GEOMETRIC ENERGY BASED ANALYSIS AND CONTROLLER DESIGN OF HYDRAULIC ACTUATORS APPLIED IN ROLLING MILLS

G. Grabmair*, K. Schlacher*,, A. Kugi+**

* Christian Doppler Laboratory for Automatic Control of Mechatronic Systems in Steel Industries, J.K.University, Altenbergerstr. 69, 4040 Linz, *Austria*.
Phone:++43-732-2468-9730 Fax:++43-732-2468-9734 Email:gernot.grabmair@jku.at
** Department of Automatic Control, J.K.University, *Austria*.
+ Chair of System Theory and Automatic Control, Saarland University, D-66123 Saarbrücken, *Germany*.

Keywords: Nonlinear control, differential geometry, energy based, Hamiltonian systems, hydraulic actuators

Abstract

The two main goals of this contribution are to demonstrate the integration of hydraulic actuator systems with PCH systems and to discuss a modification of the well known input-output linearization in connection with PCH systems. We will show that this approach is closely related to the existence of so called Casimir functions in the PCH context. Finally, industrial measurements demonstrating the implemented control law will be presented.

1 Introduction

By now, energy based modelling and control concepts are known for their astonishing robustness properties. Port Controlled Hamiltonian (PCH) systems are closely related to that idea. They are restricted to plants with special mathematical structure properties of the dynamic equations which describe the behavior of these plants. Apart from that, many nonlinear control concepts used in practical applications are based on well established methods, namely the exact input-state or input-output linearization (see, e.g., [3]). One disadvantage of these methods for industrial applications is that one has to measure the whole state. Therefore, it is of great interest from the control point of view, to integrate the energy based description with the input-output linearization under a restriction of the measurable state variables.

This contribution is organized as follows. We will start with a brief summary of the Hamiltonian and the PCH framework, followed by the incorporation of hydraulic systems. Towards this goal we have to deal with some thermodynamics. Finally, we will show that the applicability of a constrained version of the well known input-output linearization to PCH systems coincides, to some extent, with the existence of so called Casimir functions.

2 The PCH Framework

Here and further, we will denote by ∂_q the partial derivative with respect to q, by $\partial_x f$ the gradient of f and by $\dot{q}$ the total derivative of q with respect to time. A *(generalized) Hamiltonian system* can be written in local coordinates as

$$\dot{x} = J(x)\, \partial_x H(x)^T \tag{1}$$

with $x \in \mathbb{R}^n$, the Hamiltonian $H(x)$ and the skew-symmetric *structure matrix* $J(x)$. If the Jacobi identity is fulfilled and $\operatorname{rank}(J(x)) = 2m$ then a consequence of Darboux' theorem guarantees (see [5]) the existence of *canonical coordinates* $x = (q, p, z)$, $q, p \in \mathbb{R}^m$, $z \in \mathbb{R}^{n-2m}$ such that

$$\begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & I_m & 0 \\ -I_m & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \partial_x H(x)^T \tag{2}$$

with the $m \times m$ identity matrix I_m, at least locally. The quantities $z = C(x)$ of (2) are called *distinguished (Casimir) functions* and they fulfill

$$\partial_x C(x)\,(J(x)) = 0\ .$$

Trivially, these functions are invariants independent of the Hamiltonian H – they remain constant along the solutions of (2) and of course of (1). These distinguished functions can be used to simplify the analysis of (1) because they lead to a reduction in order of the ODE(ordinary differential equation)-system.

The notion of a (generalized) Hamiltonian system can be – with an increase of the range of included physical systems – extended to a so-called (generalized) Port Controlled Hamiltonian (PCH) System with Dissipation (see [8]). Let $u \in \mathbb{R}^l$, $y \in \left(\mathbb{R}^l\right)^*$ denote the the l-dimensional input and output, then the structure of a PCH system is given by

$$\begin{aligned} \dot{x} &= \left(J\left(x\right) - R\left(x\right)\right)\partial_x H\left(x\right)^T + G\left(x\right)u \\ y &= \partial_x H\left(x\right) G\left(x\right) \ . \end{aligned} \quad (3)$$

