# Fuel Consumption Reduction with a Starter-Alternator using an MPC-based Optimisation

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### Abstract

In order to minimize the fuel consumption and emissions of a soft hybrid vehicle while maintaining the SOC of the battery in a specified window, an optimising control strategy is required. An optimization developed for this purpose is introduced here. It employs a cost function that is minimized off-line to reduce the calculation power required for its implementation.

## 1. Introduction

Spurned by the increasing electrical energy consumption in modern vehicles, the implementation of the starter-alternator is being developed by the automotive industry to provide power to the coming 42 V power distibution network. It has been realized that the starter-alternator not only could be used to generate and start but also to provide boost to the combustion engine. With a starter-alternator integrated in its drivetrain, a vehicle could be classified as a soft hybrid, and through the use of an optimising strategy, it could be controlled to reduce fuel consumption and emissions as well as to keep the battery charged. In this paper, an optimisation based on model predictive control is introduced. It minimizes a cost function over a sliding time frame and could be easily implemented in a real-time application without previous knowledge of the driving cycle.

## 2. Overview of Drivetrain with Starter-Alternator

A vehicle that contains a starter-alternator integrated in its drivetrain can be treated as a parallel-hybrid-vehicle since the propulsion power could be provided by both the combustion engine and an electric machine. In drivetrains that are being developed for this technology, the starter-alternator may be placed inside the bell housing between the engine and gearbox. In this position, the starter-alternator can take the place of the engine's flywheel.

### 2.1 Modes of Operation with a Starter-Alternator

A starter-alternator is an electric machine that can function as a motor or generator, and as such it can add a positive or negative torque to the crankshaft of the combustion engine. When the electric machine is operated as a motor, it is said to boost. In this case, the electrical torque produced by the starter-alternator $\mathbf{M_E}$ has the same sign as the torque produced by the combustion engine $\mathbf{M_{CE}}$. When the electric machine is operated as a generator, it produces an additional drag on the crankshaft, and the electrical torque is negative with respect to the torque produced by the engine.

The required crankshaft torque $\mathbf{M_{req}}$ that is necessary to propel a vehicle at a certain speed and with a certain acceleration and gear is a function of the rolling friction, aerodynamic drag, climbing resistance and inertia of the vehicle. It follows that the required torque is a function of the wishes of the driver or of a prescribed driving cycle. The electrical and combustion-engine torques are added at the crankshaft, and the sum must equal the requested torque at any given time:

$$M_{req} = M_{CE} + M_E \tag{1}$$

Therefore a starter-alternator can be used to shift the operating point of the combustion engine by choosing an electrical torque for a given requested crankshaft torque. A shift in the engine torque against the backdrop of a map of specific consumption $\mathbf{b_{eff}}$ for a compression-ignition engine is illustrated in figure 1. With the electric machine functioning as a generator, the operating point of the combustion engine is shifted to higher torques with lower specific fuel consumption values. The opposite occurs when the electric machine is operated as a motor. In this case the operating point of the combustion engine sinks to a lower torque and higher specific consumption. A map of specific fuel consumption for a spark-ignition engine is similar to its compression-ignition counterpart in that $\mathbf{b_{eff}}$ sinks with rising loads, although the minimum specific consumption is not located on the maximum torque curve, as is the case with a diesel engine. The product of the specific fuel consumption and the specific calorific value of the fuel $\mathbf{C_f}$ is inversely proportional to the indicated thermal efficiency of the combustion energy $\boldsymbol{\eta_{th}}$, which defines the fraction of the chemical energy in a mass of fuel that is changed to kinetic energy [2]:

$$\eta_{th} = 1/(b_{eff} \cdot C_f) \tag{2}$$

It follows that a shift to a higher specific fuel consumption is a shift to a lower thermal efficiency, and conversely a shift to a lower specific consumption is simultaneously a shift to a higher, better efficiency. This ability to improve the operating point of the combustion engine with a starter-alternator may be exploited by an optimising control system in order to improve the global efficiency of a drive train over a driving cycle. Such a control system must take into account that every shift in operating point brings about a change in the state of charge of the battery. Therefore the state of charge must be weighed against an improvement in operating efficiency by an optimising control system.

