

# TRANSIENT CONTROL AND VOLTAGE REGULATION OF POWER SYSTEMS USING APPROXIMATE SOLUTION OF *HJB* EQUATION

**Mahdi Jalili Kharaajoo, M.J. Yazdanpanah**

*Control & Intelligent processing Center of Excellence,  
Electrical and Computer Engineering Department,  
University of Tehran, P.O. Box: 14395/515, Tehran, Iran  
E-mails: [mahdijalili@ece.ut.ac.ir](mailto:mahdijalili@ece.ut.ac.ir) and [yazdan@ut.ac.ir](mailto:yazdan@ut.ac.ir)*

**Keywords:** Power system transient stability, Voltage regulation, Optimal Control, Nonlinear feedback control, *HJB* equation.

## Abstract

In this paper a new method to design a nonlinear optimal controller using approximate solution of the *HJB* equation is presented. Using this method the power system stabilizer and voltage regulator is designed. In this paper the advantages of the controller with nonlinear feedback will be shown. Simulation results reveal the effectiveness of the proposed approach.

## 1 Introduction

A power system must be modeled as a nonlinear system for large disturbances. Although power system stability may be broadly defined according to different operating conditions [6], an important problem, which is frequently considered, is the problem of transient stability. It concerns the maintenance of synchronism between generators following a severe disturbance. By the excitation control in a generating unit transient stability can be greatly enhanced. Another important issue of power system control is to maintain steady acceptable voltage under normal operating and disturbed conditions, which is referred as the problem of voltage regulation [5,8,9,11,13].

So far, a lot of research about the design of power system stabilizers have been considered, which consist of wide-range of strategies, such as adaptive controllers [3,7] or fuzzy expert systems [1,10]. In [2] a comparison of some approaches for the design of power system stabilizers has been presented. One of these approaches is the use of *LQR* controllers. In spite of its simplicity, the use of this method has some disadvantages, like the sensitivity of the controller to the variation of system parameters or the limited range of controllable disturbances. In the other words, the domain of validity of the *LQR* controllers in contrast to the actual systems, which are nonlinear, has considerable limitations. These limitations have encouraged control engineers to introduce nonlinear controllers to design power system stabilizers. In spite of its complexity, nonlinear controllers have the advantage of increasing the region of the stability of the system. In [16] it has been proved that a controller with nonlinear feedback always has a larger domain of validity than the controllers with linear feedback. In [17] this claim has been shown for a flexible link manipulator. In this

paper we will consider the above issues by approximate solution of the *HJB* equation using Taylor's Series expansion.

In the transient stabilizing control, a common phenomenon is that the post-fault voltage value varies considerably from the pre-fault one [4,13,18]. From the practical point of view, voltage quality is a very important index of power supply in power system operation. So, the post-fault value is expected to reach the normal value as closely as possible. In [4] a global control law to maintain the transient stability and achieve satisfactory post-fault voltage level of a power system when subjected to a severe disturbance has been designed. The control signal from the global controller is the average of the signals from the local control laws, each weighted by the value of its operating region membership function. Since the membership function can be determined by direct measurable variables of power systems. In this paper we will design local controllers of global control law using approximate solution of the *HJB* equation.

## 2 Dynamic Model of Power Systems

In this paper, a simplified dynamic model of a power system, namely, a single machine to infinite bus (SMIB) power system is considered [6,13]. This model consists of a single synchronous generator connected through a parallel transmission line to a very large network approximated by an infinite bus. The model is shown in figure 1.

In recent years, most of the nonlinear excitation controllers have been developed based on the classical third order dynamic generator model and the simulation results presented them showed that such a simplification has very little effects on the performances of the designed controllers in the system presented in more details [6].

The classic third order single-axis dynamic model of the SMIB power system Fig.1 can be written as follows [4,6,13,18]:

### 2.1 Mechanical equations

$$\dot{\delta}(t) = \omega(t) - \omega_o \quad (1)$$

$$\dot{\omega}(t) = -\frac{D}{2H}(\omega(t) - \omega_o) - \frac{\omega_o}{2H}(P_e(t) - P_m) \quad (2)$$

The mechanical input power  $P_m$  is treated as a constant in the excitation controller design, i.e., it is assumed that the governor action is slow enough not to have any significant impact on the machine dynamics.

