EXPLICIT MODEL PREDICTIVE CONTROL OF GAS-LIQUID SEPARATION PLANT

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Abstract

Exact or approximate solutions to constrained linear model predictive control problems can be pre-computed off-line in an explicit form as a piecewise linear state feedback defined on a polyhedral partition of the state space. This leads to efficient real-time computations and admits implementation at high sampling frequencies in real-time systems with high reliability and low software complexity. In this paper, an explicit model predictive controller for a gas-liquid separation plant is designed and experimentally tested.

1 Introduction

Model predictive control (MPC) has become the accepted methodology to solve complex control problems related to process industries. It allows the design of multi-input multi-output (MIMO) control systems that minimize a quadratic performance index in the presence of input and output constraints imposed on the system. Recently, several methods for explicit solution of MPC problems have been developed. The main motivation behind explicit MPC is that an explicit state feedback law avoids the need for real-time optimization, and is therefore potentially useful for applications where MPC has not traditionally been used. In addition to embedded system applications, this technology is also suitable for certain small-scale process control applications. In particular, low-level control of fairly simple unit processes is typically implemented using conventional PI/PID control rather than MPC. Some reasons are the need for high reliability, fast processing and low software complexity, as these are typically executed in a highly reliable computer environment. On the other hand, explicit MPC is ideally suited for such problems as it offers the benefits of constrained MIMO control with low complexity and high reliability for such small-scale problems.

In [1] it was recognized that the constrained linear MPC problem can be posed as a multi-parametric quadratic program (mp-QP), when the state is viewed as a parameter to the problem. It was shown that the solution (the control input) has an explicit representation as a piecewise linear (PWL) state feedback on a polyhedral partition of the state space, see also [2,8,11,12], and they develop an mp-QP algorithm to compute this function.

In [6,9,10], algorithms that determine an approximate explicit PWL state feedback solution by imposing an orthogonal search tree structure on the partition, have been developed. They lead to more efficient real-time computations. The present paper considers the application of one of these algorithms to the design of an explicit model predictive controller for a laboratory gas-liquid separation plant, including experimental evaluation.

2 Approximate approach to explicit MPC

2.1 Explicit MPC and exact mp-QP

Formulating a linear MPC problem as an mp-QP is briefly described below, see [1] for further details. Consider the linear system:

$$\boldsymbol{x}(t+1) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \qquad (1)$$

where $\boldsymbol{x}(t) \in \boldsymbol{R}^n$ is the state variable, $\boldsymbol{u}(t) \in \boldsymbol{R}^m$ is the input variable, $\boldsymbol{A} \in \boldsymbol{R}^{n\times n}$, $\boldsymbol{B} \in \boldsymbol{R}^{n\times m}$ and $(\boldsymbol{A},\boldsymbol{B})$ is a controllable pair. For the current $\boldsymbol{x}(t)$, MPC solves the optimization problem:

$$V^*(\boldsymbol{x}(t)) = \min_{U \equiv \{u_t^T, \ldots, u_{t+N-1}^T\}^T} J(U, \boldsymbol{x}(t)) \qquad (2)$$

subject to:

$$\begin{aligned}
&\boldsymbol{y}_{\min} \le \boldsymbol{y}_{t+k|t} \le \boldsymbol{y}_{\max},\ k = 1, \ldots, N \\
&\boldsymbol{u}_{\min} \le \boldsymbol{u}_{t+k} \le \boldsymbol{u}_{\max},\ k = 0, 1, \ldots, N-1 \\
&\boldsymbol{x}_{t|t} = \boldsymbol{x}(t) \\
&\boldsymbol{x}_{t+k+1|t} = \boldsymbol{A}\boldsymbol{x}_{t+k|t} + \boldsymbol{B}\boldsymbol{u}_{t+k},\ k \ge 0 \\
&\boldsymbol{y}_{t+k|t} = \boldsymbol{C}\boldsymbol{x}_{t+k|t},\ k \ge 0
\end{aligned} \qquad (3)$$

with the cost function given by:

$$J(U, \boldsymbol{x}(t)) = \sum_{k=0}^{N-1}\left[\boldsymbol{x}_{t+k|t}^T \boldsymbol{Q}\boldsymbol{x}_{t+k|t} + \boldsymbol{u}_{t+k}^T \boldsymbol{R}\boldsymbol{u}_{t+k}\right] + \boldsymbol{x}_{t+N|t}^T \boldsymbol{P}\boldsymbol{x}_{t+N|t} \qquad (4)$$