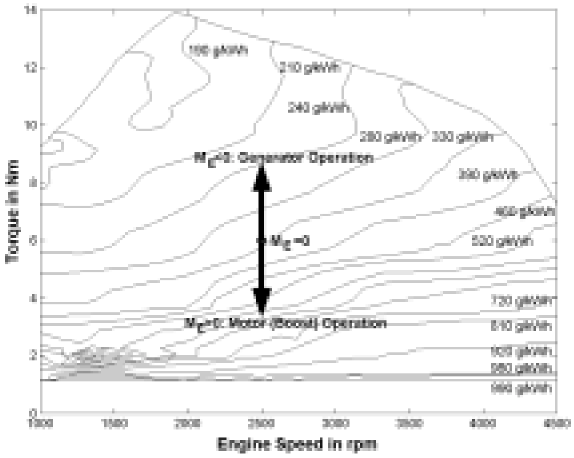


Figure 1: Shifting Operating Point of Combustion Engine with Electric Torque

## 2.2 Vehicle and Combustion Engine Model

A Ford Focus with a 1.25 liter, three cylinder spark-ignition engine and a starter-alternator was built up for testing purposes, and this vehicle is used as the basis for the simulations. The combustion engine produces a maximum mechanical power of 56 kW, while the starter-alternator can produce a maximum of 3 kW continuous mechanical power. In generator mode, it can produce a maximum of 5 kW electrical power. The starter-alternator is an asynchronous machine designed to operate on a 42 V power distribution network. In the test vehicle, a 36 V, 27 Ah AGM (absorbent glass matt technology) battery is used as an energy storage device.

The vehicle is modeled in Simulink. The dynamic behavior of the spark-ignition engine is modeled by a differential equation of the first order, and the fuel consumption is calculated using a fuel mass-flow map.

## 2.3 Starter-Alternator Model

Like the combustion engine, the dynamic behavior of the electric machine is also modeled by a differential equation of the first order. However, its time constant is approximately one hundred times faster than that of the combustion engine. The starter-alternator changes electrical to mechanical power as a motor and mechanical to electrical power as a generator. The positive or negative electrical power that is generated or used by the electric machine $\mathbf{P_E}$ can be expressed as a product of the mechanical power $\mathbf{P_{Mech}}$ and an efficiency factor $\mathbf{K_E}$:

$$P_E = K_E \cdot P_{Mech} \tag{3}$$

The efficiency factor describes the total efficiency of the electric machine and its drive circuit (inverter) and is a function of the rotational speed and mechanical torque of the device. For the operation as a generator, the electrical power produced by the machine is defined positive and can be expressed as follows:

$$P_E = \eta_{E,gen} \cdot P_{Mech}, \quad K_E = \eta_{E,gen} \quad and \quad K_E \leq 1 \tag{4}$$

Similarly, the electrical power used when the electric machine is operated as a motor is defined as being negative and can be expressed as follows:

$$P_E = \frac{1}{\eta_{E,mot}} \cdot P_{E,mech}, \quad K_E = \frac{1}{\eta_{E,mot}} \quad and \quad K_E \geq 1 \tag{5}$$

In the simulation, the efficiencies $\boldsymbol{\eta_{E,mot}}$ and $\boldsymbol{\eta_{E,gen}}$ are obtained from maps.

## 2.4 Battery Model

Electrical power charging a battery is transformed into chemical energy and stored. When a battery is discharged, the stored chemical energy is transformed back into electrical energy.

By every transformation there occurs an energy loss that is a function of the internal ohmic resistance of the battery and the square of the charging or discharging current [5]. A schematic diagram of an external electrical power source or sink connected to a battery is illustrated in figure 2.

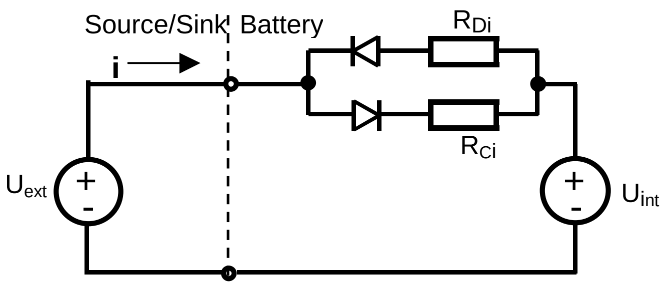


Figure 2: Schematic Diagram of Battery and external Voltage Source or Sink

Analog to the electric power, the current **i** is defined as positive when it is charging and negative when it is discharging. The internal ohmic resistance during charging is represented in the figure by $\mathbf{R_{Ci}}$, and $\mathbf{R_{Di}}$ represents the internal ohmic resistance during discharging. The battery efficiency is defined as the useable power that is stored or discharged divided by the sum of this power and the power dissipated in the internal battery resistance [5]. In the case that the battery is being charged, the battery efficiency $\boldsymbol{\eta_{BC}}$ can be formulated as follows:

