ddot{\phi}_i = 0 \quad (2)$$

$$m_\varphi^T \cdot \begin{bmatrix} \ddot{x}_G \\ \ddot{\theta}_G \end{bmatrix} + i_a \cdot \ddot{\eta}_i + n_\varphi \cdot \ddot{\phi}_i + K_f \cdot \dot{\phi}_i + K_m \cdot \phi_i = K_m \cdot C_{G,y_{sat}} \quad (3)$$

A simple manipulation of the above equations allows to obtain the generalized M , C , K state form (generalized mass matrix, generalized damping matrix, generalized stiffness matrix):

$$\begin{bmatrix} \sum_i M_i & L_i^T & m_\varphi \\ L_i & Id & i_a^T \\ m_\varphi^T & i_a & n_\varphi \end{bmatrix} \cdot \begin{bmatrix} \ddot{x}_G \\ \ddot{\theta}_G \\ \ddot{\eta}_i \\ \ddot{\phi}_i \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2\xi\omega_i & 0 \\ 0 & 0 & K_f \end{bmatrix} \cdot \begin{bmatrix} \dot{x}_G \\ \dot{\theta}_G \\ \dot{\eta}_i \\ \dot{\phi}_i \end{bmatrix} \quad (4)$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & \omega_i^2 & 0 \\ 0 & 0 & K_m \end{bmatrix} \cdot \begin{bmatrix} x_G \\ \theta_G \\ \eta_i \\ \phi_i \end{bmatrix} = \begin{bmatrix} T_{i,G,Rsat} \\ 0 \\ T_3 \end{bmatrix}$$

To complete the representation of the complete closed-loop behaviour of the system, the control laws are added. Inherited from the SPOT spacecraft family, these algorithms involve a complex MIMO multiloop architecture. The satellite is a three-axis stabilized platform through a complex MIMO and multi-loop control architecture. Command torques are produced by reaction wheels, and the SADM provides the required torques to rotate the Solar Array at constant speed. Sensors measurements are produced by gyrometers and updated at low frequency by digital Sun and Earth sensors. Initially conceived as a positive system where all of the 10 significant solar array flexible modes were phase stabilised by gyros feedback, the increase in the platform size and the demands in performance improvement have

contributed in the past 15 years of SPOT existence to make more complex the ACS architecture and more intricate the stability issues. In particular, as the SADM is actuated in "open-loop" at a constant rate, it induces harmonic internal perturbations that can damage platform stability, without being controllable from the reaction wheels actuators. To cope with this problem, a SADM control loop has been introduced as a local feedback loop around the gyro measurement, to modulate the «open-loop» constant rate command by a zero-mean compensation term. In addition, the rotation of the SA makes the system non-stationary, as the flexible modes impact on the platform dynamics evolves correspondingly to their relative geometry.

2.2 Assembling the elements into a complete LFT representation

The dynamics system and the control laws are converted into the so-called M- Δ representation or LFT form (Linear Fractional Transformation), on which the tools for μ -analysis rely. This can be assimilated to a parallel connection of a block Δ and a matrix M .

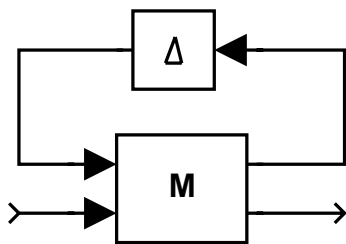


Figure 2: Classical representation of the M- Δ form

The plant dynamics, the control laws, the loop delays and the parameters uncertainties are all individually transformed into LFT objects. The tools available at Astrium then allow to manipulate these objects in MATLAB™ just as easily as better-known LTI (Linear Time-Invariant) objects: they can be connected in series, in parallel, etc., thus allowing to build a complete LFT model of the closed loop system, ready for analysis. The complete LFT model is represented in the figure below.

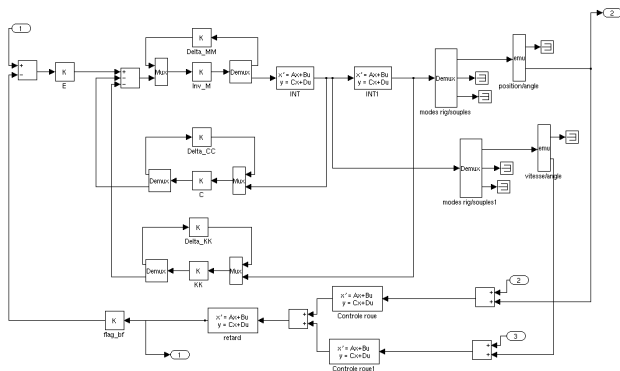


Figure 3: Block-diagram of the AOCS control loop with the plant dynamics under the M, C, K formalism

Removing the 'Delta_' blocks in the above diagram amounts to considering the linear model in its nominal operating condition. The 'Delta_' blocks introduce the structured description of the impact of parametric variations on the plant dynamics.