## 2.2 Generator electrical dynamics

$$\dot{E}'_q(t) = \frac{1}{T'_{do}}(E_f(t) - E_q(t)) \quad (3)$$

## 2.3 Electrical equations (Assumed $x'_d = x_q$ )

$$E_q(t) = E'_q(t) + (x_d - x'_d)I_d(t) \quad (4)$$

$$E_f(t) = k_c u_f(t) \quad (5)$$

$$P_e(t) = \frac{E'_q(t)V_s}{x'_{ds}} \sin \delta(t) \quad (6)$$

$$I_d(t) = \frac{E'_q(t) - V_s \cos \delta(t)}{x'_{ds}} \quad (7)$$

$$I_q(t) = \frac{V_s}{x'_{ds}} \sin \delta(t) \quad (8)$$

$$Q(t) = \frac{E'_q(t)V_s}{x'_{ds}} \cos \delta(t) - \frac{V_s^2}{x'_{ds}} \quad (9)$$

$$E_q(t) = x_{ad} I_f(t) \quad (10)$$

$$V_t(t) = [(E'_q(t) - x'_d I_d(t))^2 + (x'_d I_q(t))^2]^{\frac{1}{2}} \quad (11)$$

More details about power system modeling are in [6]. The definition of the above parameters is as follows:

- $\delta(t)$  Power angle of the generator, radians
- $\omega(t)$  Rotor speed of the generator, radian/s
- $\omega_o$  Synchronous machine speed, radian/s
- $P_m$  Mechanical power, p.u.
- $P_e(t)$  Active electrical power delivered by the generator, p.u.
- $E'_q(t)$  Transient EMF in the quadratic axis of the generator, p.u.
- $E_q(t)$  EMF in the quadratic axis of the generator, p.u.
- $E_f(t)$  Equivalent EMF in the excitation coil, p.u.
- $V_t(t)$  Generator terminal voltage, p.u.
- $u_f(t)$  Input of the SCR amplifier of the generator, p.u.
- $V_s$  Infinite bus voltage,
- p.u.  $x_{ds} = x_d + x_T + x_L$
- $x'_{ds} = x'_d + x_T + x_L$
- $x_s = x_T + x_L$

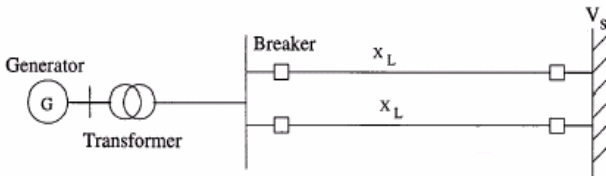


Fig. 1: A single machine infinite bus power system.

The fault considered in this paper is a symmetrical three-phase short circuit fault, which occurs on the middle of one of the transmission lines. When the fault on the transmission lines is removed, the breakers of the lines are opened.

## 3 Design of Optimal Nonlinear Feedback Controller Based on Approximate Solution of the HJB Equation

Optimal control is the determination of control signals in order to minimize or maximize a definite cost function while fulfilling some constraints. Using dynamic programming and optimality principle, results in a nonlinear partial differential equation known as the **HJB** equation. This equation has the following form [12,15,14]:

Assume a system with the following differential equation:

$$\dot{\underline{x}} = \underline{a}(\underline{x}(t), \underline{u}(t), t) \quad (12)$$

where  $\underline{x}$  is the state variables vector and  $\underline{u}$  is the system input vector. The problem of optimal control design is to control the above system such that the following cost function is minimized:

$$J = h(\underline{x}(t_f), t_f) + \int_{t_o}^{t_f} g(\underline{x}(\tau), \underline{u}, \tau) d\tau \quad (13)$$

where  $g, h$  are definite functions and  $t_o, t_f$  are constants. So, according the HJB equation:

$$J_t^*(\underline{x}(t), t) + H(\underline{x}(t), \underline{u}^*(\underline{x}(t), J_x^*, t), t) = 0 \quad (14)$$

where  $J^*$  is the minimum of cost function and  $\underline{u}^*$  is the input vector that minimize  $H$ , and  $H$  is Hamilton function that is defined as follows:

$$H(\underline{x}(t), \underline{u}^*(\underline{x}(t), J_x^*, t), t) = g(\underline{x}(t), \underline{u}(t), t) + J_x^{*T}(\underline{x}(t), t) \cdot [\underline{a}(\underline{x}(t), \underline{u}(t), t)] \quad (15)$$

As it can be seen the HJB equation is a partial differential equation, and finding exact analytical solution for  $J^*$  is too difficult. However, there are methods to find approximate solution for  $J^*$ . One of these possible solutions is using *Taylor's Series Expansion* of desired order. According to (14)  $\underline{u}^*$  is a function of  $J_x^*$ , so expressing  $J_x^*$  in the form of Taylor expansion of order  $n$  leads to a controller of order  $(n-1)$ . In order to obtain  $J_x^*$  in the form of Taylor expansion of order  $n$ , we will use the method proposed in [17].