and symmetric $R > 0$, $Q \geq 0$, $P > 0$. The final cost matrix P may be taken as the solution of the algebraic Riccati equation. With the assumption that no constraints are active for $k \geq N$ this corresponds to an infinite horizon LQ criterion, and the MPC is stabilizing [5]. It is shown in [1] that by substituting:

$$x_{t+k|t} = A^k x(t) + \sum_{j=0}^{k-1} A^j B u_{t+k-1-j} \qquad (5)$$

in the optimization problem defined by (2), (3) and (4), this can be rewritten in the form:

$$V^*(x(t)) = \frac{1}{2} x^T(t) Y x(t) + \min_U \left\{ \frac{1}{2} U^T H U + x^T(t) F U \right\} \qquad (6)$$
$$\text{subject to } GU \leq W + Ex(t)$$

The column vector $U \equiv \{u_t^T, \dots, u_{t+N-1}^T\}^T \in R^s$, $s = m \cdot N$, is the optimization vector, $H = H^T > 0$, and H, F, Y, G, W, E are easily obtained from Q, R, and (2) – (5) (since only the optimizer U is needed, the term involving Y is usually removed from (6)). Further, by defining $z \equiv U + H^{-1} F^T x(t)$, $z \in R^s$, the optimization problem (6) is transformed into the following equivalent problem [1]:

$$V_z^*(x) = \min_z \frac{1}{2} z^T H z \qquad (7)$$
$$\text{subject to } Gz \leq W + Sx$$

where $V_z^*(x) = V^*(x) - \frac{1}{2} x^T (Y - FH^{-1} F^T) x$ and $S \equiv E + GH^{-1} F^T$. The vector x is the current state, which can be treated as a vector of parameters. The number of inequalities is denoted q and the number of free variables is $s = m \cdot N$. Then $H \in R^{s \times s}$, $G \in R^{q \times s}$, $W \in R^{q \times 1}$, $S \in R^{q \times n}$.

It has been shown that the optimization problem (7) is a multi-parametric quadratic program (mp-QP) and its solution can be found in an explicit form $z^* = z^*(x)$ [1]:

Theorem 1 *Consider the mp-QP (7) and suppose $H > 0$. The solution $z^*(x)$ (and $U^*(x)$) is a continuous PWL function of x defined over a polyhedral partition of the parameter space, and $V_z^*(x)$ is a convex (and therefore continuous) piecewise quadratic function.*

Algorithms for iteratively constructing a polyhedral partition of the state space and computing the PWL solution are given in [1,2,8,11,12].

2.2 Approximate mp-QP algorithm

In [9,10], an algorithm that determines an approximate explicit PWL state feedback solution of possible lower complexity is developed. The idea is to require that the state space partition is represented as a binary search tree (quad-tree [4], cf. Figure 1), i.e. to consist of orthogonal hypercubes organized in a hierarchical data-structure. This allows extremely fast real-time search. When searching the tree, only n scalar comparisons are required at each level.

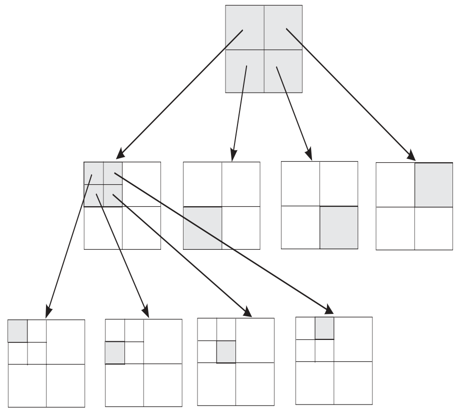

Figure 1: Quad-tree partition in a 2-dimensional state space.

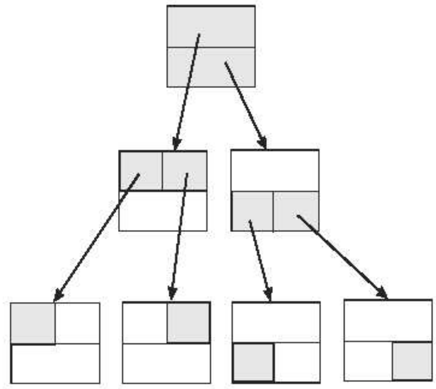

Figure 2: *k – d* tree partition in a 2-dimensional state space.