The distinguished (Casimir) functions $C\left(x\right)$ are generalized to solutions of $\partial_x C\left(x\right)\left(J\left(x\right) - R\left(x\right)\right) = 0$. In the case of a Hamiltonian equal to the internally stored energy, the skew symmetric matrix J is related to the lossless internal flow of energy, whereas one describes the dissipative effects with the positive semi-definite matrix R. $\langle y, u\rangle$ is the power supplied to the plant via its ports. Therefore, the output y is often called the output collocated to u. Besides the simple damping injection there are more elaborate controller designs like IDA-PBC, which leads to state feedback laws such that the matrices R, J or the Hamiltonian H take desired values R_d, J_d, H_d. More challenging is, whether one can solve the previous problem by help of the restricted control law $u = u\left(\hat{y}\right)$, $\hat{y} = \hat{y}\left(x\right)$ such that u depends on the measurable quantity $\hat{y} \in \mathbb{R}^k$, $k < l$ only. It has turned out that several controller design problems for hydraulic systems belong to this class.

2.1 PCH Structure of Hydraulic Systems

As a descriptive example we will use an arrangement of a single acting piston and a linear mass-spring-damper system as load. Towards this end, we will have to deal with systems which have an interchange of mass with the environment.

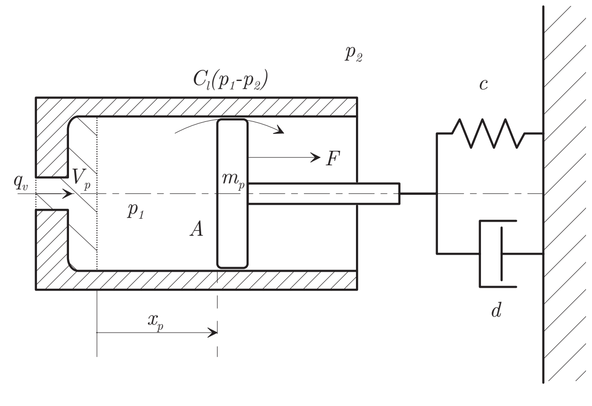

Figure 1: The hydraulic actuator system.

2.1.1 Thermodynamic Interlude

As already mentioned, PCH systems strongly depend on an accurate treatment of the energy storage inside and the power flows into and out of a plant. It is not really astonishing that equilibrium thermodynamics can contribute a lot to an energy based modelling approach, because its foundation is mainly based on the energy balance principle. The authors want to point out that even if equilibrium thermodynamics is a quite axiomatic branch of physics there does exist a well developed mathematical calculus – namely the calculus of Pfaffian forms – which handles the numerous relations of differentials in a quite appealing systematic manner. The complete description of a thermodynamic system is given by its *fundamental equation* which relates the internal energy U with the other state variables. Usually, the system is given in form of an *equation of state*, e.g., the ideal gas, the van der Waals gas or the isentropic fluid. In the following, we will deal with *open thermodynamic systems* and *reversible processes*. Let us use (U, T, S, p, V, h, m) – the internal energy, the temperature, the entropy, the pressure, the volume, a function h which has to be determined and the mass of the fluid – as the thermodynamical coordinates. Therefore, the first law of thermodynamics has to be written with the independent variables S, V and m as

$$\omega = \mathrm{d}U - T\mathrm{d}S + p\mathrm{d}V - h\mathrm{d}m \ .[1] \quad (4)$$

The additional term $h\mathrm{d}m$ can be considered as an *ansatz* for the change of energy due to the in- and outward fluid flows. The positive sign has been assigned to the inward flow. We will confine ourselves to an isentropic process without any external heat supply $T\mathrm{d}S$. That is the reason, why (4) reduces to

$$\mathrm{d}U + p\mathrm{d}V - h\mathrm{d}m \quad (5)$$

in only two independent variables. One often uses specific quantities in the field of isentropic fluid mechanics, namely the set of coordinates $\left(\rho, m, u_s\left(\rho\right), p\left(\rho\right)\right)$ – with the independent coordinates ρ (the mass density of the fluid) and m. The symbol u_s denotes the specific internal energy. These quantities are given by the transformation $\left(U = u_s m, V = \rho^{-1}m\right)$. One obtains from

[1] One can simply set this and the following forms equal to zero in order to obtain the common notation. The exterior forms are understood as a generator of a contact ideal over a seven dimensional contact bundle $(\mathcal{E}, \pi, \mathcal{B})$ with the projection $\pi : \mathcal{E} \rightarrow \mathcal{B}$, $(S, V, m, U, T, p, h) \longmapsto (S, V, m)$, the base manifold $\mathcal{B}$ and the total manifold $\mathcal{E}$ (see [1]). A valid section $\psi : (S, V, m) \longmapsto (S, V, U, T, p, h)$ has to fulfill $\psi^*\omega = 0$ in order to describe a thermodynamic system.