Considering the introduction presented above, we will follow the design of optimal nonlinear control for power systems. For simplicity, the design of a first order controller will be considered, that is equal to estimate of  $J^*$  up to order 2. For this case the deviation of variables  $\delta, \omega$  and  $E'_q$  from their steady state initial values are selected as control variables. A similar procedure can be used to design of higher order controllers. In fact the first order controller is the *LQR* controller that can be obtained directly by solving Riccati equation.

### 3.1 Nonlinear Feedback Transient Controller Design

In order to design an optimal controller, first, a cost function should be considered. For this system, because of the significance of the variation of power angle, rotor relative speed, and control signal, the following cost function is considered.

$$J = \int_0^\infty \left( \frac{1}{2} q_1 (\delta - \delta_0)^2 + \frac{1}{2} q_2 \omega^2 + \frac{1}{2} r u^2 \right) \quad (16)$$

where  $r, q_1$  and  $q_2$  are positive constants. For this work we will use the following values for them:

$$q_1 = 100, q_2 = 50, r = 1$$

Control variables are deviation of power angle, rotor relative speed and generator transient EMF from their steady state initial values:

$$u = [x_1 \ x_2 \ x_3] = [\Delta\delta \ \omega \ \Delta E'_q] \quad (17)$$

In order to obtain a first order controller for the system, the Taylor's Series expansion of  $J^*$  of order 2 is needed. For this, we will use the procedure discussed before. So the first order presentation of power system (1)-(3) is:

$$\begin{cases} \frac{dx_1}{dt} = x_2 \\ \frac{dx_2}{dt} = c_3 x_{3o} \cos(x_{1o}) x_1 + c_2 x_2 + c_3 \sin(x_{1o}) x_3 \\ \frac{dx_3}{dt} = -c_5 \sin(x_{1o}) x_1 + c_4 x_3 + c_6 u \end{cases} \quad (18)$$

where

$$x_1 = \delta - \delta_o, x_2 = \omega - \omega_o, x_3 = E'_q - E'_{qo}$$

The definition of the variables  $x_{1o}, x_{2o}, x_{3o}$  and also the constants  $c_1, \dots, c_5$  are as follows:

$$\begin{aligned} c_1 &= \frac{\omega_o P_{mo}}{2H} & c_2 &= -\frac{D}{2H} & c_3 &= -\frac{\omega_o V_s}{2H x'_{ds}} \\ c_4 &= -\frac{x_{ds}}{T'_{do} x'_{ds}} & c_5 &= \frac{V_s (x_d - x'_d)}{T'_{do} x'_{ds}} & c_6 &= \frac{k_c}{T'_{do}} \\ x_{1o} &= \delta_o & x_{2o} &= \omega_o & x_{3o} &= E'_{qo} \end{aligned}$$

So, the Hamilton function is:

$$H = 50x_1^2 + 25x_2^2 + .5u^2 + J^*_{x_1} x_2 + J^*_{x_2} (c_3 x_{3o} \cos(x_{1o}) x_1 + c_2 x_2 + c_3 \sin(x_{1o}) x_3) + J^*_{x_3} (-c_5 \sin(x_{1o}) x_1 + c_4 x_3 + c_6 u) \quad (19)$$

To find  $u^*$  that minimize  $H$ , we should obtain derivative of  $H$  at  $u$  and get it zero:

$$\frac{\partial H}{\partial u} = 0 \Rightarrow u^* = -\frac{c_6}{r} J^*_{x_3} \rightarrow \frac{\partial^2 H}{\partial u^2} = r > 0 \quad (20)$$

Driving (19) by  $u^*$ , obtain the minimum value of  $H$ . Expressing of  $J^*$ , that is second ordered polynomial of control variables, can be as follows:

$$J^* = k_1 x_1^2 + k_2 x_2^2 + k_3 x_3^2 + k_4 x_1 x_2 + k_5 x_1 x_3 + k_6 x_2 x_3 \quad (21)$$

In (21) the coefficient of  $k_1, \dots, k_6$  are unknown. In this case  $J^*_x$  is:

$$\begin{aligned} J^*_{x_1} &= 2k_1 x_1 + k_4 x_2 + k_5 x_3 \\ J^*_{x_2} &= 2k_2 x_2 + k_4 x_1 + k_6 x_3 \\ J^*_{x_3} &= 2k_3 x_3 + k_5 x_1 + k_6 x_2 \end{aligned} \quad (22)$$

