LINEAR-QUADRATIC REGULATORS APPLIED TO SEWER NETWORK FLOW CONTROL

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Abstract

The problem considered addresses the optimal wastewater distribution to several retention reservoirs in an urban sewer network during rainfall in the aim of protecting the quality of receiving waters via minimization of overflows. To this end, a linear multivariable feedback regulator is developed using the linear-quadratic design procedure. Inflow predictions are accommodated via suitable feedforward terms in the control law. A study for a real large-scale combined sewer network using this method is presented on the basis of a realistic simulation model. Results demonstrate the efficiency of the developed methodology.

1 Introduction

The construction of treatment plants for sewage protects the quality of the waters that receive the outflows of the sewer networks. However, urban combined sewer networks do not have separated collectors for the domestic and industrial sewage and the rainwater drainage. Therefore, during rainfall, networks and/or treatment plants may be overloaded, and overflows of untreated wastewater may take place upstream of overloaded stretches, causing pollution of receiving waters. Placing retention reservoirs at appropriate locations of the network is a cost-efficient way to avoid overflows at moderate rain events and to reduce them at stronger rainfall as the water is stored in the reservoirs during the rainfall and is directed towards the treatment plant after the rainfall stops.

Optimal operation of the combined sewer network (that contains retention reservoirs) implies that for each rain event the whole retention capacity of all reservoirs will be used before overflows take place somewhere in the network. This, however, cannot be guaranteed by fixed gate settings, such as fixed weirs or manually adjustable gates for the filling and emptying of the storage spaces. Especially if the rainfall is distributed unevenly over the urban area, there may be reservoirs that are not totally filled, while overflows occur elsewhere in the network. In these cases, a further reduction of overflows can be obtained via real-time operation of the reservoirs, e.g. by use of controllable gates that are driven by an automatic control strategy. On the other hand, an efficient control strategy may lead to substantial cost savings, as the number and storage capacities of the reservoirs required to keep overflows below a certain (usually legislatively defined) limit, depends upon the efficiency of the applied control strategy.

A real-time control structure for sewer networks that combines high efficiency and low implementation cost, may be composed of a number of control layers (multilayer control structure). Such a flexible, reliable, and efficient hierarchical control structure for real-time control of sewer networks has been proposed e.g. in [13]: An *adaptation layer* is responsible for rain and/or inflow prediction (if needed) and for real-time estimation of the system state. An *optimization layer* is responsible for the central, overall network control, i.e. for specifying reference trajectories for the reservoir storages and outflows. A decentralized *direct control layer* is responsible for the realization of the reference trajectories. With regard to the optimization layer, several approaches have been proposed in the past, like:

- Nonlinear optimal control [12, 4, 9, 5, 14].
- Multivariable feedback control [8, 4, 6].
- Methods based on dynamic [2] or linear programming [10].
- Expert systems, fuzzy control [1], and further heuristic approaches.

This paper focuses on the multivariable feedback control approach to central control of sewer network flow. Several improvements, modifications and extensions introduced to previously developed versions of the method [8] in order to increase its efficiency are included. The main paper focus compared to previous studies [6] is on application and testing of this method by use of a realistic simulation model, the program KANSIM.

2 Control Problem Formulation

2.1 Mathematical model and constraints

For the study of the sewer network control problem, two mathematical models of the sewer network are employed, a realistic simulation model (KANSIM) and a simpler control design model that is referred to as *simplified model*. The simplified model is used for the design of the multivariable regulator while KANSIM [7] is used for testing the performance and sensitivity of the control method. Within

KANSIM all the dynamic phenomena that take place in the different elements of the sewer network are modeled in detail using known laws of hydraulics such as the Saint-Venant equations for the sewer stretches, the Poleni formula for overflows, etc., while the simplified model has lower accuracy and complexity.

Combined sewer networks consist of a set of elements in which different processes take place, as for example storage (in the reservoirs or in the sewers), transport (in the sewers), merging of flows (in the nodes). The typical elements upon which a combined sewer network may be built according to the simplified model [3, 6] are reservoirs, nodes, external inflows, link elements and treatment plants (Figure 1).

