

PREDICTIVE CONTROL OF DISSOLVED OXYGEN IN AN ACTIVATED SLUDGE WASTEWATER TREATMENT PLANT

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Abstract

This paper reports on the design of several model predictive controllers for dissolved oxygen (DO) in an activated sludge wastewater treatment plant (WWTP). The paper explores several possibilities for obtaining appropriate models for prediction using subspace identification methods. The paper also illustrates the use of a number of possibilities for rearranging the models to include actuator limitations. Finally, three model predictive controllers (MPCs) are designed, tested and the results are compared with a PI controller. A well established simulation benchmark is used as a test bench for this study.

1 Introduction

The efficiency and economics of wastewater treatment have become an important issue for water companies in the UK and in the rest of Europe due to new, more stringent EU directives for environmental protection. The most common wastewater treatment process is the Activated Sludge technology. The costs of wastewater treatment using this technology include chemicals, energy, and human resources for the process and its operation. In order to minimise these costs, the wastewater industry has been led into the development and use of sophisticated strategies for process control. For example the use of intermittent aeration to minimise energy consumption has been reported in several publications [6, 10, 12], strategies to increase hydraulic capacity to cope with rain or storm events as reported in [9], or improved optimisation by efficient handling of information collected by the control system as reported in [15].

A recent study of four treatment plants in Scotland, Denmark, Germany, and Poland reported in [2], concluded that dissolved oxygen control loops contained P or PI controllers, which were usually mis-tuned and performed poorly. Some side effects of poor tuning are instability and limit cycles, which in turn lead to blower and valve saturation, and wear and tear.

This paper reports on the design of several predictive controllers for dissolved oxygen. The problem of finding suitable models for prediction and estimation of dissolved oxygen is solved by using subspace identification [14, 13]. Furthermore, the data for the identification is collected on-line and in closed-loop in order to minimise changes in the working system.

The paper is organised in the following way: the first part is an introduction into the activated sludge process and the simulation benchmark employed in this study. Following the introduction the identification and validation of models is discussed. This section also presents the rearrangements made to the models to include disturbance models as well as the methods used to extract (estimate) the states necessary for the prediction. Section 4 presents the design of the model predictive controllers (MPC). Comparative results for dynamic behaviour and long-term behaviour are presented in section 5, and finally conclusions are drawn at the end of the paper.

2 Activated sludge wastewater treatment plants and models

Activated sludge wastewater treatment processes are complex biological systems that are difficult to describe mathematically. A first mathematical model was formally introduced in [4], and is known as the Activated Sludge Model No.1 (ASM1). The ASM1 model describes the degradation of carbonaceous material as well as nutrient removal. Mathematically, the model is composed of a set of 13 non-linear differential equations. In particular, one of them describes the dissolved-oxygen dynamics. The WWTP simulation benchmark employed in this work uses ASM1 to represent the biological treatment.

The WWTP simulation benchmark was developed by the COST action 624 & 682 research group [1]. The benchmark is composed of five cascade biological reactors and a non-reactive settling tank. The first two reactors are anoxic, while the last three are aerobic. The model also has a recirculation flow and a return sludge flow. In this configuration, it has been assumed that only the last aerobic reactor has dissolved oxygen control (PI controller), while the other two reactors have fixed aeration.

The dissolved oxygen sensor utilised in the simulations has a 1-minute time delay and 1 minute sampling time. Actuators have been modelled as physical limitations in the air supply equivalent to a maximum oxygen transfer (k_{La}) of $360 [day^{-1}]$. The simulation benchmark also provides three files of dynamic influent data for dry, rain and storm conditions, and a file of constant influent data used to stabilise the plant. The plant layout is presented in Figure (1).

The purpose of the controllers designed and tested in this paper is to improve the system performance for setpoint manoeuvring and compensate for external disturbances like load changes due

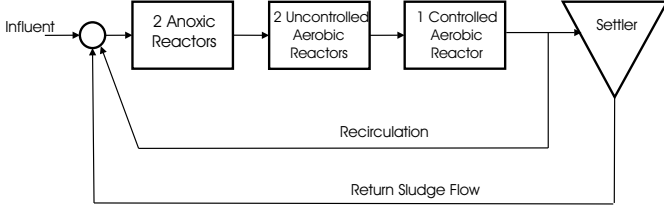


Figure 1: COST Simulation Benchmark - Plant Layout.