So,

$$u^* = -\frac{c_6}{r} (2k_3 x_3 + k_5 x_1 + k_6 x_2) \quad (23)$$

By substituting  $u^*$  and  $J^*_x$  from (23) and (22) in (19) and sorting the coefficients, after setting equivalent to zero all terms, we will have the following nonlinear equations:

$$\begin{cases} -q_1 + \frac{k_4^2}{r} c_6^2 - 2k_3 c_3 \cos(x_{1o}) + 2k_4 c_5 \sin(x_{1o}) = 0 \\ -q_2 + \frac{k_6^2}{r} c_6^2 - 2k_3 - 4k_2 c_2 = 0 \\ -2k_2 c_3 x_{3o} \cos(x_{1o}) - 2k_1 - k_3 c_2 + \frac{k_6 k_4}{r} c_6^2 + k_6 c_5 \sin(x_{1o}) = 0 \\ -k_6 c_3 x_{3o} \cos(x_{1o}) - 2k_2 c_3 x_{3o} \cos(x_{1o}) - 2k_1 - k_3 c_2 + \frac{k_6 k_4}{r} c_6^2 + k_6 c_5 \sin(x_{1o}) = 0 \\ -2k_5 c_4 + 2\frac{k_5^2}{r} c_6^2 - k_6 c_3 \sin(x_{1o}) = 0 \\ -k_4 + 2\frac{k_6 k_5}{r} c_6^2 - k_6 c_4 - k_6 c_2 - 2k_2 c_3 \sin(x_{1o}) = 0 \end{cases} \quad (24)$$

As it is seen, equations (24) are nonlinear equations of unknown parameters  $k_1, \dots, k_6$  that can be solved using symbolic software like MAPLE or MATHEMATICA. In this work we have used MAPLE.5 for that.

### 3.2 Nonlinear Feedback Voltage Controller Design

To design a voltage controller we define new control variables as follows:

$$x_1 = V_t - V_{to}, x_2 = \omega - \omega_o, x_3 = E'_q - E'_{qo}$$

Using (11) we can rewrite the system equations (1)-(3) as follows:

$$\begin{aligned} \dot{V}_t &= \left[ \left( \frac{k_c}{T_{do}} u_f - \frac{x_{ds}}{T_{do} x'_{ds}} E'_q + \frac{V_s (x_d - x'_d)}{T_{do} x'_{ds}} A_1 \right) (E'_q x_s^2 + x_s x'_d V_s A_1) \right. \\ &\quad \left. - x_s x'_d E'_q V_s (\omega - \omega_o) A_2 \right] / V_t x'_{ds} \\ \dot{\omega} &= -\frac{D}{2H} (\omega - \omega_o) + \frac{\omega_o}{2H} P_m - \frac{\omega_o V_s}{2H x'_{ds}} E'_q A_2 \\ \dot{E}'_q &= \frac{k_c}{T_{do}} u_f - \frac{x_{ds}}{T_{do} x'_{ds}} E'_q + \frac{V_s (x_d - x'_d)}{T_{do} x'_{ds}} A_1 \end{aligned} \quad (25)$$

where

$$\begin{aligned} A_1 &= \frac{V_t^2 x'^2_{ds} - E'^2_q x_s^2 - V_s^2 x'^2_d}{2x_s x'_d E'_q V_s} \\ A_2 &= (1 - A_1^2)^{\frac{1}{2}} \end{aligned} \quad (26)$$

We will design the voltage controller with the same procedure that the transient controller designed. In here the cost function is:

$$J = \int_0^\infty \left( \frac{1}{2} q_1 (V_t - V_{to})^2 + \frac{1}{2} q_2 \omega^2 + \frac{1}{2} r u^2 \right) \quad (27)$$

For this case we consider the following parameters:

$$q_1 = 100, q_2 = 50, r = 1$$

### 4 Global Control of Power Systems

As stated in the introduction, our global control objective is to achieve good control performance over of the wide range for the anticipated operating region. Specifically, we have the following control task:

Global Control Problem: design a smooth nonlinear feedback control law for the excitation system (1)-(3) such that the closed-loop power system is transiently stable when subjected

to a fault, and restores the steady prefault voltage value after the disturbance.