A particular network can be assembled from these elements. The whole flow process may be considered to have a vector input **u** including all controllable reservoir outflows, a disturbance vector **d** including all external inflows, and a state vector **x** including all reservoir storages and link outflows. Then, the model equations may be expressed in the following general form [3]

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{x}(k-1),...,\mathbf{x}(k-\kappa_x), \mathbf{u}(k), \mathbf{u}(k-1),...,\mathbf{u}(k-\kappa_u), \mathbf{d}(k), \mathbf{d}(k-1),...,\mathbf{d}(k-\kappa_d)] \quad (1)$$

where k= 0,1,... is the discrete time index and κ_x, κ_u, κ_d are the longest time delays of **x**, **u** and **d**, respectively.

The control constraints have the form

$$\mathbf{u}_{min} \le \mathbf{u}(k) \le \mathbf{u}_{max}(\mathbf{x}(k),k) \quad (2a)$$

$$\sum_{\kappa=1}^{\kappa_u} \boldsymbol{A}(k)\mathbf{u}(k-\kappa) \le \mathbf{c}(\mathbf{x}(k),k) \quad (2b)$$

where $\boldsymbol{A}(k)$ is a matrix of zero/one coefficients. In contrast to KANSIM, the simplified model does not consider backwater effects.

2.2 Control Objectives

The main task of the control system is *the minimization of overflows* for any rainfall event. This can be achieved by:

- Using all available storage space before allowing an overflow to occur somewhere in the network. Moreover, if, due to strong rainfall, overflows are unavoidable, they should be distributed as homogeneously as possible over time and over the network reservoirs. However, if there are storage elements without overflow capability (no overflow weirs), the avoidance of overloading of these storage elements is of even higher importance.
- Emptying the network as soon as possible (by fully using the inflow capacity of the treatment plant) so as to provide free storage space for a possible future rainfall.

A direct way of considering these main objectives, along with some secondary operational objectives, is via minimization of a nonlinear objective function [3]. In this paper, an alternative approach that leads to a quadratic objective criterion is taken [3].

3 Regulator Design

3.1 Linear-quadratic formulation

For the sewer network flow control problem, application of the linear-quadratic-regulator (LQR) methodology appears most convenient [8].

The LQR methodology is not directly applicable in presence of time delays, like those appearing in the process model (1). This difficulty may be readily circumvented by introducing some auxiliary variables $\tilde{x}_i$ [6]. Thus, if a control variable u_j appears in the model equations with time delay κ_{uj}, one may introduce the additional auxiliary state equations

$$\begin{aligned} \tilde{x}_1(k+1) &= u_j(k) \\ \tilde{x}_2(k+1) &= \tilde{x}_1(k) \\ &\dots \\ \tilde{x}_{\kappa_{uj}}(k+1) &= \tilde{x}_{\kappa_{uj}-1}(k) \end{aligned} \quad (3)$$

and substitute $\tilde{x}_{\kappa_{uj}}(k)$ in all model equations where $u_j(k-\kappa_{uj})$ appears. This modification can be performed for all time-delayed control and state variables of the process model. The auxiliary variables $\tilde{\mathbf{x}}$ are regarded as additional state variables that are incorporated in the state vector **x**. With this modification, (1) obtains the simpler form

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k), \mathbf{d}(k)]. \quad (4)$$

To facilitate the application of LQR design, linearization around a stationary nominal point is required. For the definition of this point, a nominal rainfall is considered that leads to constant nominal external inflow values $\mathbf{d}^N$ such that $d_1^N + d_2^N + ... + d_{nd}^N = r_{max}$ (r_{max} is the plant's maximum capacity) results in absence of any control actions (all gates opened). Under nominal conditions no overflows occur, because we have assumed that the sum of external inflows equals r_{max}. Using these values, the nominal values for each reservoir's outflow u_i^N, the nominal reservoir storages V_i^N, the nominal values of the link outflows q_i^N and of the auxiliary variables $\tilde{x}_i^N$ are obtained [3]. The nominal steady-state just described corresponds to a steady-state form of (4)

$$\mathbf{x}^N = \mathbf{f}(\mathbf{x}^N, \mathbf{u}^N, \mathbf{d}^N). \quad (5)$$