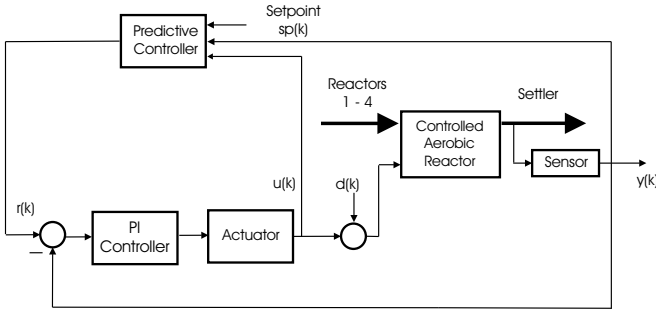


Figure 2: Predictive control scheme.

to daily variations in influent composition or weather changes as rain events. In this context, the proposed controller structure would have a hierarchical architecture as the one presented in Figure (2).

3 Identification and validation of models for dissolved oxygen

3.1 Identification

In this paper subspace identification has been adopted as the method to generate models for prediction and estimation for dissolved oxygen control. In general terms, these algorithms allow the identification of multivariable systems, however for the purpose of this paper a single input - single output case is considered. The identification algorithms are not described in this paper, but the reader is encouraged to review the referenced publications [13, 14].

As explained in section 2, only the last reactor allows manipulation of the air compressor. Figure (3) shows a diagram of the control structure in the last aeration tank. In order to obtain a representative model of the dynamics, it is necessary to

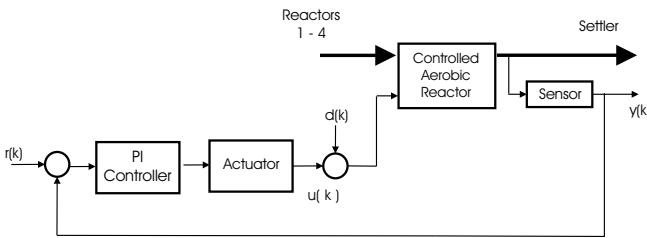


Figure 3: Low-level dissolved oxygen control loop.

No.	Model	Order	Algorithm	Notes
1	open loop	2	[13]	1st algorithm
2	closed-loop	3	[14]	SV
3	closed-loop	3	[14]	CVA

Table 1: Identified Models

have data of a persistently excited system. An option to generate data sets with such characteristics is to externally excite the system, as in [7, pp.156], by employing a pseudo random binary signal (PRBS). For this study, the last reactor was excited with a PRBS signal of 1 [mg/l] amplitude around an operating point of 2 [mg/l] during 1 day.

Initially three models were identified using the algorithms in [13, 14, pp.131]. The first model is an open-loop model (from $u(k)$ to $y(k)$) identified from closed-loop data using the first algorithm presented in [13]. The following two models are closed-loop models, identified from $r(k)$ to $y(k)$ using the SV and the CVA options of the combined deterministic-stochastic robust identification algorithm presented in [14]. Table (1) presents a summary of the data in the table is that the maximum order is of the third degree.

3.2 Actuator and disturbance models

None of the previously identified models include the actuator output signals. In order to include the physical limitations of the aeration system it is necessary to somehow include them into the model. Additionally, to compensate for the plant-model mismatch and un-modelled disturbances given by the changing influent load (daily variations and weather effects), it is necessary to introduce a disturbance model. The section will begin by including the actuator (controller) dynamics into account and therefore formulating three new models which will be denoted as composite for the open-loop case and augmented for the closed-loop cases.

3.2.1 Composite and augmented models

Let the loop-controller be described by equation (1) and the plant model No.1 by equation (2). The loop-controller is a PID type controller represented in state-space.