(5)

$$\left(u_s + \frac{p}{\rho} - h\right) \mathrm{d}m + m\left(\frac{\partial u_s}{\partial \rho} - \frac{p}{\rho^2}\right) \mathrm{d}\rho \,.$$

From this, one derives the relations (compare, e.g., with [2])

$$h = u_s + \frac{p}{\rho}, \qquad p = \rho^2 \frac{\partial u_s}{\partial \rho} \,. \tag{6}$$

The first equation can be used as a definition of the specific (free) enthalpy h of the fluid flow. The specific internal energy of the supplied fluid is therefore equal to the specific internal energy of the portion of fluid under consideration. The second one is a condition for the fundamental equation of the material. For fluids one usually assumes that the – possibly implicit – constitutive equation (the equation of state in the thermodynamic sense) $f(p, \rho, T) = 0$ is independent of T. Therefore, one can use the isothermal definition of the bulk modulus E (see Merrit 1967)

$$\frac{\partial p}{\partial \rho} = \frac{E}{\rho} \tag{7}$$

as the equation of state. Further, hydraulic fluids meet the assumption of a constant bulk modulus E quite well as long as the pressure changes are reasonable small. Equation (7) can now be integrated in closed form

$$p = \ln\left(\frac{\rho}{\rho_0}\right) E + p_0 \,,$$

where p_0 is the reference pressure and ρ_0 the density of the fluid at the reference pressure. Because $\mathrm{d}U$ is a total differential, its integral is path independent. Therefore, one can integrate first along $m = \mathrm{const}$ and then along $\rho = \mathrm{const}$. That results finally in

$$\begin{aligned}\Delta U = U_1 - U_0 &= -\int_{\rho_0}^{\rho_1} p \mathrm{d}\left(\frac{m_0}{\rho}\right) + \int_{m_0}^{m_1} h(\rho_1) \mathrm{d}m \\ &= \frac{m_0}{\rho_1}\left(E \ln\frac{\rho_0}{\rho_1} + (E + p_0)\left(\frac{\rho_1}{\rho_0} - 1\right)\right) + \\ &+ \int_{m_0}^{m_1} h \mathrm{d}m \,.\end{aligned} \tag{8}$$

The symbol Δ expresses the fact that only changes of internal energy are considered. One rediscovers for $m_1 = m_0$ the *fundamental equation* of a closed thermodynamic system. Obviously, one can proceed in the same way if E is replaced by a function of the pressure p and therefore of ρ, but possibly, the evaluation of the integrals becomes more involved.

In hydraulic systems the fluid flow is characterized usually by the volume flow q_v. Due to the balance of mass, the total amount of fluid m inside the chamber fulfills $\dot{m} = \rho q_v$ and one obtains the ODE for the coordinate m as

$$\dot{m} = \rho q_v = \frac{m}{V} q_v \,. \tag{9}$$

The integral in (8) results with $\mathrm{d}m = \dot{m}\mathrm{d}t$ in

$$\int_{m_0}^{m_1} h \mathrm{d}m = \int_{t_0}^{t_1} h \rho q_v \mathrm{d}t \,.$$

This is nothing else than the time integral of the power of the fluid flow $P_h = \dot{m}h$. In the following, we will drop the index 1 for the reason of simplicity.

2.1.2 PCH Structure of the SAP

In order to study a practical example let us consider the hydraulic actuator depicted in Fig. 1, with a single acting piston connected to a spring-mass-damper system. The surrounding pressure p_2 is assumed to be constant and for the sake of simplicity we neglect the leakage effects ($C_l = 0$). Additionally, the hydraulic servo valve is assumed to be servo compensated such that the valve flow q_v acts as the one-dimensional input. This is no principal restriction because the compensation of the valve characteristic – a state dependent static nonlinearity – is a straight forward task. It is worth mentioning that the amount of fluid which is currently inside the cylinder with geometric volume $V = V_p + Ax_p$ (with V_p the volume of the pipe and A the effective area of the piston) is indicated by m. It has turned out to be convenient to distinguish between two sets of state variables namely canonical coordinates $x^T = [x_p, p_p, m]$ with the momentum $p_p = m_p v_p$ and velocity $v_p = \dot{x}_p$, and sensor coordinates $\breve{x}^T = [x_p, v_p, p_1]$. The naming of the first set will become clear immediately and the naming of the second is due to its (partially) simple availability by the deployment of standard sensor equipment.