Linearization of (4) around this steady-state leads to

$$\Delta\mathbf{x}(k+1) = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\Big|_N \Delta\mathbf{x}(k) + \frac{\partial \mathbf{f}}{\partial \mathbf{u}}\Big|_N \Delta\mathbf{u}(k) + \frac{\partial \mathbf{f}}{\partial \mathbf{d}}\Big|_N \Delta\mathbf{d}(k) \quad (6)$$

where $\Delta\mathbf{x}(k)=\mathbf{x}(k)-\mathbf{x}^N$, $\Delta\mathbf{u}(k)=\mathbf{u}(k)-\mathbf{u}^N$, and $\Delta\mathbf{d}(k)=\mathbf{d}(k)-\mathbf{d}^N$ are the linearized variables, and $\mathbf{A}=\partial\mathbf{f}/\partial\mathbf{x}\,|_N$, $\mathbf{B}=\partial\mathbf{f}/\partial\mathbf{u}\,|_N$, $\mathbf{C}=\partial\mathbf{f}/\partial\mathbf{d}\,|_N$ are the state, control, and disturbance matrices, respectively, of the linearized system.

For reasons not detailed here, the original system (6) is not fully controllable [3]. To obtain a *fully controllable* linear model, the n_x state variables and according state equations corresponding to the n_x reservoirs, are replaced by n_x-1 new state variables and state equations. The new state variables and state equations are obtained by building n_x-1 independent differences of the old state equations. For example, if the linearized conservation equations of reservoirs i and j are

$$\Delta V_i(k+1) = \Delta V_i(k) - T[\Delta u_i(k) + \Delta d_i(k)] \quad (7)$$
$$\Delta V_j(k+1) = \Delta V_j(k) - T[\Delta u_j(k) + \Delta u_l(k)] \quad (8)$$

respectively, where $V_i(k)$ is the storage in reservoir i and T is the discrete time interval, a new state equation may be obtained with new state variable

$$\Delta x_i'(k+1)= \frac{\Delta V_i(k+1)}{V_{i,max}-V_i^N}-\frac{\Delta V_j(k+1)}{V_{j,max}-V_j^N}. \quad (9)$$

Although other schemes may be envisaged, we consider a specific reference reservoir No. j in (9) while $i=1,\ldots,n_x$, $i\neq j$. Note that the modification (9) is applied only to the reservoir state equations, while the other state equations (for the link outflows and for the auxiliary variables) remain unchanged.

A quadratic criterion that considers the control objectives mentioned previously has the general form (for simplicity $\Delta\mathbf{x}$ is used in the following to denote $\Delta\mathbf{x}'$)

$$J=\sum_{k=0}^{\infty}\left(\|\Delta\mathbf{x}(k)\|_{\mathbf{Q}}^2+\|\Delta\mathbf{u}(k)\|_{\mathbf{R}}^2\right) \quad (10)$$

where $\|\boldsymbol{\eta}\|_{\mathbf{S}}^2=\boldsymbol{\eta}^T\mathbf{S}\boldsymbol{\eta}$ while $\mathbf{Q}$ and $\mathbf{R}$ are nonnegative definite, diagonal weighting matrices. The infinite time horizon in (10) is taken in order to obtain a time-invariant feedback law according to the LQ optimization theory [11].

Due to the definition of $\Delta\mathbf{x}(k)$, the first term in (10) penalizes relative storage differences between reservoirs. The diagonal elements of $\mathbf{Q}$ corresponding to the reservoir storages Δx_i are set equal to 1, while the diagonal elements of $\mathbf{Q}$ corresponding to the link outflows Δq_i and those corresponding to the auxiliary variables $\Delta\tilde{x}_i$ are set equal to zero. A controller designed to minimize this criterion, will automatically tend to equalize the relative storage distribution between reservoirs. This is an indirect way of achieving overflow minimization for the sewer network.

By the choice of the weighting matrix $\mathbf{R}$, i.e. its diagonal elements, the magnitude of the control reactions can be influenced. This is necessary in order to avoid high feedback parameters that would lead to nervous control behavior. Moreover, it provides the possibility to consider, to a certain extent, indirectly, the constraints (2), because increased values of the diagonal weighting parameters will lead to lower deviations of outflows from their nominal values. However, the strict consideration of the constraints (2) is not guaranteed by the quadratic criterion and must be imposed after the feedback law calculations, i.e. the control variables must be truncated according to (2). The choice of the diagonal matrix $\mathbf{R}$ is performed by a *trial-and-error* procedure so as to achieve a satisfactory control behavior for a given application network.