We used the following trapezoid-shaped like membership functions which are able to indicate different operating stages [4]:

$$\mu_\delta = \left(1 - \frac{1}{1 + \exp(-120(z - .08))}\right). \quad (28)$$

$$\mu_v = 1 - \mu_\delta \quad (29)$$

where

$$z = \sqrt{a_1 \omega^2 + a_2 (\Delta V_t)^2} \quad (30)$$

Where  $a_1$ ,  $a_2$  are positive design constants providing appropriate scaling which can be chosen according to the different sensitivity requirement of power frequency and voltage.

Membership function (28) is plotted in Fig. 2. It can be seen that  $\mu_\delta(z)$  gets its dominant value when  $z$  is far away from the origin, which corresponds to the transient period: on the other hand,  $\mu_v(z)$  does so when  $z$  is close to the origin, which indicates the post-transient period. Since the membership function values are determined by the directly measurable variables,  $\omega$  and  $\Delta V_t$ , the fault sequence need not to be known beforehand.

Therefore, the whole operating region is partitioned into the following two subspaces by the membership functions, where  $S_1$  indicates the transient period and  $S_2$  indicates the post transient period.

$$S_1 = \{(w, \Delta V_t) | \mu_v \leq \mu_\delta\} \quad (31)$$

$$S_2 = \{(w, \Delta V_t) | \mu_\delta \leq \mu_v\}$$

The characteristic function of each subspace  $S_l (l=1,2)$  defined by:

$$\tau_l = \begin{cases} 1 & z \in S_l \\ 0 & \text{otherwise} \end{cases} \quad (32)$$

Note that  $\tau_1 + \tau_2 = 1$ .

It should be pointed out that  $\omega$  and  $\Delta V_t$  are chosen as the index variables in (30) since they sufficiently represent operating status for the problem transient stability and voltage regulation. If the problem under consideration is voltage stability, reactive power could be included in the index. Similarly the proposed method can be extended to other power system control issues. The chosen membership functions have a trapezoid-shaped like which is well known in fuzzy control to separate operating conditions. The system performance is not sensitive to different parameters  $a_1$  and  $a_2$ .

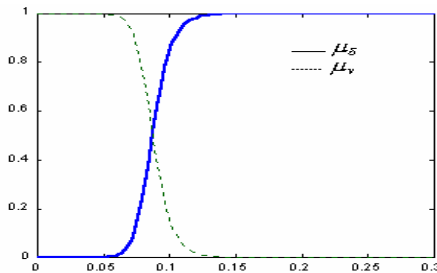


Fig. 2: Membership function,  $\mu_\delta$  "——",  $\mu_v$  "-----".

The global control law is the average of the individual control laws, weighted by the operating region membership functions, i.e., the input  $u_f$  takes the form:

$$u_f = \mu_\delta u_{f1} + \mu_v u_{f2} \quad (33)$$

where  $u_{f1}$  is the transient controller and  $u_{f2}$  is the voltage controller.

The global control (33) has the following interpretation: in the transient period system states are far away from the equilibrium, the primary control is to regulate them to enter a neighborhood of the equilibrium without large oscillations; then in the post-transient period around the equilibrium the voltage need to be tuned to reach the prefault level. The membership function plays the role of appropriate weighting and smooth interpolation of the two controllers. One of the appealing abilities of the method is that the operating status is automatically distinguished by the membership functions that are functions of directly measurable variables. The form of the control law (33) is such that a smooth transfer between the local controllers is automatically achieved.

## 5 Simulation Results

The prefault conditions of the system are:

$$x_d = 1.863 \quad x'_d = 0.257 \quad x_T = 0.127$$

$$x_L = 0.4853 \quad x_{ad} = 1.712 \quad H = 4$$

$$D = 5 \quad \omega_o = 314.159 \quad k_c = 1$$

$$x_{ds} = 2.23265 \quad x'_{ds} = 0.62665 \quad x_s = 0.36965$$

$$T'_{do} = 6.9$$

$$\delta_o = 34.2^\circ \quad P_{mo} = .8 \quad V_{to} = 1$$

$$c_1 = 31.4159 \quad c_2 = -0.625$$

$$c_3 = -44.05994393 \quad c_4 = -0.5163527708$$

$$c_5 = 0.2611444928 \quad c_6 = 0.1449275362$$

The fault is a symmetrical three-phase short circuit fault with its sequences described as:

*Case 1.* A permanent fault occurs:

Stage 1: The system is in the prefault steady states;

Stage 2: A fault occurs at  $t = t_o$ ;