$$\eta_{BC} = \left(1 + \frac{R_{Ci} \cdot i}{U_{\text{int}}}\right)^{-1}, \quad i \geq 0 \tag{6}$$

The battery efficiency during a discharge can be expressed as follows:

$$\eta_{BD} = 1 + \frac{R_{Di} \cdot i}{U_{\text{int}}}, \qquad i < 0 \qquad (7)$$

As in the case of the electric machine, the battery efficiency can be expressed by an efficiency factor describing the efficiency of an energy transformation. While the efficiency factor of the electric machine describes the efficiency of the transformation between mechanical and electrical energy, the efficiency factor of the battery, $\mathbf{K_B}$, describes the efficiency of the transformation between electrical and chemical energy. The change in the energy stored in the battery **E** can be expressed with a simple first-order differential equation:

$$\dot{E} = K_B \cdot P_E \qquad (8)$$

When the battery is being charged and the input power is positive, the differential equation has the form:

$$\dot{E} = \eta_{BC} \cdot P_E, \qquad K_B = \eta_{BC} \qquad (9)$$

Conversely, when the input power is negative and the battery is being discharged, the equation has the following form:

$$\dot{E} = \frac{1}{\eta_{BD}} \cdot P_E, \qquad K_B = \frac{1}{\eta_{BD}} \qquad (10)$$

The nominal state of charge is defined to be less than the maximum charge in order to allow for the storage of  cheaply won energy such as that from regenerative braking [3]. In the optimisation, the deviation in the nominal state of charge is to be minimized. It is defined as the quotient of the energy stored in the battery $\mathbf{E_{Bat}}$ to the energy stored at the nominal state of charge $\mathbf{E_{Bat,nom}}$:

$$SOC = \frac{E_{Bat}}{E_{Bat,nom}} \cdot 100\% \qquad (11)$$

The battery model used in the simulations was created using impedance spectroscopy. With this technique, the complex impedance of the battery over a range of state of charge values and currents is determined experimentally on a test rig and used to create a parameterized equivalent circuit. With this circuit, both the ohmic and imaginary components of the battery's internal resistance are calculated and used to predict the energy losses and terminal voltage.

## 3 Derivation of an MPC-Based Optimisation

An optimisation is required that reduces the fuel consumption of a vehicle whose drivetrain includes a starter-alternator without previous knowledge of the speed and gears chosen during a drive. A further goal may be to reduce the exhaust gas emissions, and in the following derivation, the inclusion of individual exhaust gas types is included in the optimisation. Because these goals are achieved by shifting the operating point of the combustion engine with the starter-alternator, which effects the energy stored in the battery, it is necessary to balance an improvement in the engine’s performance with a change in the state of charge of the battery. The fuel consumption, exhaust gas production and state of charge could be used to define a cost function, and the goal of the optimisation can be stated as minimizing the cost function over the driving cycle.

Because the driving cycle is unknown, it is necessary to make a prognosis of the vehicle’s operation during a limited time frame. This optimisation horizon is chosen to be longer than the sampling time of the vehicle's powertrain controller, and a new optimisation over the horizon is calculated after each sampling interval. Because the optimisation is updated at a much shorter interval than the horizon length, the influences of the length of the horizon and unexpectedly changing conditions are minimized [6].

The optimisation presented here is based on the basic theory of model predictive control in which a prediction of vehicle states is calculated over a sliding horizon using a plant model and predicted inputs [1]. In the optimisation it is assumed that the vehicle speed remains constant in the time frame. This assumption works well with the NEDC test cycle, which consists of intervals of constant acceleration followed by intervals of constant speed, but for use in actual driving conditions, better results may be achieved by taking the acceleration of the vehicle into account. Simulations using optimisation horizons with several lengths were carried out, and a horizon of 80 seconds brought the best results.