In order to derive the mathematical model for the system under consideration, the first principle required additionally is the balance of momentum

$$m_p \dot{v}_p = -F_c - F_d + Ap_1 \tag{10}$$

with the piston and load mass m_p, the forces F_c, F_d caused by the spring and the damper and the pressure in the cylinder head chamber p_1. For the sake of simplicity, we neglect the inertia of the oil in (10) as already stated. Finally, we choose the relations $F_c = c(x_p - x_{p,0})$ and $F_d = dv$ for the mass damper system with the coefficients c, d and $x_{p,0} = \tilde{x}_{p,0} - c^{-1}Ap_2$.

Following the approach of the first section, we derive the mathematical model with coordinates $[x_p, p_p, m]^T$ in the form (3) from

$$\begin{aligned} G^T &= \left[0, 0, m\left(V_p + Ax_p\right)^{-1}\right] \\ 2H &= 2\Delta H + m_p v_p^2 + c\left(x_p - x_{p,0}\right)^2 \\ y &= \Delta H\left(V_p + Ax_p\right) + p_1 \end{aligned} \quad (11)$$

and

$$J = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (12)$$

with the internal energy of a closed thermodynamic system with $V_p + Ax_0 = m\left(\rho_0\right)^{-1}$

$$\begin{aligned} \Delta H = &\left(V_p + Ax_p\right) E \ln \frac{\left(V_p + Ax_p\right)}{\left(V_p + Ax_0\right)} + \\ &A\left(E + p_0\right)\left(x_0 - x_p\right) \end{aligned}$$

as well as from the relations (8, 9, 10). One can easily discover the form of (2) in (12) and therefore $[x_p, p_p, m]^T$ are already canonical coordinates. Further, m is a distinguished variable and it remains obviously constant along the trajectories of the system as long as q_v vanishes. This is clear from the point of view of an engineer because as long as no fluid flow is supplied to the hydraulic chamber the amount of fluid inside itself has to remain constant. This fact demonstrates how physical knowledge can help to find canonical coordinates. Apart from that, the Hamiltonian can be used as an energy based Lyapunov function candidate in order to investigate the stability of the uncontrolled plant.

3 Controller Design

It is clear that we have from now on the full realm of systematic passivity or better PCH based control system design techniques at ours disposal (see [6]). Nevertheless, we will focus in this paper on the similarities between the *input-output linearization with constrained measurements (with constraints)* as introduced in [7] applied to a subclass of PCH systems and the energy based controller design. As a central feature we are able to restrict the necessary measurements for the implementation of the designed control law on a set of easily available ones. E.g. we prohibit the occurrence of the generalized momenta p in the control law. Further, we can even prevent certain parameters from appearing in the input-output linearizing control law by an virtual extension of the state vector.

3.1 Input-Output Linearization with Constraints for PCH Systems

Here we can only sketch an extension of the well known input-output linearization method (see, e.g., [3]) presented in [7], where the multi-input case is treated too. The key idea of this approach is to construct the determining equations of all feedback laws or, equivalently, to find all output functions such that the input-output map is linear and the feedback law depends on a fixed set of measurable variables only. Towards this end, one has to consider two sets of distributions – in the differential geometric sense – Δ_i and Λ_i and their involutive closure $\overline{\Delta_i + \Lambda_i}$. The Δ_i's give the relations for an input-output linearizing output h and the Λ_i's give the restrictions for h in order that the undesired variables do not appear in the control law.

We consider a single-input-single-output (SISO) PCH system in canonical coordinates $x = (q, p, z)$ with $G(x)^T = \left[0, 0, g^{\tilde{z}}\right]$ with regular $g^{\tilde{z}}$ and $\dim(z) = 1$. The key assumption here is how the input u enters into the system. Now, we are looking for outputs h for which the input-output linearization results in a state feedback law independent of the generalized momenta p. We introduce the indices $\tilde{q} = 1, \ldots, m$, $\tilde{p} = m+1, \ldots, 2m$, $\tilde{z} = 2m+1, \ldots, n$ and $i = 1, \ldots, n$ and e.g. $\operatorname{span}\{\partial_{\tilde{q}}\}$ is used as an abbreviation for $\operatorname{span}\{\partial_1, \partial_2, \ldots, \partial_m\}$ where ∂_j, $j = 1, \ldots, m$ indicates the vectorfield in the j-direction.