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + B_c e(k) \\ u(k) &= C_c x_c(k) + D_c e(k) \end{aligned} \quad (1)$$

$$\begin{aligned} x_o(k+1) &= A_o x_o(k) + B_o u(k) \\ y(k) &= C_o x_o(k) + D_o u(k) \end{aligned} \quad (2)$$

where $e(k)$ is the error between the oxygen measurement $y(k)$ and the reference signal $r(k)$. Closing the loop and rearranging the matrices the following composite system is found:

$$\begin{aligned} X(k+1) &= A \cdot X(k) + B \cdot r(k) \\ Y(k) &= C \cdot X(k) + D \cdot r(k) \end{aligned} \quad (3)$$

where:

$$X(k) = \begin{bmatrix} x_o(k) \\ x_c(k) \end{bmatrix} \quad (4)$$

$$Y(k) = \begin{bmatrix} y(k) \\ u(k) \end{bmatrix} \quad (5)$$

$$A = \begin{bmatrix} A_o - B_oMD_oC_o & B_oMC_o \\ B_cD_oMD_cC_o - B_cC_o & A_c - B_cD_oMC_c \end{bmatrix} \quad (6)$$

$$B = \begin{bmatrix} B_oMD_c \\ B_c - B_cD_oMD_c \end{bmatrix} \quad (7)$$

$$C = \begin{bmatrix} C_o - D_oMD_cC_o & D_oMC_c \\ -MD_cC_o & MC_c \end{bmatrix} \quad (8)$$

$$D = \begin{bmatrix} D_oMD_c \\ MD_c \end{bmatrix} \quad (9)$$

$$M = (I + D_cD_o)^{-1} \quad (10)$$

For the case of the closed-loop models No.2 and 3, the controller dynamics and therefore the actuators limits are included in a different way. Considering the same loop-controller state-space representation and the closed-loop model of Equation (11), and using the error definition it is possible to define an augmented model described by Equations (12-18).

$$\begin{aligned} x_{cl}(k+1) &= A_{cl}x_{cl}(k) + B_{cl}r(k) \\ y(k) &= C_{cl}x_{cl}(k) + D_{cl}r(k) \end{aligned} \quad (11)$$

$$\begin{aligned} X(k+1) &= A \cdot X(k) + B \cdot r(k) \\ Y(k) &= C \cdot X(k) + D \cdot r(k) \end{aligned} \quad (12)$$

$$X(k) = \begin{bmatrix} x_{cl}(k) \\ x_c(k) \end{bmatrix} \quad (13)$$

$$Y(k) = \begin{bmatrix} y(k) \\ u(k) \end{bmatrix} \quad (14)$$

$$A = \begin{bmatrix} A_{cl} & 0 \\ -B_cC_{cl} & A_c \end{bmatrix} \quad (15)$$

$$B = \begin{bmatrix} B_{cl} \\ B_c(I - D_{cl}) \end{bmatrix} \quad (16)$$

$$C = \begin{bmatrix} C_{cl} & 0 \\ -D_cC_{cl} & C_c \end{bmatrix} \quad (17)$$

$$D = \begin{bmatrix} D_{cl} \\ D_c(I - D_{cl}) \end{bmatrix} \quad (18)$$

Validation of these models is performed by measuring the *percentage variance accounted for* (vaf) between the measured and predicted signals (19). The *vaf* coefficient is only a measure of the degree of similarity between the two signals and does not measure biases. Table (2) presents a summary of the obtained *vaf* coefficients for the oxygen concentration and actuator output predictions. Figures (4) and (5) show the signals of the three models validated against the DO measurement from the plant.

$$vaf\% = \left(1 - \frac{var(y - \hat{y})}{var(y)}\right) * 100 \quad (19)$$

Model	y vaf (%)	u vaf (%)
composite	96.809	81.9695
augmented 1	99.95716	86.55406
augmented 2	99.95720	86.55406

Table 2: VAF coefficients for composite and augmented models

3.2.2 Disturbance model

There are two important reasons to include a disturbance model in the DO dynamics description. The most obvious one is for compensation of changing load conditions due to influent concentrations and flow variations during the day and in meteorological events as rain or storm. There is however a second reason. The models which are being used have been recovered from an identification procedure. Therefore, they are just an approximation to the real plant dynamics. Due to this plant-model mismatch, the augmented and the composite models will give significant errors in the prediction of the actuator (controller) output, Figures (4-5).

In general terms, disturbance models have to be chosen accordingly to the expected load. It could be argued that in the case of WWTPs, the most common disturbance will have a cyclic daily fluctuation. However, the prediction horizon of a variable like dissolved oxygen is in the range of fractions of an hour. Therefore it is more realistic to assume either a constant disturbance or a slowly decaying model like in [7]. This paper assumes a constant disturbance model.