### 3.1 Cost Function Components

The state of charge **SOC** can be expressed as a state variable, and the change in the state of charge with respect to the nominal value **ΔSOC** over the optimisation horizon is to be minimized. Using equations (3), (8) and (11), the change in the state of charge can be expressed as an integrated function of the rotational speed **n**, the torque and efficiency factors of the electric machine $\mathbf{M_E}$ and $\mathbf{K_E}$, the efficiency factor of the battery $\mathbf{K_B}$, and the nominal charge $\mathbf{E_{Batt,nom}}$:

$$\Delta SOC = \int_{T_0}^{T_0+T_H} \frac{K_B(t) \cdot M_E(t) \cdot n(t)}{E_{Batt,nom} \cdot K_E(t)} \cdot dt \qquad (12)$$

The limits of the integration are the start time $\mathbf{T_0}$ and the sum of the start time and the optimisation horizon length $\mathbf{T_H}$. In order to carry out an optimisation, the performance index should always be positive. Therefore the absolute value of the change in the state of charge is used in the cost function.

The masses of fuel consumed and exhaust gases produced are also factors that are to be minimized over the optimisation horizon. This can be accomplished by maximizing the average combustion efficiency and similar efficiencies expressing the energy production with respect to the exhaust gas masses that result, or by minimizing the reciprocals of the average efficiencies over the time horizon. A weighted sum of the reciprocals of these efficiencies can be expressed as a single value that is to be minimized. Such an expression, which characterizes the performance of the system over the optimisation horizon, is referred to in the literature as a Lagrange index [4].

The combustion efficiency defined in equation (2) is inversely proportional to the product of the specific fuel consumption $\mathbf{b_{eff}}$ and the specific heating value of the fuel $\mathbf{C_f}$, which is a constant. Maximizing the average combustion efficiency can be achieved by minimizing the average specific fuel consumption. This component of a Lagrange performance index can be expressed as follows:

$$L_{Fuel} = K_{Fuel} \cdot \left[ \frac{C_f}{T_H} \cdot \int_{T_0}^{T_0+T_H} b_{eff}\left(M_V(t), n(t)\right) \cdot dt + R_{Fuel} \right] \quad (13)$$

The offset $\mathbf{R_{Fuel}}$ and the weighting factor $\mathbf{K_{Fuel}}$ do not affect the minimum but can be used to set the range of the Lagrange fuel component between 0 and 1, which corresponds to 0 and 100%. They will be used later to assign weights to the minimization of individual exhaust gasses or fuel consumption. Similarly, the specific production of an exhaust gas in g/kWh is inversely proportional to an exhaust gas efficiency expressing the ratio of kinetic energy released to the mass of gas produced. As with the consumption efficiency, the average of a given exhaust gas efficiency can be maximized by minimizing the average specific gas production, and the range of possible values of each Lagrange exhaust gas component $\mathbf{L_G}$ can be set using a weighting factor $\mathbf{K_G}$:

$$L_G = K_G \cdot \left[ \frac{C_G}{T_H} \int_{T_0}^{T_0+T_H} b_{G,eff}\left(M_V(t), n(t)\right) \cdot dt + R_G \right] \quad (14)$$

Components for the fuel and exhaust gases $CO_2$, CO, HC and $NO_X$ are added together to form the complete Lagrange index **L** that is to be minimized:

$$L = L_{Fuel} + L_{CO_2} + L_{CO} + L_{HC} + L_{NO_X} \quad (15)$$

The engine maps for the specific consumption and exhaust gas production can be seen as describing surfaces with the coordinates torque $\mathbf{M_V}$ and rotational speed **n**. Because the high and low areas of these surfaces don't always match, it is sometimes necessary to make compromises in the minimization in order to meet emissions or fuel consumption goals for a certain driving cycle. Compromises could be made by emphasizing certain components through their weighting constants $\mathbf{K_{Fuel}},\ldots\mathbf{K_{NOx}}$. The individual components of the Lagrange performance index could be expressed as the sum of a single integral and a constant:

$$\frac{K_{Fuel} \cdot C_f}{T_H} \cdot \int_{T_0}^{T_0+T_H} b_{eff}(M_V(t), n(t)) \cdot dt + \frac{K_{CO} \cdot C_{CO}}{T_H} \cdot \int_{T_0}^{T_0+T_H} b_{CO,eff}(M_V(t), n(t)) \cdot dt$$