The application of the mentioned algorithm (see [7]) results with the additional assumption $\partial_{\tilde{p}}\left(g^{\tilde{z}} u - R^{\tilde{z}i}\partial_i H\right) = 0$ and the extended vector field $f_e = f^i \partial_i + \partial_t = \partial_t + \left(J^{\tilde{q}\tilde{p}}\partial_{\tilde{p}} H - R^{\tilde{q}i}\partial_i H\right)\partial_{\tilde{q}} - \left(J^{\tilde{p}\tilde{q}}\partial_{\tilde{q}} H + R^{\tilde{p}i}\partial_i H\right)\partial_{\tilde{p}} + \left(g^{\tilde{z}} u - R^{\tilde{z}i}\partial_i H\right)\partial_{\tilde{z}}$, for short $f_e = \partial_t + \lambda^{\tilde{q}}\partial_{\tilde{q}} - \lambda^{\tilde{p}}\partial_{\tilde{p}} + \lambda^{\tilde{z}}\partial_{\tilde{z}}$, in the distribution sets

$$\Delta_1 = \operatorname{span}\{\partial_u\}, \quad \Delta_2 = \operatorname{span}\{\partial_u, \partial_{\tilde{z}}\}$$

and

$$\begin{aligned} &\Lambda_0 = \operatorname{span}\{\partial_t, \partial_{\tilde{p}}\}, \quad \Lambda_1 = \operatorname{span}\left\{\partial_t, \partial_{\tilde{p}}, \partial_{\tilde{p}}\lambda^{\tilde{q}}\partial_{\tilde{q}}\right\}, \\ &\Lambda_2 = \operatorname{span}\left\{\partial_t, \partial_{\tilde{p}}, \partial_{\tilde{p}}\lambda^{\tilde{q}}\partial_{\tilde{q}}, \partial_{\tilde{p}}\lambda^{\tilde{q}}\partial_{\tilde{q}}\lambda^{\tilde{q}}\partial_{\tilde{q}}, \partial_{\tilde{p}}\lambda^{\tilde{q}}\partial_{\tilde{q}}\lambda^{\tilde{z}}\partial_{\tilde{z}}\right\}. \end{aligned}$$

If $\operatorname{rank}\left(\partial_{\tilde{p}}\lambda^{\tilde{q}}\right) = m$ then $\Lambda_1 = \operatorname{span}\{\partial_t, \partial_{\tilde{p}}, \partial_{\tilde{q}}\}$ and $\Lambda_2 = \operatorname{span}\left\{\partial_t, \partial_{\tilde{p}}, \partial_{\tilde{q}}, \partial_{\tilde{p}}\lambda^{\tilde{q}}\partial_{\tilde{q}}\lambda^{\tilde{z}}\partial_{\tilde{z}}\right\}$. Clearly $\mathrm{d}h\left(\overline{\Delta_2 + \Lambda_2}\right)$ admits only constant outputs h. Therefore, the maximal possible rank $r_{\max} = 1$ and $\overline{\Delta_1 + \Lambda_1} = \operatorname{span}\{\partial_t, \partial_u, \partial_{\tilde{p}}, \partial_{\tilde{q}}\}$, which gives $\partial_t h = 0$, $\partial_u h = 0$ and

$$\mathrm{d}h\left(\operatorname{span}\{\partial_{\tilde{p}}, \partial_{\tilde{q}}\}\right) = 0\,. \quad (13)$$

The first two conditions mean that h is independent of the time and that its relative degree has to be at least one. Condition (13) coincides with the definition of a distinguished function C – written in canonical coordinates – as $\partial_x C J(x) = 0$, what demands that C is a function in the invariants z only or again $\mathrm{d}C(\mathrm{span}\{\partial_{\tilde{p}}, \partial_{\tilde{q}}\}) = 0$. In other words the input-output linearization with constrained measurements applied to PCH systems is closely related to the existence of Casimir functions as long as the maximal possible relative degree is one. In this case therefore, we can give a clear physically motivated interpretation of the output h. If $\mathrm{rank}\left(\partial_{\tilde{p}}\lambda^{\tilde{q}}\right) < m$ one has to investigate the further distributions and a higher relative degree than one becomes possible. In this case the connection between constrained input-output linearization and Casimir function is not longer obvious in the general case.