Stage 3: The fault is removed at  $t = t_o + t_f$ ;

Stage 4: The system is in the post fault state;

*Case 2.* A permanent fault occurs and the mechanical input power increases:

Stage 1: The system is in the prefault steady states;

Stage 2: A fault occurs at  $t = t_o$ ;

Stage 3: The fault is removed at  $t = t_o + t_f$ ;

Stage 4: The mechanical input power of the generator has a 20% step increase at  $t = t_1$ ;

Stage 5: The system is in the post fault state;

*Case 3.* A temporary fault and a permanent fault occur:

Stage 1: The system is in the prefault steady states;

Stage 2: A fault occurs at  $t = t_o$ ;

Stage 3: The fault is removed at  $t = t_o + t_f$ ;

Stage 4: The system conditions restore to the prefault conditions at  $t = t_2$ ;

Stage 5: Another fault occurs on the same situation at  $t = t_3$ ;

Stage 6: The fault is removed at  $t = t_3 + t_f$ ;

Stage 7: The system is in the post fault state;

We choose the following parameters in the simulation:

$$t_o = 0.1s, t_1 = 1s, t_2 = .8s, t_3 = 1.2s$$

we will chose for  $t_f$  different values.

In this section we consider the fault of Case 1 with  $t_f = 0.1$ sec. In Fig. 3 the power system responses using transient controllers are shown. As it can be seen all of the controllers can stabilize the system, but in the post-transient period the level of the generator terminal voltage is not its prefault value and is higher than it. So, we should design a controller due to regulate the generator terminal voltage in addition to stabilize the system. In order to further investigating the performance of each controller, we use the energy of vibrations as a criterion to evaluate the quality of control action. The energy of power angle vibration can be defined as follows:

$$E_\delta(t) = \int_0^t (\delta(\tau) - \delta_o)^2 d\tau \quad (34)$$

With the above criterion we can compare the ability of controllers to damp the vibrations. This comparison is shown in Fig. 4. It can be seen that among these controllers the third order controller has the best performance in controlling the energy of the vibrations.

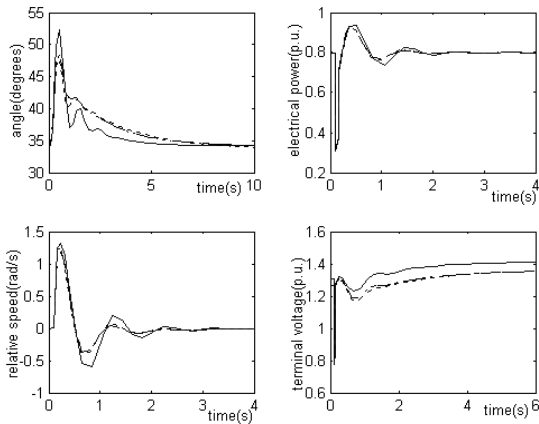


Fig. 3: Power system response for the fault of Case 1.

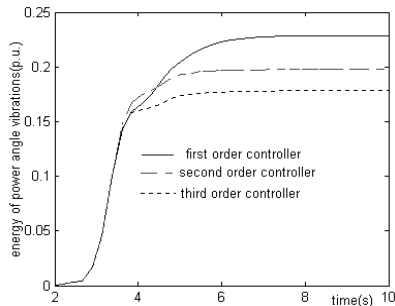


Fig. 4: The energy of power angle vibrations using different order controllers.

In order to investigate the effect of the nonlinear feedback controllers to the system response, consider the fault of Case 2 that mechanical input power increases after a determined time. In Fig. 5 the closed loop response with the second and third order controllers are shown. As it can be seen in this case the system with the second order controller is unstable (dotted line), but the third order controller can stabilize the system (solid line). For further investigation consider a fault of Case 3 on the infinite bus with  $t_f = 0.1$ sec. The power system response for this case using different order controllers is shown in Fig. 6 We can see that the first order controller can not stabilize the system (dotted line), but the system with the second order controller is stable (solid line).

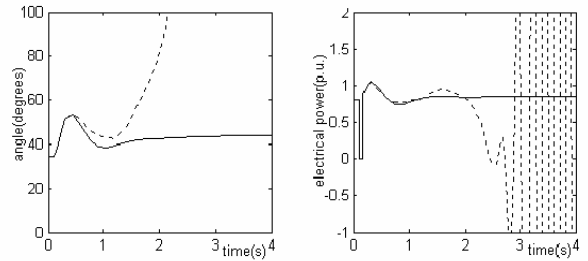


Fig. 5: Power system response for the fault of Case 2 with  $t_f = 0.1$ sec using the second order “-----” and the third order “\_\_\_\_\_” controllers.