The main idea of the approximate mp-QP algorithm is to compute the solution of the problem (7) at the 2^n vertices of a considered hypercube X_0 by solving up to 2^n QPs. Based on these solutions, a feasible local linear approximation $\hat{z}_0(x)$ to the PWL optimal solution $z^*(x)$, valid in the whole hypercube X_0, is computed by using the following result [3]:

Lemma 1. *Consider the bounded polyhedron $X_0 \subseteq X_f$ with vertices $\{v_1, v_2, \dots, v_M\}$ (here X_f is the feasible set: $X_f = \{x(t) \in R^n \mid \exists\, U \text{ satisfying } (3)\}$). If K_0 and g_0 solve the QP:*

$$\min_{K_0, g_0} \sum_{i=1}^{M} \left(z^*(v_i) - K_0 v_i - g_0\right)^T H \left(z^*(v_i) - K_0 v_i - g_0\right) \qquad (8)$$

subject to:

$$G(K_0 v_i + g_0) \leq S v_i + W, \quad i \in \{1, 2, \dots, M\} \qquad (9)$$

then the least squares approximation $\hat{z}_0(x) = K_0 x + g_0$ is feasible for the mp-QP (7) *for all $x \in X_0$.*

If the maximal cost function error in the hypercube X_0 is smaller than some prescribed tolerance, no further refinement of X_0 is needed. Otherwise, X_0 is partitioned into 2^n equal-sized hypercubes and the procedure described above is repeated for each of these.

In this paper, an improved version of the approximate mp-QP algorithm is used which is based on a *k - d* tree partition of the state space (Figure 2) as a more flexible and powerful alternative to the generalized quad-tree (Figure 1). With the *k - d* tree [4], a hyper-rectangle is split into two equal parts and thus only ***one*** scalar comparison is required at each level when searching the tree. Also, the *k - d* tree allows the incorporation of heuristic rules that split the hyper-rectangle at the axis along which the change of error is maximal (before splitting). It has been shown in [6] that the use of such heuristics reduces the complexity of the partition significantly.

The complexity is further reduced by implementing control input trajectory parameterization as it is described in [13]. The idea is to use an input trajectory parameterization with less degrees of freedom in order to reduce the dimension of the optimization problem. The most common approach is to pre-determine the time-instants at which the control input $\boldsymbol{u_i}$ is allowed to change (input blocking):

$$\boldsymbol{N}^{\boldsymbol{u_i}}_{\boldsymbol{change}} = [1 \;\; \boldsymbol{N}^{\boldsymbol{u_i}}_1 \;\; \boldsymbol{N}^{\boldsymbol{u_i}}_2 \;\; \dots \;\; \boldsymbol{N}^{\boldsymbol{u_i}}_{\boldsymbol{l}}] \tag{10}$$

The described approximate mp-QP algorithm is guaranteed to terminate with an approximate feasible solution that satisfies a specified maximum allowed error in the cost function [10].

3 Model of the gas-liquid separation plant

We consider a sub-process within a semi-industrial installation which is used for reduction of NO_x in effluent gasses and technological waste water treatment by means of neutralisation with CO_2 contained in flue gasses [14]. The role of the separation unit (Figure 3 from [14]) is to capture flue gasses under low pressure from effluent channels by means of water flow and to carry them over under high enough pressure to the downstream (neutralisation) stage.

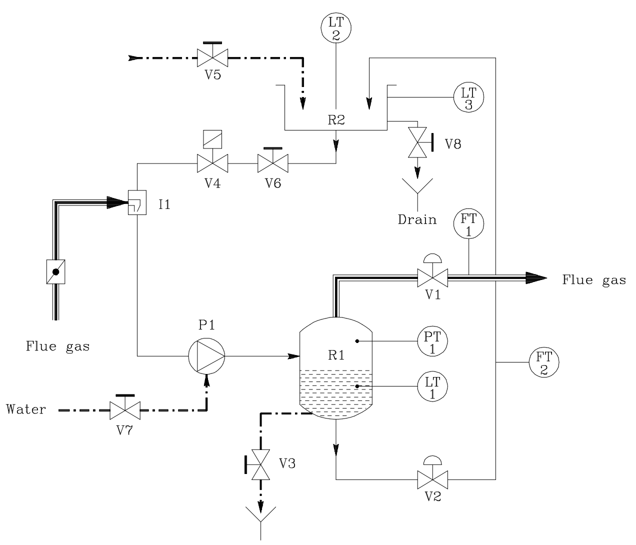

Figure 3: Process scheme of the separation unit.