$$+ \ldots + K_{Fuel} \cdot R_{Fuel} + K_{CO} \cdot R_{CO} + \ldots =$$

$$\int_{T_0}^{T_0+T_H} \frac{K_{Fuel} \cdot C_f \cdot b_{eff}(M_V(t), n(t)) + K_{CO} \cdot C_{CO} \cdot b_{CO,eff}(M_V(t), n(t)) + \ldots}{T_H} \cdot dt$$

$$+ \ldots + K_{Fuel} \cdot R_{Fuel} + K_{CO} \cdot R_{CO} + \ldots = \int_{T_0}^{T_0+T_H} P(M_V(t), n(t)) \cdot dt + Q \quad (16)$$

The dependant variable $\mathbf{P(M_V(t),n(t))}$ is obtained by adding the products of the individual engine maps and their weights together point by point and can be referred to as a performance index map or surface. A sample performance index map used in simulations to optimise a drivetrain with a spark-ignition engine is illustrated in Figure 3.

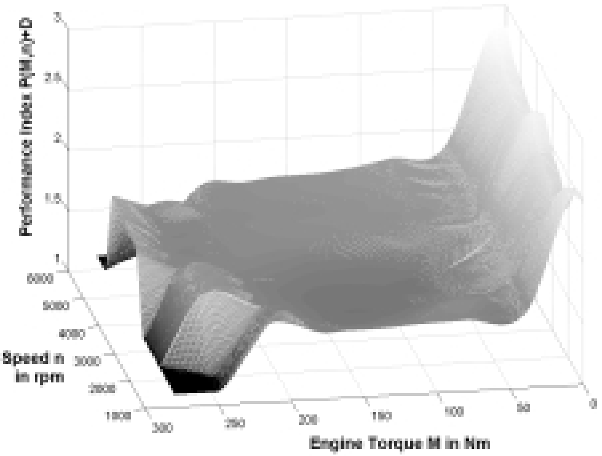


Figure 3: Performance Index Map for Spark-Ignition Engine

The total cost function is made up of the sum of the SOC and Lagrange components:

$$J = K_1 \cdot \left| \int_{T_0}^{T_0+T_H} \frac{K_B(t) \cdot M_E(t) \cdot n(t)}{K_E(t) \cdot Q_{Bat}} \cdot dt \right| +$$

$$K_2 \cdot \int_{T_0}^{T_0+T_H} P(M_V(t), n(t)) \cdot dt + Q \quad (17)$$

The factors $\mathbf{K_1}$ and $\mathbf{K_2}$ are weighting factors that can used to emphasize the battery's state of charge or engine performance in the total cost function. An optimising control system is sought that minimizes the cost function at each sampling time in order to find the best compromise between the two sides. Finding a solution by solving a variation problem defined by the system describing the drivetrain and the combined performance index is difficult because of the non linearities represented by the engine maps and the efficiency characterisctics of the electric machine and battery. However, by implementing a number of simplifications, it is possible to find a closed solution to the minimizing problem that can be used by a control system to optimise the operation of a drivetrain in real time.

## 3.2 Simplification through Restriction of System Dynamics and Variables

The state equations of the system describing a hybridized drivetrain are made up of the equations of motion of the combustion engine, electrical machine and vehicle (18), (19) and (20) and the energy state equation of the battery (21). Here **ρ** represents the air density, $\mathbf{C_W \bullet A}$ represents the product of the vehicle's air drag coefficient and frontal area, **v** represents the vehicle speed, **g** represents the earth's acceleration, $\mathbf{m_{Veh}}$ represents the vehicle's mass and $\mathbf{m_{Inirtia}}$ represents an equivalent additional mass that is effectively added to the vehicle mass during acceleration.

$$\dot{M}_{CE} = -\frac{1}{\tau_{CE}} M_{CE} + \frac{M_{CE\_In}}{\tau_{CE}} \tag{18}$$

$$\dot{M}_{E} = -\frac{1}{\tau_{E}} M_{E} + \frac{M_{E\_In}}{\tau_{E}} \tag{19}$$

$$\dot{v} = \frac{m_{Veh}}{m_{Veh} + m_{Inertia}} \left( \frac{1}{R_{Wheel} \cdot m_{Veh}} M_{req} - \frac{\rho C_W A v^2}{2} - f_R m_{Veh} g \right) \tag{20}$$

$$\dot{E} = K_B \cdot \left( \frac{25 \ \cdot K_E \cdot v}{3 \cdot \pi \cdot \cdot R_{Wheel} \cdot i} M_E - P_{Loads} \right) \tag{21}$$

Equation (1), which defines the requested torque $\mathbf{M_{req}}$ as the sum of the torques from the combustion engine $\mathbf{M_{CE}}$ and electrical machine $\mathbf{M_E}$, is a boundary condition to the system of equations.