2.3 Final model: nominal and disturbed cases

The final model has a large size, which is generally the case when dealing with 'real' satellites, in order to reach a sufficient level of representativity.

The figures below represent the diagonal open-loop transfer functions between control input and dynamics output on each of the spacecraft axes, parameterised with the angular position of the solar array drive mechanism (0 to 180 degrees - the other 180 degrees inducing a symmetrical behaviour).

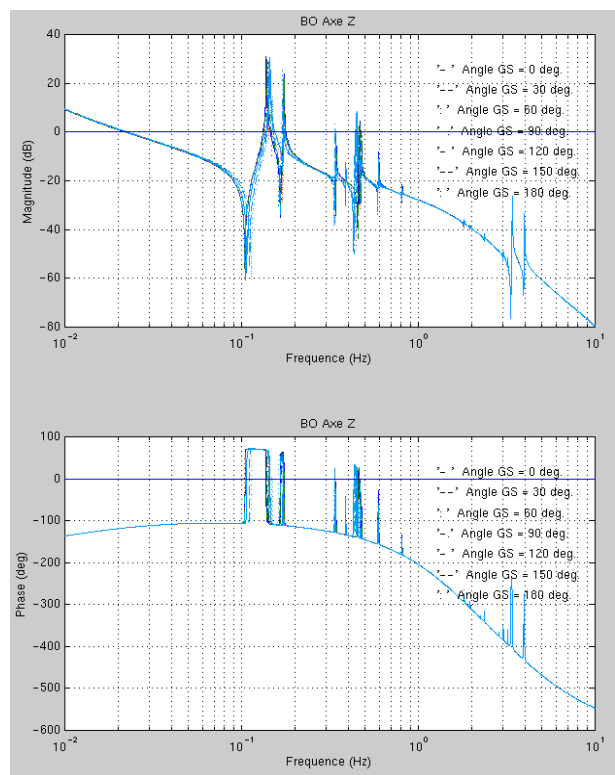


Figure 4: Envelope of the frequency response of the open loop for the Z axis (Bode plots)

A large set of flexible modes is observed, significantly impacting the loop stability up to the 10th mode due to the gyro rate feedback.

All flexible modes are phase stabilised through gyro rate feedback.

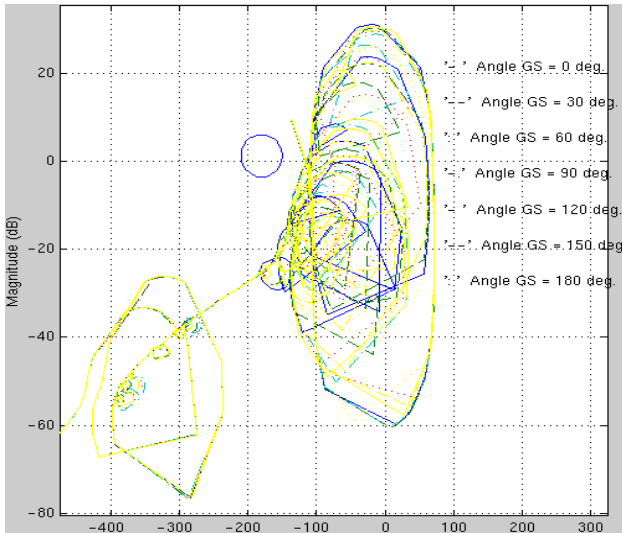


Figure 5: Envelope of the frequency response of the open loop for the Z axis (Nichols chart)

3 Results of the campaign analysis

3.1 Global approach for the campaign analysis

The analysis was divided into three incremental steps:

1. Construction and validation (in nominal and disturbed cases) of the LFT representation of the model, taking into account the uncertainties on the various relevant parameters. All approximations are duly recorded for later expertise. This preliminary step (described in the previous section) is not fully automated, and requires a very good knowledge of the system considered.
2. Automatic search of the worst-case combinations of uncertain parameters that induce instability, on gradually more complete parameter sets. It is indeed recommended to start with homogeneous sets of simple parameters for which the sensitivity of the system is *a priori* known. Then these reduced sets are progressively enriched with extra parameters, until the full dimension of the uncertainty space is covered.
3. The ultimate step is more ambitious and aims at finding the combination of parameter values that lead to a degradation in performances down to a certain predefined level (specified by a criterion similar to a module margin, or more generally an H_∞ template). This quantitative information is even more pertinent to the control systems engineer, as it determines the boundaries of the uncertainty domain in which the system can ‘nominally’ evolve while keeping the required performance levels. This analysis is however not trivial, as it requires to conduct dichotomy operations.