The flue gasses coming from the effluent channels are "pooled" by the water flow into the water circulation pipe through the injector $\boldsymbol{I_1}$. The water flow is generated by the pump $\boldsymbol{P_1}$ (water ring). The speed of the pump is kept constant. The pump feeds the mixture of water and gas into the separator $\boldsymbol{R_1}$ where gas is separated from water. Hence the accumulated gas in $\boldsymbol{R_1}$ forms a sort of "gas cushion" with increased internal pressure. Owing to this pressure, flue gas is blown out from $\boldsymbol{R_1}$ into the next neutralisation unit. On the other side the "cushion" forces water to circulate back to the reservoir $\boldsymbol{R_2}$. The quantity of water in the circuit is constant. If for some reason additional water is needed, the water supply path through the valve $\boldsymbol{V_5}$ is utilised.

The complete non-linear model of the gas-liquid separator is given in [14]. A linearized model can be obtained from the existing non-linear model:

$$\begin{bmatrix} \Delta\dot{\boldsymbol{p}}_1 \\ \Delta\dot{\boldsymbol{h}}_1 \end{bmatrix} = \boldsymbol{A_c} \begin{bmatrix} \Delta\boldsymbol{p}_1 \\ \Delta\boldsymbol{h}_1 \end{bmatrix} + \boldsymbol{B_c} \begin{bmatrix} \Delta\boldsymbol{v}_1 \\ \Delta\boldsymbol{v}_2 \end{bmatrix} \tag{11}$$

where $\Delta\boldsymbol{p_1}$ and $\Delta\boldsymbol{h_1}$ denote the change of separator gas pressure $\boldsymbol{p_1}$ and liquid level $\boldsymbol{h_1}$ from the steady-state values ($\Delta\boldsymbol{p}_1 = \boldsymbol{p}_1 - \boldsymbol{p}_{1s}, \Delta\boldsymbol{h}_1 = \boldsymbol{h}_1 - \boldsymbol{h}_{1s}$), and $\Delta\boldsymbol{v_1}$ and $\Delta\boldsymbol{v_2}$ are respectively the changes in the positions $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ of the two valves ($\Delta\boldsymbol{v}_1 = \boldsymbol{v}_1 - \boldsymbol{v}_{1s}, \Delta\boldsymbol{v}_2 = \boldsymbol{v}_2 - \boldsymbol{v}_{2s}$). The linear model corresponds to the following steady state:

$$\boldsymbol{p}_{1s} = 0.5\,\boldsymbol{bar}\ ,\ \boldsymbol{h}_{1s} = 1.4\,\boldsymbol{m}\ ,\ \boldsymbol{v}_{1s} = 0.4152\ ,\ \boldsymbol{v}_{2s} = 0.7462 \tag{12}$$

and the way to compute the elements of the matrices $\boldsymbol{A_c}$ and $\boldsymbol{B_c}$ is given in details in [14]. From the continuous-time model, a linear discrete-time model corresponding to sampling interval $\boldsymbol{T_s} = 1\boldsymbol{s}$ is obtained, with the following state and control matrices:

$$\boldsymbol{A} = \begin{bmatrix} 0.9719 & -0.0001 \\ -0.0006 & 0.9999 \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} -0.0832 & -0.0041 \\ 0 & -0.0023 \end{bmatrix} \tag{13}$$

The state variables are $\boldsymbol{x}_1 = \Delta\boldsymbol{p}_1\,[\boldsymbol{bar}]$ and $\boldsymbol{x}_2 = \Delta\boldsymbol{h}_1\,[\boldsymbol{m}]$, and the control variables are $\boldsymbol{u}_1 = \Delta\boldsymbol{v}_1$ and $\boldsymbol{u}_2 = \Delta\boldsymbol{v}_2$. The following input and rate constraints are imposed on the valve positions $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$:

$$0 \le \boldsymbol{v}_1 \le 1,\ 0 \le \boldsymbol{v}_2 \le 0.8625 \tag{14}$$

$$-0.33 \le \dot{\boldsymbol{v}}_1 \le 0.66,\ -0.33 \le \dot{\boldsymbol{v}}_2 \le 0.66 \tag{15}$$