With the restriction that the torques $\mathbf{M_{CE}}$ and $\mathbf{M_E}$ remain constant in the interval between the sampling times of the driving cycle or requested torque, the cost function can be simplified. This is done by neglecting their rise-times, which can be done, because the dynamics of the combustion engine and electric machine are much quicker than the dynamics of the input signal describing the drive cycle.

A further simplification can be achieved through the use of constant battery and electric machine efficiency factors $\mathbf{K_B}$ and $\mathbf{K_E}$, and a final simplification is the assumption that the vehicle speed remains constant over the optimisation horizon. As a result, the rotational speed of the engine **N** becomes a constant in the cost function. This of course produces deviations from an optimal solution during accelerations, but the deviation is minimized by the frequent updating of the optimisation. Using these simplifications, the integrals in the performance index equation (17) could be replaced by simpler expressions. Because **Q** in (17) is a constant, it can be eliminated from the equation. The simplified cost function can be expressed as follows:

$$J = K_1 \cdot \left| E_0 + \frac{K_B \cdot M_E \cdot N \cdot T_H}{K_E(M_E, N)} \right| + K_2 \cdot P(M_V, N) \tag{22}$$

The energy $\mathbf{E_0}$ is the difference between the nominal energy capacity of the battery and the amount of energy stored in the battery at the sampling time $\mathbf{T_0}$. The SOC- (first-) component of the cost function is minimized to zero when the electrical energy added to or taken from the battery equals the negative of $\mathbf{E_0}$. This is accomplished by boosting or generating with the electrical torque $\mathbf{M_E}'$, which is defined as follows:

$$M'_E = -\frac{E_0 \cdot K_E(M'_E, N)}{K_B \cdot N \cdot T_H} \tag{23}$$

For a given required torque $\mathbf{M_{req}}$, the combustion engine torque $\mathbf{M_{CE}}'$ corresponding to $\mathbf{M_E}'$ can be calculated using the condition defined by equation (1):

$$M'_{CE} = M_{req} - M'_E \tag{24}$$

A deviation in the electrical torque $\mathbf{M_E'+\Delta M}$ causes a reciprocal deviation in the torque produced by the combustion engine according to equation (25):

$$M'_{CE} + \Delta M = M_{req} - M'_E + \Delta M \tag{25}$$

The cost function (22) can be rewritten in terms of the electrical and combustion engine torques and torque derivations $\mathbf{\Delta M}$ and simplified by using the relationship between $\mathbf{E_0}$ and $\mathbf{M_e'}$ as expressed in equation (23):

$$J = K_1 \cdot \left| \frac{K_B \cdot \Delta M \cdot N \cdot T_H}{K_E(M'_E - \Delta M, N)} \right| + K_2 \cdot P(M'_{CE} + \Delta M, N) \tag{26}$$

### 3.3 Closed Solution to Optimisation

In order to obtain a closed control law that performs the optimisation, it is necessary to estimate the average electric machine efficiency factor $\mathbf{K_E(M_E,N)}$ and treat it as a constant in the calculation of $\mathbf{M_E}'$ that is carried out in equation (23) and in the cost function. In this case the cost function can be expressed as follows:

$$J = K_1 \cdot \left| \frac{K_B \cdot \Delta M \cdot N \cdot T_H}{K_E} \right| + K_2 \cdot P(M'_{CE} + \Delta M, N) \tag{27}$$

The minimization of the cost function expressed by equation (27) can be carried out off-line on each element of the performance index map described by $\mathbf{P(M_{CE}',N)}$, and the result is an optimisation map that gives an optimised combustion engine torque $\mathbf{M_{CE}^*}$ for a given $\mathbf{M_{CE}}'$. An optimisation map is illustrated in Figure 4.