3.2 Choice of the analysis campaigns

The objective of the campaigns was primarily to prove the pertinence of the worst-case analysis approach for such a

complex system. The campaigns were therefore split into gradual levels of complexity (see step 2 above):

- Campaign 1: worst-case analysis in *stability* on a reduced set of parameters with an *a priori* high sensitivity. This includes the resonance frequencies of the flexible appendages (17 for the solar array, 2 for each antenna). The order of the uncertainty block (Δ block) is therefore 21.
- Campaign 2: worst-case analysis in *stability* on an augmented set of parameters specified by the development team. This includes in addition to the above set the inertia of the solar array and its mass, the mass of the antennas, as well as the uncertain stiffness of the SADM. The order of the Δ block is now 31.
- Campaign 3: worst-case analysis in *stability* on all nonzero parameters with a nonzero variation (null terms or null variations impose a separate processing in order to avoid ill conditioning of the LFT forms, and therefore to ensure proper numerical behaviour of the search algorithms). These parameters include all the previous ones plus the mass of the central body, the nonzero position errors of the central body and of the solar array, as well as all inertia terms. The order of the Δ block is finally 73.

A single M- Δ form is used to perform these stability worst-case analyses. This M- Δ form is directly inherited from previous worst-case analyses on spacecraft attitude control systems since the tools to build the LFT lead to very generic re-usable elements. For each campaign, the M- Δ representation form is either complete (campaign 3) either ‘truncated’ (i.e. only some of the whole varying/uncertain parameters are part of the Δ set. The other potentially varying/uncertain parameters are fixed to their nominal values). Therefore it is straightforward to obtain several reduced LFTs from a complete LFT taking into account all the varying /uncertain parameters. The analyses can range from the determination of a precise multi-parameter worst-case combination (obtained from the complete LFT) to dedicated sensitivity worst-case analyses (obtained thanks to reduced LFTs).

Campaign	Uncertain Parameters	Δ block order ¹	Model order
1	- 17 cantilever frequencies of SA - 2 cantilever frequencies of the ASCAT antennas	21	63
2	- Idem campaign 1 - Central body inertia - SA mass - ASCAT antennas mass - SADM stiffness	31	178
3	- Idem campaign 2 - Central Body mass - CoM position (central body & SA) - SA and antennas inertia	73	335

Table 1: Characteristics of the LFTs for the μ -analysis

¹ without repeatability

The order of the model takes into account the repeatability of the varying parameters and influences the computational time.

Table 1 describes the characteristics of the three LFTs used in the study: the first one is the homogeneous and simplest form dedicated only to resonance frequency variations whereas the third and last one is the complete form. The continuity observed in the characteristics of the "worst-case" detected while gradually using more and more complete LFT representation confirms the physical meaning of the worst-case and the power of the μ -analysis tools used here.

3.3 Final results

The final results for each campaign are recapitulated in Table 2. Note that the solar array angular position was discretised, so that the campaign involved five distinct analyses. This discretisation (0, 45, 90 135 and 180 degrees) was more convenient from an algorithmic point of view, as the large variation of the angle is not easily represented in the form $\alpha_0(1 + \delta_\alpha)$. We verified afterwards that the evolution of the worst-case combinations found was sufficiently continuous to allow a discretisation by steps of 45 degrees.

Campaign	SA angle (deg)	Number of combinations ²	Number of relevant combinations
#1	0, 45, 90, 135, 180	4	3
#2	0, 45, 90, 135, 180	3	3
#3	0, 45, 90, 135, 180	4	2

Table 2: Results for the various analyses

The 'relevant' worst-case are derived from the complete set of worst-cases by excluding 'irrelevant' situations such as a negative variation of a resonance frequency greater than -100% .