which by taking into account the steady state values (12) are represented as the following constraints on the control inputs $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$:

$$\begin{gathered} -0.4152 \le \boldsymbol{u}_1(\boldsymbol{t}+\boldsymbol{k}) \le 0.5848 \\ -0.7462 \le \boldsymbol{u}_2(\boldsymbol{t}+\boldsymbol{k}) \le 0.1163 \\ \boldsymbol{k} = 0,1,\dots,\boldsymbol{N}-1 \end{gathered} \tag{16}$$

$$\begin{gathered} -0.33\boldsymbol{T_s} \le \boldsymbol{u}_1(\boldsymbol{t}+\boldsymbol{k}) - \boldsymbol{u}_1(\boldsymbol{t}+\boldsymbol{k}-1) \le 0.66\boldsymbol{T_s} \\ -0.33\boldsymbol{T_s} \le \boldsymbol{u}_2(\boldsymbol{t}+\boldsymbol{k}) - \boldsymbol{u}_2(\boldsymbol{t}+\boldsymbol{k}-1) \le 0.66\boldsymbol{T_s} \\ \boldsymbol{k} = 0,1,\dots,\boldsymbol{N}-1 \end{gathered} \tag{17}$$

In order to avoid the steady state offset of the model predictive controller, two more states are added to the model (13), which take into account the integral error:

$$x_3(t+1) = x_3(t) + T_s x_1(t) \,,\; x_4(t+1) = x_4(t) + T_s x_2(t) \quad (18)$$

Thus, the linear discrete-time model of the gas-liquid separation unit becomes:

$$A = \begin{bmatrix} 0.9719 & -0.0001 & 0 & 0 \\ -0.0006 & 0.9999 & 0 & 0 \\ T_s & 0 & 1 & 0 \\ 0 & T_s & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -0.0832 & -0.0041 \\ 0 & -0.0023 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (19)$$

4 Real-time performance of explicit model predictive controller for the gas-liquid separation plant

4.1 Design of explicit model predictive controller for the gas-liquid separation plant

The approximate mp-QP approach [10] is applied to design an explicit MPC controller for the gas-liquid separation plant. The MPC minimizes the cost function (4) subject to the system equation (19) and the input constraints (16). The rate constraints (17) are not taken into account during the design of the MPC controller. Instead, a rate limiter is placed at the output of the controller in its real-time implementation, that guarantees the satisfaction of the rate constraints. In (4), $\boldsymbol{P}$ is chosen as the solution of the discrete algebraic Riccati equation and the cost matrices are:

$$Q = \mathrm{diag}\{0.05,\, 100,\, 0.005,\, 0.0001\},\; R = \mathrm{diag}\{1,\, 1\} \quad (20)$$

The horizon is $N = 500$ and the time instants at which the input variables can change are:

$$N_{u_1} = [1\ 5\ 10\ 15\ 20\ 25\ 30\ 35\ 40\ 45\ 50\ 100\ 102\ \ 104\ 106\ 108\ 110\ 300\ 302\ 304\ 306\ 308\ 310] \quad (21)$$

$$N_{u_2} = [1\ 5\ 10\ 15\ 20\ 25\ 30\ 35\ 40\ 45\ 50\ 100\ 300] \quad (22)$$

which makes totally 36 optimization variables. The state space to be partitioned is 4-dimensional and is defined by $X = [-0.5, 0.5] \times [-0.2, 0.2] \times [-3, 3] \times [-10, 60]$. The size of the regions on each of the state variables is restricted to be larger than $\Delta_{x1} = 0.01$, $\Delta_{x2} = 0.004$, $\Delta_{x3} = 0.06$ and $\Delta_{x4} = 0.7$. The prescribed tolerance on the cost function approximation error is $\bar{\varepsilon} = 0.5$.

The resulting MPC controller has 2693 regions in its state space partition and 24 levels of search. With one scalar comparison required at each level of the *k-d* tree, 24 arithmetic operations are required in the worst case to determine which region the state belongs to. Totally, 40 arithmetic operations are needed in real-time to compute the two control inputs with this MPC controller (24 comparisons, 8 multiplications and 8 additions).

4.2 Real-time experiments

The real-time experiments were pursued in the environment schematically shown in Figure 4. This environment encompasses supervisory control on two levels: upper level with Factory Link SCADA system and lower procedural and basic control levels implemented in two PLCs. This is one of possible configurations of control, which can be found in industry.