To introduce the disturbance effect into the composite and the augmented model formulations it is only necessary to redefine the state and output equation as in Equations (20-25).

$$\begin{aligned} \xi(k+1) &= A_d\xi(k) + B_dr(k) \\ Y(k) &= C_d\xi(k) + D_ar(k) \end{aligned} \quad (20)$$

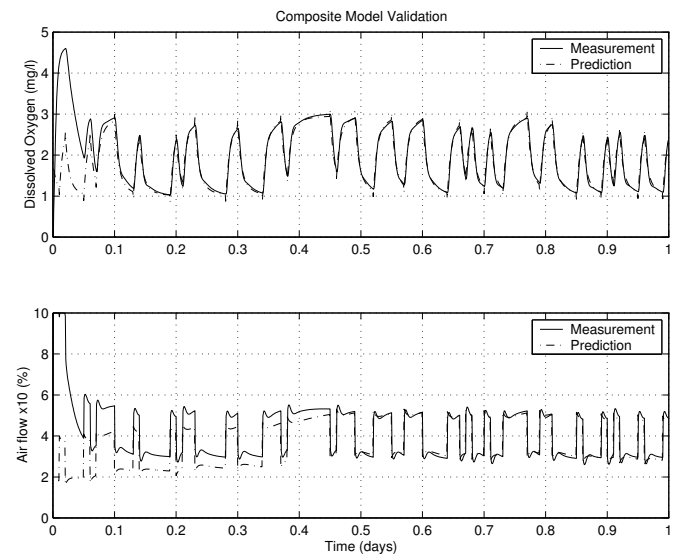


Figure 4: Composite model prediction

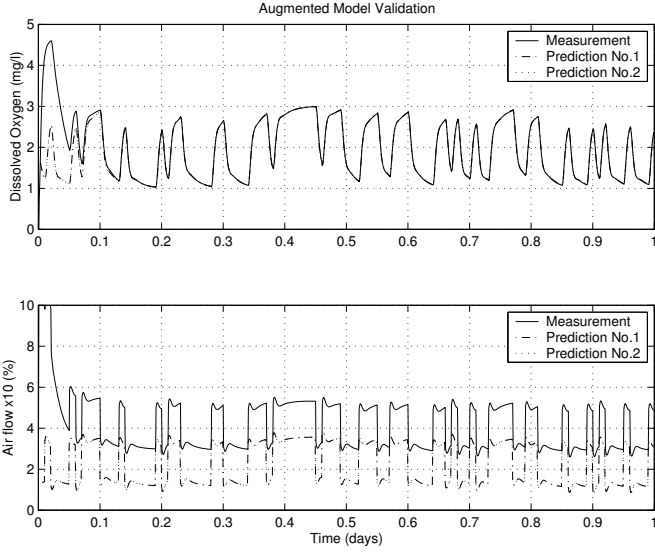


Figure 5: Augmented models predictions

where

$$\xi(k) = \begin{bmatrix} X(k) \\ d(k) \end{bmatrix} \quad (21)$$

$$A_d = \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} \quad (22)$$

$$B_d = \begin{bmatrix} B \\ 0 \end{bmatrix} \quad (23)$$

$$C_d = \begin{bmatrix} C & I \end{bmatrix} \quad (24)$$

$$D_d = D \quad (25)$$

Notice however, that this approach can only be implemented in conjunction with a state-observer which provides the initial plant-model mismatch with which it is possible to calculate the predictions. Therefore in order to evaluate these models it is necessary to implement them through state observers or estimators.

3.3 State observers and estimation

A fundamental part of the design of a MPC structure has to do with the state extraction or estimation. In the case of deterministic systems the most common approach is to design a state observer if the system is observable. If the system is of stochastic nature, the optimal solution would be a Kalman filter. However, in this process as in many industrial processes the noise characteristics are not known. Due the lack of this information it might be time consuming and difficult to calibrate a Kalman filter compared to a state observer. Therefore this paper will not address the advantages or disadvantages of using Kalman filtering. Instead state observers designed using pole placement are employed.