The three campaigns resulted into quite similar physical results and the worst-case combinations could be categorized into three types:

- Destabilizing interaction between the flexible modes around the cutoff frequency. This is a classical combination, well known to the designers, and the control laws were robust enough to handle it (as long as the parameters remain within their expected variation domain, no stability nor loss of performance could be encountered).
- A worst-case involving primarily the first mode of the second antenna. This case was less well known, as it involved flexible modes absent in the SPOT family. Statistical analyses confirmed that a reduction of 26% in the resonance frequency of the first flexible mode of

the second antenna could destabilize the system. The mixed- μ analysis toolbox had allowed to corroborate an unexpected anomaly observed during statistical extensive validation campaign, as well as to provide valuable information (parameters involved in the destabilising combination, frequency of the instability, ...) useful for the design of a more robust control system.

- A third case involved a combined evolution of the flexible modes for the solar array and both antennas. This case was hardly intuitive as multivariable and totally unexpected, and just as troublesome as the above case. As a representative illustration, we present in Table 3 & Figure 6 a worst-case combination of this third kind obtained during the second campaign (and with a solar array angle of 90 degrees).

Uncertain parameters with a variation > 1 %	Variation
Main inertia around X axis of the central body	-36.5 %
Main inertia around Y axis of the central body	-36.5 %
Main inertia around Z axis of the central body	-36.9 %
Solar array cantilever frequency # 1	+4.5 %
Solar array cantilever frequency # 2	+2.75 %
Solar array cantilever frequency # 5	+1.8 %
Solar array cantilever frequency # 6	+14.65 %
Solar array cantilever frequency # 12	+49.51 %
Solar array cantilever frequency # 13	+6.4 %
Solar array cantilever frequency # 14	+1.43 %
Solar array cantilever frequency # 16	2.8 %
Solar array cantilever frequency # 17	1.54%
Mass of the antenna 1	+3.36 %
Cantilever frequency # 1 of the antenna 1	+13.3 %
Cantilever frequency # 2 of the antenna 1	-57.4 %
Cantilever frequency # 1 of the antenna 2	+43.2%
Cantilever frequency # 2 of the antenna 2	+7.2 %
Frequency of the instability	1.64 Hz

Table 3: Destabilizing combination involving a non-intuitive distributed evolution of the frequencies for the antennas and the solar array

The above worst-case destabilizes the control on the Z axis, and involves the inertia of the central body (-35%), the twelfth flexible mode of the solar array (+50%) as well as a combined variation of the resonance frequencies for the second mode of the first antenna (-57%) and the first mode of the second antenna (+43%), presenting a complex combination of multiple variations of heterogeneous parameters.

² Mean over the SA tilts

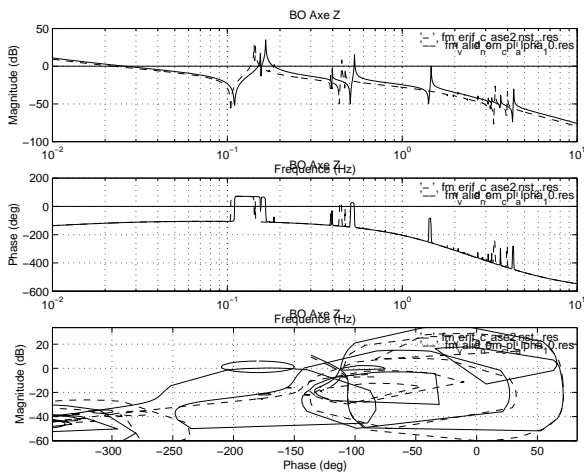


Figure 6: Nichols chart of the open-loop transfer function for the destabilizing case

4 Conclusions and research directions

4.1 Conclusions on the analysis of MetOp ACS

The worst-case analysis conducted for the MetOp attitude control system was very interesting in a twofold way:

- It showed that with a reduced modelling effort, the known worst combinations of parameter variations could be detected, thus providing an easy way to validate analyses conducted with different and often heavier methods (Monte-Carlo campaigns for instance).
- It allowed to discover non-intuitive worst cases that had not been detected earlier. The results facilitated the retuning of the control laws.

The campaign also showed that the tools, method and approach were sufficiently mature to efficiently support the control design and validation process for full-fledged spacecraft problems, and that stability worst-case analysis are no longer an R&D topic.

4.2 Worst-case analysis in ‘performance’

The analyses in terms of *performance*, providing combinations of parameters that do not destabilize the system but bring it to its performance limits for a given H_∞ criterion will be a valuable continuation to the design and validation of complex control systems such as attitude control for spacecraft or space structure. We are continuing the effort in this domain through dedicated R&D studies.

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