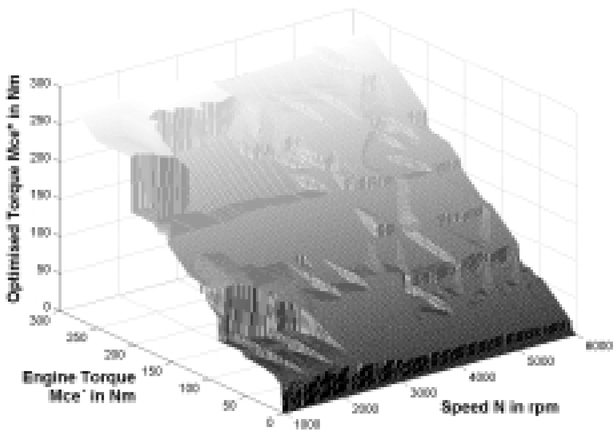


Figure 4: Optimisation Map of Spark-Ignition Engine

In order to implement the map in a control system, the electrical torque $\mathbf{M_E}'$ and the corresponding combustion engine torque $\mathbf{M_{CE}}'$ are calculated with equations (23) and (24) for a given required torque $\mathbf{M_{req}}$ and state of charge. The optimised combustion engine torque $\mathbf{M_{CE}}^*$ for the measured engine speed **N** is determined from the optimisation map, and the corresponding optimised torque to be produced by the starter-alternator is calculated using equation (28):

$$M_E^* = M_{req} - M_{CE}^* \qquad (28)$$

## 4 Simulations with Optimisation Method in Closed Form

Simulations using the optimisation method outlined above were carried out with and without regenerative braking for electrical loads between 100 W and 1 kW and compared with voltage control in order to judge whether improvements in fuel consumption occur. The relative improvements using the closed solution optimisation over voltage control are shown in chart 1.

The reference strategy is voltage control, which is used in conventional vehicles on the road today. Voltage control is accomplished by controlling the generator to maintain a constant voltage on the power distribution network. This indirectly causes the generator to provide all electrical load power, and so voltage control could be referred to as a load-following mode. In the case of the 42V power distribution system, the generator is simply regulated to maintain 42V.

Regenerative braking was implemented by controlling the starter-alternator to provide as much negative torque as possible when deceleration was requested by the driver. The negative torque was limited by the maximum torque line of the electric machine and a maximum allowable voltage of 45 V on the power distribution network. With these limitations it was found that approximately 30 Wh or an average power of approximately 90 W over the cycle could be generated with regenerative braking.

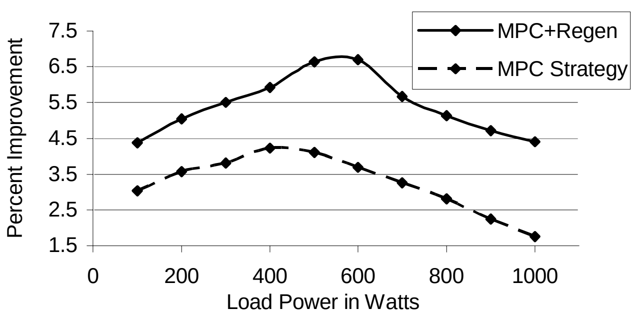


Chart 1: Improvement in Fuel Consumption with respect to Voltage-Control Strategy

As might be expected, it could be seen that the combination of the optimisation and regenerative braking produces the best results. The relative improvement decreases as the load power increases, because the improvement from the strategy is due to the ability of the drivetrain to decouple the energy production schedule from the engine operating points prescribed by the drive cycle and use the battery as an energy buffer. As the load power rises, this ability to decouple production from request decreases, because the starter-alternator is forced to constantly generate in order to keep up with demand.

## 5 Conclusion

An optimisation based on model predictive control was introduced in this paper. It uses no previous knowledge of the driving cycle but employs a simple prognosis of the vehicle’s speed in a sliding horizon to minimize a cost function that balances improvements in fuel consumption and emissions with a change in the state of charge of the battery. In order to create a closed form of the optimisation that could easily be implemented in real time, the system equations had to be simplified by assuming constant efficiencies of the battery and electrical machine.

The optimisation was examined in simulations alone and in combination with regenerative braking. Despite its simplicity it was found to perform well with both applications, although the degree of improvement was dependant on the loading of the power distribution network.

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