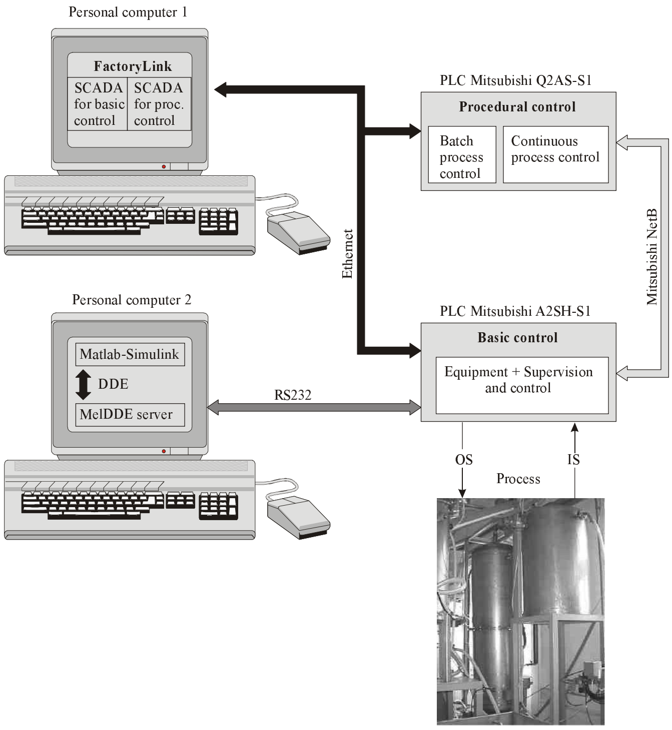

Figure 4: Scheme showing environment for control and experimentation.

User-friendly experimentation with the process plant is enabled through interface with Matlab/Simulink environment. This interface enables PLC access with Matlab/Simulink using DDE protocol via Serial Communication Link RS232 or TCP/IPv4 over Ethernet IEEE802.3. Control algorithms for experimentation can be prepared in Matlab code or as Simulink blocks and extended with functions/blocks, which access PLC. This interface also enables user-friendly data acquisition for Matlab users. In our case all control schemes were put together as Simulink blocks and tested at the plant operating points as described in the previous section.

In Figures 5 to 8, the real-time performance of the approximate explicit MPC controller, in closed-loop with the plant is shown. The set point is $p_1^* = p_{1s} = 0.5\,bar$ and $h_1^* = h_{1s} = 1.4\,m$. The experimental results (the solid line) are compared with the exact MPC trajectory (the dotted line) computed by solving the optimization problem at each time instant, based on the process model and with the simulated approximate trajectory (the dashed line). The latter two curves (the dotted curve and the dashed curve) are difficult to distinguish since they are almost matching. It can be seen from the figures that the explicit MPC controller brings the plant to the desired set-point despite of the error in the steady state process model and the transient performance is close to that of the optimal trajectory. The set point changes are handled by using the new set-point values p_1^* and h_1^* when determining the values of the state variables $x_1 = p_1 - p_1^*$ and $x_2 = h_1 - h_1^*$, where p_1 and h_1 are the measured variables.

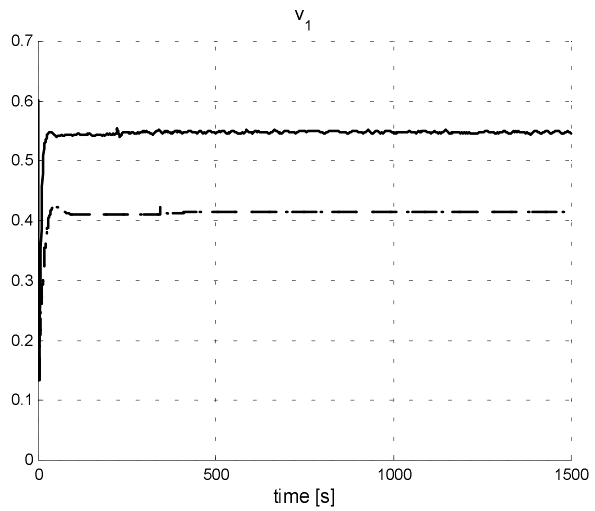

Figure 5: Trajectory of v_1 (position of valve 1).