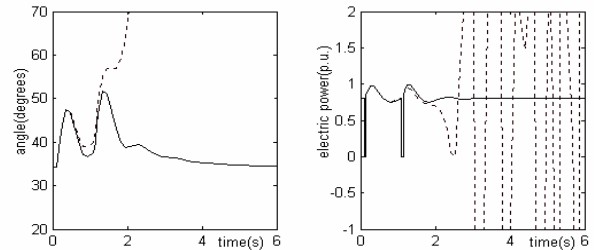


Fig. 6: Power system response for the fault of Case 3 with  $t_f = 0.1$ sec using the first order “-----” and the third order “\_\_\_\_\_” controllers.

In this section we will use the global control law obtained before. In Fig. 7 the closed-loop system response using the first order global control law for the fault of Case 1 is shown. It is seen that the closed-loop system is stable and the post-fault generator terminal voltage regulated to its prefault value. In Fig. 8 the power system response for the fault of Case 2 with the third order global controller is shown, in this case using the first order controller or the second order one can not stabilize the system. As it can be seen the closed-loop system with this control action has good transient and post-fault performance. In Fig. 9 the power system response for the fault of Case 3 with the second order global control law is shown, in the previous section it has been shown that using the first order controller can not stabilize the closed-loop system. We can see that, this controller can stabilize the system and regulate the voltage to the prefault value of that in this situation.



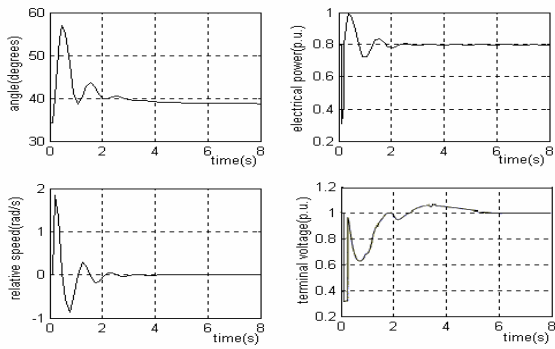


Fig. 7: Power system response for the fault of Case 1 on the transmission line using the first order global controller.

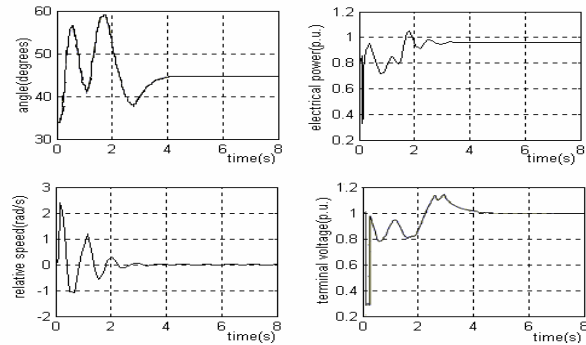


Fig. 8: Power system response for the fault of Case 2 on the transmission line using the third order global controller.

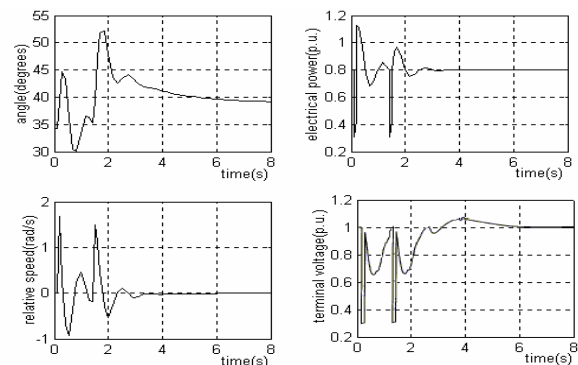


Fig. 9: Power system response for the fault of Case 3 on the transmission line using the second order global controller.

## 6 Conclusion

In this paper the design of global nonlinear feedback controller for power systems based on the approximate solution of the *HJB* equation was presented. First a transient controller designed by solving the *HJB* equation of the system to enhance the system's stability. We formed the *HJB* equation to obtain the optimal input that minimized the cost function. The corresponding index was defined as the sum of weighted squares of power angle, rotor relative speed and transient EMF. The *HJB* equation is a partial differential equation for which an analytical solution is difficult to be found. So, we used an approximate method using Taylor's Series expansion. The procedure of obtaining the first order approximation of the solution was presented with details.