3.2 Application to the Hydraulic Ram – Position Control without Velocity Measurement

We directly obtain $\overline{\Delta_1 + \Lambda_1} = \mathrm{span}\left\{\partial_t, \partial_{q_v}, \partial_{p_p}, \partial_{x_p}\right\}$ and $\overline{\Delta_2 + \Lambda_2} = \mathrm{span}\left\{\partial_t, \partial_{q_v}, \partial_{p_p}, \partial_{x_p}, \partial_m\right\}$ for the SAP. Clearly the maximal possible rank is $r_{\max} = 1$. Therefore the IO-linearizing output has to be of the form $h = f(m)$. Of course, this output has relative degree one and the resulting system can be stabilized with standard linear techniques. Or, one can perform an IDA-PBC controller design with the desired Hamiltonian

$$H_d = \frac{c\left(x_p - x_{p,e}\right)^2}{2} + \frac{p_p^2}{2m_p} + \Gamma\left(E \ln\left(\frac{m}{m_e}\right)\right)^2 + EA\int_0^{x_p - x_{p,e}} \ln\left(\frac{V_p + A\left(x_{p,e} + x\right)}{V_p + Ax_{p,e}}\right)\mathrm{d}x$$

with the prescribed equilibrium $\left[x_{p,e}, 0, m_e\right]^T$ and the desired structure and dissipation matrices in the corresponding coordinates

$$J_d = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & \beta \\ 0 & -\beta & 0 \end{bmatrix}, \; R_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & d & -\beta \\ 0 & -\beta & \frac{m^2\alpha}{2E^2\Gamma} \end{bmatrix}$$

with $\beta = \frac{mA}{4E\Gamma}$ and $\Gamma = 1\mathrm{m}^5\mathrm{N}^{-1}$ which leads to the control law (already in sensor coordinates)

$$q_v = -\alpha\left(V_p + Ax_p\right)\left(\ln\left(\frac{V_p + Ax_p}{V_p + Ax_{p,e}}\right) + \frac{1}{E}\left(p_1 - p_{1,e}\right)\right),$$

which stabilizes the closed loop asymptotically. The desired Hamiltonian H_d is derived from the original H mainly by the neglect of a cross term in x_p and m. We want to point out, that the same control law can be obtained be the input-output linearization with constraints. This connection to PCH systems gives an explanation for the robustness of the control law.

Nevertheless, this control law first of all needs the measurement of the pressure p_1 and secondly the stiffness c of the spring must be known. To overcome this partial drawback, a closer look on the system unveils that the system dynamics can be decomposed into the behavior along a *slow* and a *fast manifold*. A further inspection shows that $p_1 - p_{1,e}$ does only appear as a small perturbation term due to the smallness of αE^{-1}. Again, this term can be given a nice physical meaning. Namely, it is nothing else than a laminar leakage flow towards a chamber at constant pressure $p_{1,e}$ with a position dependent leakage coefficient $\alpha E^{-1}\left(V_p + Ax_p\right)$. Nevertheless, the challenging question is if this term can be neglected without destroying the excellent behavior of the IDA-PBC controller. An answer concerning the stability has already been given in [4] where the reduced version of the control law

$$q_v = -\alpha\left(V_p + Ax_p\right)\ln\left(\frac{V_p + Ax_p}{V_p + Ax_{p,e}}\right) \tag{14}$$

had been developed by pure differential geometric methods and where a proof of stability for the reduced control law can be found, too.

This control law has been already implemented many times by our industrial partner in steel manufacturing plants as the inner most position control loop. There, two rolls have to be pressed one against the other in order to achieve the desired strip thickness. Due to the heuristic character of the so called roll force models, which try to establish a relation between the roll force and some other quantities (e.g., entry and exit thicknesses of the strip, stiffness of the strip, strip tensions, etc.), the load model for the actuator is hardly available in a simple manner. This load, which the hydraulic piston is acting on, can be modelled with sufficient accuracy by a linear spring and a constant force, because for a high strip quality the quantities of the roll force model have to be held approximately constant.