4 Control Design

Several subspace predictive control methods have been developed within the last few years. The technique itself is considered to be fairly new and it has been just recently that some possibilities of implementing model predictive controllers (MPCs) directly from a subspace framework are being explored as for example in [3, 5, 11]. This paper, however, does not approach the implementation of the MPC controllers in this way, but uses a state space formulation as described later in the paper.

In this section, three predictive controllers are designed. The predictor is formulated for the composite, composite with disturbance model and augmented with disturbance model. For the augmented model, only the CVA case is considered since results are very similar to the SV algorithm.

4.1 Cost function and prediction

Considering any of the proposed models (12, 3, 20), the predictions over an output horizon H_p with control horizon H_u can be described by equation (26).

$$\mathcal{Y}(k) = \Psi \hat{x}(k) + (\Upsilon + \Omega)r(k-1) + (\Theta + \Phi)\Delta R(k) \quad (26)$$

where:

$$\Psi = \tilde{C} \cdot \begin{bmatrix} A \\ \vdots \\ A^{H_u} \\ \vdots \\ A^{H_p} \end{bmatrix} \quad (27)$$

$$\Upsilon = \tilde{C} \cdot \begin{bmatrix} B \\ \vdots \\ \sum_{i=0}^{H_u} A^i B \\ \vdots \\ \sum_{i=0}^{H_p-1} A^i \end{bmatrix} \quad (28)$$

$$\Theta = \tilde{C} \cdot \begin{bmatrix} B & \dots & 0 \\ AB+B & \dots & 0 \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{H_u-1} A^i B & \dots & B \\ \sum_{i=0}^{H_u} A^i B & \dots & AB+B \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{H_u} A^i B & \dots & \sum_{i=0}^{H_p-H_u} A^i B \end{bmatrix} \quad (29)$$

$$\Phi = \begin{bmatrix} D & D & 0 & \dots & 0 \\ D & D & D & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ D & D & D & \dots & D \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ D & D & D & \dots & D \end{bmatrix} \quad (30)$$

Model	Hp	Hu	\mathcal{R}	\mathcal{Q}
composite	50	10	1×10^2	diag[15 1×10^{-5}]
comp. & dist.	50	10	1×10^3	diag[15 1×10^{-5}]
aug. & dist.	50	7	30	diag[15 1×10^{-5}]

comp. & dist.: composite model with disturbance estimation
aug. & dist.: augmented model with disturbance estimation

Table 3: Controller parameters.

$$\Omega = \begin{bmatrix} D \\ \vdots \\ D \end{bmatrix} \quad (31)$$

$$\tilde{C} = \begin{bmatrix} C & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C \end{bmatrix} \quad (32)$$

The increment of the control inputs over the prediction horizon are calculated by minimising the following cost function:

$$J(k) = \|\mathcal{Y}(k) - S(k)\|_{\mathcal{Q}}^2 + \|\Delta R(k)\|_{\mathcal{R}}^2 \quad (33)$$

Define then the following expression:

$$\mathcal{E}(k) = S(k) - \Psi \hat{x}(k) - (\Upsilon + \Omega)r(k-1) \quad (34)$$

By substituting (26) and (34) into (33), the cost function can be written as:

$$J(k) = \|(\Theta + \Phi)\Delta R(k) - \mathcal{E}(k)\|_{\mathcal{Q}}^2 + \|\Delta R(k)\|_{\mathcal{R}}^2 \quad (35)$$

If \mathcal{Q} and \mathcal{R} are factored as $\mathcal{Q} = S_{\mathcal{Q}}S_{\mathcal{Q}}^T$ and $\mathcal{R} = S_{\mathcal{R}}S_{\mathcal{R}}^T$, then $\Delta R(k)$ can be calculated as the least-squares solution to:

$$\begin{bmatrix} S_{\mathcal{Q}}(\Theta + \Phi) \\ S_{\mathcal{R}} \end{bmatrix} \Delta R(k) = \begin{bmatrix} S_{\mathcal{Q}}\mathcal{E}(k) \\ 0 \end{bmatrix} \quad (36)$$

Table (3) summarises the values for the prediction horizon (Hp), control horizon (Hu) and weightings \mathcal{Q} and \mathcal{R} that produced the best results.