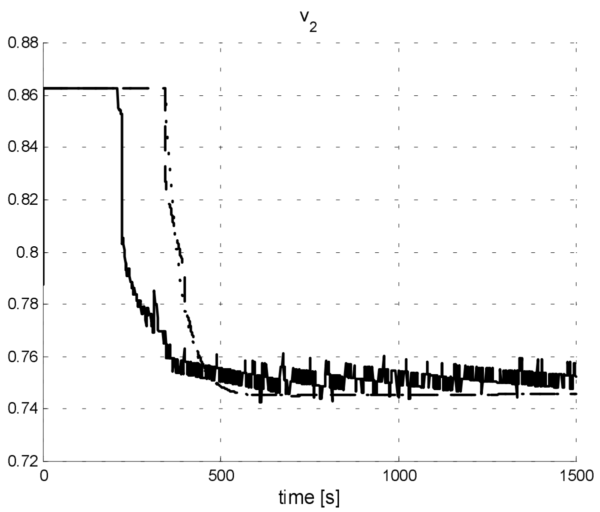

Figure 6: Trajectory of v_2 (position of valve 2).

It can be seen from Figure 6 that there is a slight chattering of the signal for the second valve. This can be explained as follows. The signal we depict has come out of analog digital converter which has 10 bit A/D converter resolution that means approximately 0.1 % of quantization noise on the range 0 to 1 which was used in our case. Afterwards this signal has gone through the controller with gain of about 10 which amplified the quantization noise to about 1% (as it can be seen in the figure). This is not a problem for the valve and actually even helps beating the small dead zone contained in valve and makes it react faster when the change of control signal occurs.

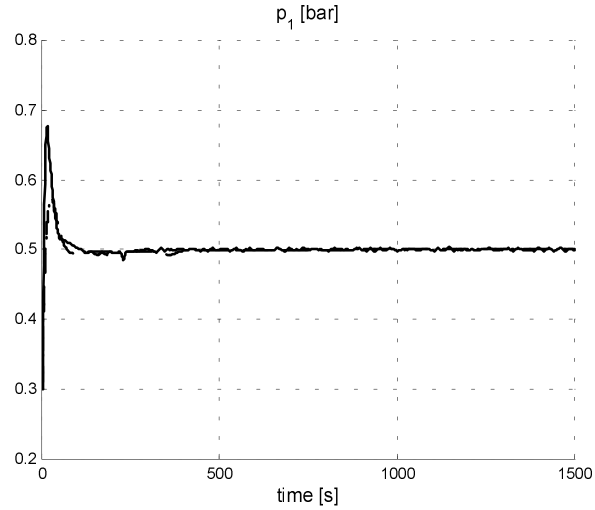

Figure 7: Trajectory of p_1 (pressure in the separator).

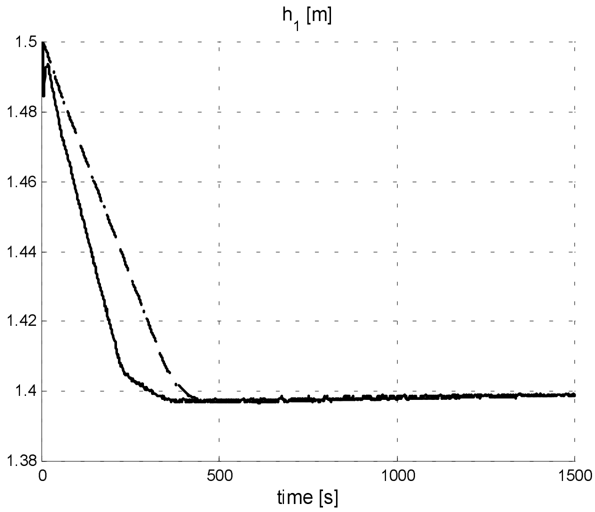

Figure 8: Trajectory of h_1 (liquid level in the separator).

5 Conclusions

An approximate explicit MPC approach has been experimentally tested on a two-input two-output laboratory gas-liquid separation plant. The approach achieves performance close to that of conventional MPC, but requires only a fraction of the real-time computational machinery, leading to fast and reliable computations. It therefore provides an interesting alternative to conventional PI/PID control for low-level process unit control. A further benefit of the approach is that the use of a nonlinear plant model does not appear to make the real-time implementation more complex [7], in strong constrast to conventional MPC. The main limitation of the approach is that the complexity of state space partition tends to increase rapidly with the state space dimension, which makes the approach unsuitable for larger problems.

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