Although, using designed transient controller could stabilize the system, the final generator terminal voltage was increased, which is not desirable for power systems. So, we

designed the voltage controllers with the same procedure. In this case the cost function was defined as the sum of weighted squares of terminal voltage, rotor relative speed and transient EMF. To achieve the global control action that guarantee both transient stability and voltage regulation we used a membership function that was a function of measurable parameters of the system (rotor relative speed and generator terminal voltage). Then the global control law was formed as a weighted summation of both transient controller and voltage controller with respect to their membership functions. In this way in the transient period the transient controller is more effective, and in the post-transient period the voltage controller is the dominant controller. So, we could design the global control law for power systems using approximate solution of the *HJB* equation. Simulations results showed the effectiveness of the proposed approach.

## References

- [1] K.S. Al-Olimat , A.A. Ghadaky,"Intelligent parallel controller based on fuzzy linguistic rules" South Eastern Symposium On System Theory-2001,pro,33<sup>rd</sup>,pp.359-383.
- [2] G.E. Boakarim,S. Wang,J.H. Chow,C.N. Taranto,N. Martins"A comparison of classical , robust and decentralized control design for multiple power systems stabilizers" IEEE Trans. On Power Systems,vol.15,No.4,pp.1287-1792,2000.
- [3] G.P. Chen,O.P. O.P. Malik,Y.H. Qin,G.Y. Xu"AN adaptive power system stabilizer based on the self optimizing pole-shifting control strategy"IEEE Trans.,1993,EC-8(4),pp.639-645.
- [4] Y. Guo,D.J. Hill,Y. Wang "Global transient stability and voltage regulation for power systems" IEEE Trans. On Power Systems,vol.16,No.4,pp.678-688,2001.
- [5] Y. Guo,D.J. Hill,Y. Wang "Nonlinear decentralized control of large-scale power systems" Automatica,36(2000),pp.1275-1289.
- [6] P. Kundur " Power system stability and control" In the EORI Power System Engineering Series,New York,Mc Grow Hill,1994.
- [7] M. Lown,E. Swidenbank,B.W. Hogg,R. Smith "Adaptive fuzzy reference model control of multi-machine power systems" Fuzzy Sets And Systems,102(1999),pp.59-70.
- [8] Q. Lu,Y. Zhang"Nonlinear stabilizing control off multimachine systems" IEEE Trans. On Power Systems,vol.4,No.1,pp.236-241,1989.
- [9] Q. Lu,S. Mei,W. Hu,F.F. Wu,T. Shen "Nonlinear decentralized disturbance attenuation excitation control via new recursive design for multi machine power systems" IEEE Trans. On Power Systems,vol.16,No.4,pp.279-286,2001.
- [10] S.M. Sadeghzadeh,M. Ehsan,N. Hadj Said,R. Feuillet "Transient stability improment of multimachine power systems using on-line fuzzy control of SEMS" Control Engineering Practice,7(1999),pp.531-536.
- [11] Y. L. Tan,Y. Wang "Nonlinear excitation and phase shifter controller for transient stability enhancement of power systems using adaptive control law"INT.J. Electrical Power And Energy Systems,vol.18,No.6,pp.397-403,1996.
- [12] R. Uinter "Optimal Control" Birkhaus,Boston,2000.
- [13] Y. Wang,D.J. Hill,R.H. Middleton,L. Gao "Transient stability enhancement and voltage regulation for power systems" IEEE Trans. On Power Systems,vol.8,pp.620-627,1993.
- [14] J. Yang,X.Y. Zhou "Stochastic controls-Hamiltonian systems and HJB equations" Springer,New York,1999.
- [15] M.J. Yazdanpanah,E. Fallah Chulabi "Nonlinear PSS design based on solving HJB equation for a single machine system(In Farsi)" published at J. of Amir-Kabir University.
- [16] M.J. Yazdanpanah,K. Khorasani,P.V. Patel "On the estimate of the domain of validity of nonlinear  $H_\infty$  control"INT. J. Control,vol.72,No.12,pp.1097-1105,1999.
- [17] M.J. Yazdanpanah,K. Khorasani,P.V. Patel "Uncertainly compensation for a flexible link manipulator using nonlinear  $H_\infty$  control"INT. J. Control,vol.6,pp.753-771,1998.
- [18] C. Zhu,R. Zhou,Y. Wang "A new nonlinear voltage controller for power systems" INT.J. Electrical Power And Energy Systems,vol.19,No.1,pp.19-27,1996.