Some measurements taken from a steel rolling mill at a plant of the Bethlehem Steel Corporation (Bethlehem, Pennsylvania) can be seen in (fig 2). The working rolls of the mill stand are in touch and rotate. One can see one upward and one downward step in position with different external loads. The position signals are nearly

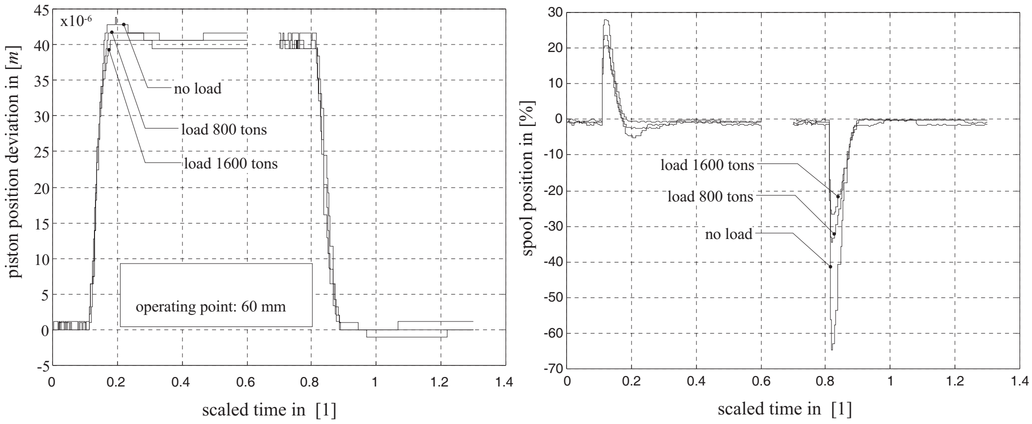

Figure 2: Step responses of the nonlinear controller at different loads.

equal for different loads, whereas the valve spool position shows, that the nonlinearity is compensated well by the control law. The noise due to quantization is about $1.1\ \mu\mathrm{m}$.

Remark: If the hydraulic valve is not already servo compensated, this has to be achieved by an extension of the control law and for this task the measurement of the pressure becomes unavoidable.

4 Conclusions

We have shown how hydraulic actuators fit into the language of PCH systems. Even if that has been demonstrated on quite a simple example, it should have become clear how this has to be done for more general hydraulic equipment. The connection of a refinement of the well known input-output linearization with the Casimir functions of the PCH perspective has been established in this paper. This method has been used to motivate a position control law for a single acting hydraulic ram. Finally we have been able to present some measurements from a real plant – a steel rolling mill. We do not want to withhold the fact, that the control law has already become a quasi standard for the control of hydraulic actuators for our industrial partner.

The field for future research is quite large. On the one hand the possibilities of the structural properties do not seem to be exploited by now. E.g., the structural properties of the achieved description can bee used for the stability analysis of the plant. Therefore, the mathematical background, which has been only touched in this contribution, is worth further reading. On the other hand the presented treatment can be applied to hydraulic valves, drives and so on.

5 Acknowledgements

We gratefully acknowledge the financial support from the *Voest Alpine Industrieanlagenbau* (VAI) within the scope of the *Christian Doppler Laboratory (CDL) for Automatic Control of Mechatronic Systems in Steel Industries*. This work has been partially supported by the European sponsored project GeoPlex with reference code IST-2001-34166.

References

[1] William L. Burke. *Applied differential geometry*. Cambridge, 1997.

[2] A. J. Chorin and Jerold E. Marsden. *A mathematical introduction to fluid mechanics*. Springer, 1990.

[3] Alberto Isidori. *Nonlinear control systems*. Springer, 3rd edition, 2001.

[4] Andreas Kugi. *Non-linear control based on physical models*. Springer, 2001.

[5] Peter J. Olver. *Applications of lie groups to differential equations*. Springer, 2nd edition, 1993.

[6] Romeo Ortega, Arjan van der Schaft, Bernhard Maschke, and Gerardo Escobar. Interconnection and damping assignment passivity-based control of port-controlled hamiltonian systems. *Automatica*, 38(4), 2002.

[7] Kurt Schlacher, Andreas Kugi, and Rainer Novak. Input to output linearization with constrained measurements. In *5th IFAC Symposium NOLCOS'01*, pages 471 – 476, St. Petersburg, Russia, 2001.

[8] Arjan van der Schaft. *L2-gain and passivity techniques in nonlinear control*. Springer, 1999.