4.2 Constraints

The inclusion of constraints is fundamental in this problem. It is in this way that the physical limitations arising from the actuators (air compressors) are included when solving the optimisation. Constraints also allow the inclusion of operation conditions that are necessary for the process to work. For example, in many WWTPs it is necessary to keep a minimum aeration regardless of the oxygen concentration, just to keep the reactors fully mixed. It is also evident that constraints allow limits to be imposed over variables which in practice cannot go under or over certain limits, as for example the oxygen concentration cannot be less than zero. To implement such restrictions, all the variables must be written as a function of the optimisation variable, that is in this case $\Delta R(k)$. A good discussion of how to do

this can be found in [8, pp.43-45,81-82]. For the optimisation problem, inequality constraints have the form of equation (37). For this study, the physical limits are tabulated in Table (4).

$$W\Delta R(k) \leq w \quad (37)$$

Limit/variable	DO (mg/l)	Q (%)	r (mg/l)
Lower limit	0	1	0
Upper limit	8	100	10

Table 4: Physical Limits

5 Results

Simulations for a PI controller, and the three proposed MPC are presented in this section. The simulation scenarios include constant influent, dry weather influent, rain influent, and storm influent as defined in [1]. Within the simulations, the constant influent is utilised to assess the transient response to changes in setpoint and disturbance rejection, while the dynamic influent files are used to provide a statistical evaluation of the performance in the long term. Tables (5-6) show the results for setpoint tracking and disturbance rejection, while Tables (7-9) show the statistics for dynamic performance under the specified weather conditions.

Results show that even though the performance of the composite model with disturbance estimation is acceptable in the transient analysis, its performance is significant better when the simulation is run for dynamic influent. These results show the benefit of including a second level of control over the common PI loop.

Case	Overshoot (%)	Settling Time (min)
composite	0	>360
composite w. disturbance	2.11	55
augmented w. disturbance	1.277	27
PI	0.99	74

Table 5: Dynamic performance

Case	Peak (%)	Rejection Time (min)
composite	-43.78	>360
composite w. disturbance	-34.72	55
augmented w. disturbance	-35.58	44
PI	-37.92	98

Table 6: Disturbance rejection

6 Conclusions

This paper reports on the identification of models for dissolved oxygen using subspace identification, and the design of three predictive controllers for oxygen control in an activated sludge

Case	Max	Min	Mean	Var	St.Dev.
composite	2.71	1.37	2.00	0.087	0.295
comp. & dist.	2.04	1.91	1.99	3.2×10^{-4}	0.018
aug. & dist.	2.24	1.67	1.99	0.007	0.086
PI	2.49	1.40	1.99	0.030	0.174

comp. & dist.: composite model with disturbance estimation
aug. & dist.: augmented model with disturbance estimation

Table 7: Dry weather statistics

Case	Max	Min	Mean	Var	St.Dev.
composite	2.70	1.38	2.02	0.070	0.265
comp. & dist.	2.04	1.89	1.99	2.4×10^{-4}	0.015
aug. & dist.	2.24	1.68	1.99	0.006	0.075
PI	2.49	1.42	2.00	0.024	0.153

comp. & dist.: composite model with disturbance estimation
aug. & dist.: augmented model with disturbance estimation

Table 8: Rainy weather statistics

wastewater treatment plant. The COST Simulation Benchmark is used as a testing plant. The paper contains transient and statistical evaluation of the controllers for constant, dry, rainy and storm weather conditions. The paper also includes an analysis into three identification algorithms, and provides an insight into which approach is more suitable for this application. Simulation results also indicate that models for dissolved-oxygen can be valid even under changing weather conditions.

Results show that all the controllers perform very well and improve performance compared to a single PI loop. The control structure also facilitates implementation due to only minor modifications to the plant computer system.

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Case	Max	Min	Mean	Var	St.Dev.
composite	2.70	1.38	1.99	0.080	0.282
comp. & dist.	2.05	1.91	1.99	3.0×10^{-4}	0.017
aug. & dist.	2.24	1.67	1.99	0.007	0.086
PI	2.49	1.41	1.99	0.027	0.165

comp. & dist.: composite model with disturbance estimation
aug. & dist.: augmented model with disturbance estimation

Table 9: Storm weather statistics

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