The inflow r(k) into the treatment plant is not included in the control vector $\mathbf{u}$, but is set $r(k) = r_{max}$.

3.2 Multivariable control law

Two multivariable controllers, one with and another without feedforward terms, were designed via the LQR methodology in order to investigate both the reactive and anticipatory regulator behavior. The minimization of the performance criterion (10) subject to the linearized state equation, when inflow predictions are available ($\Delta\mathbf{d}(k)\neq\mathbf{0}$), leads to the control law

$$\mathbf{u}(k)=\mathbf{u}^N-\mathbf{L}\cdot\Delta\mathbf{x}(k)-\mathbf{U}(k) \quad (11)$$

where $\mathbf{L}$ is a constant feedback gain matrix calculated from the well-known Riccati equation. The time-variant (feedforward) vector $\mathbf{U}(k)$ is calculated in real time using at each time instant k the predictions $\Delta\mathbf{d}(\kappa)$, $\kappa=k,\ldots,k+K-1$, where K is the prediction horizon. In the case of sewer network control, K corresponds to the horizon of the real-time available inflow predictions K_p (taken from a predictive rainfall-runoff model) plus the inflow predictions obtained by the use of a simple extrapolation scheme [3]. In the present study, the extrapolation scheme uses the known values of the last three time intervals K_p-1, K_p-2, K_p-3 to predict, using linear regression, the inflow values for the next 20 min after which the inflows are assumed to move towards dry weather flow values, which they reach 20 min later. If $\Delta\mathbf{d}(k)=\mathbf{0}$, the time-variant vector $\mathbf{U}(k)$ vanishes leading to a purely feedback control law in (11).

The *state feedback regulator* (11) requires availability of measurements for *all* state variables in real time. In the sewer network context, measurements are typically available for the reservoir storages and possibly for some link outflows, but not necessarily for the retarded auxiliary variables. Thus, if full real-time measurements are lacking, some sort of state estimator may have to be developed in order to estimate the missing measurements in real time [3].

4 Application Example

4.1 Application Network

To assess the efficiency of the described methodology in reducing the overflows and more generally in satisfying the control objectives when applied to a real sewer network, an extended investigation was performed for the sewer network of Obere Iller (Bavaria, Germany). This network connects five neighboring cities to one single treatment plant. The network has very long sewer stretches with accordingly long flowing times in the link elements. The simplified model of this network is depicted in Figure 1 whereby reservoir 7 is a storage element created by installing a control gate to regulate the flow at the end of a voluminous sewer in the network without overflow capability. There is, however, for emergency needs, a bypass of the control gate (a weir over the gate), so that in case of an overload, an overflow $q_{over,7}$ is created that enters the sewer 5 through nodes 4 and 5.

For the simplified model of this sewer network the equations (1) are used. The discrete time interval T is taken equal to 180 s for the control and 60 s for the KANSIM-simulation. The treatment plant has maximum capacity $r_{max} = 2\ m^3/s$. For the LQ formulation of the present problem, the auxiliary variables (3) are used in order to take into account the time delays, and the corresponding equations are added to the state equations. Thus, taking into account the time delays, we have 159 state variables for the present problem (10 for the reservoirs, 6 for the link outflows, and 143 for the auxiliary variables) [3].

Various scenarios of external inflows [3] were used in order to investigate the efficacy of the multivariable regulator for the particular network under different circumstances. The control results for one of these scenarios, which has fairly inhomogeneous external inflows, are presented in this paper.

4.2 Applied methodologies

The multivariable controllers were programmed and were connected as an additional module to the simulation program KANSIM. For the design procedure, reservoir 7 of the particular application network is considered as the reference reservoir j as this reservoir is geographically in the center of this sewer network. Thus we have as state variables $\Delta x_i = (V_i - V_i^N)/(V_{i,max} - V_i^N) - (V_7 - V_7^N)/(V_{7,max} - V_7^N)$, i=1,...,11, i≠7.

After the calculation of the control variables from (11) for the LQ regulators with or without feedforward terms, a water level control scheme is activated if necessary to keep the water level in reservoir 7 near the value $h_{w,7}$ (height of the overflow weir of reservoir 7) by appropriate operation of the control gates of the reservoirs upstream of reservoir 7 and by this way to avoid the overloading of this storage element [3].

KANSIM also simulates the underlying actions of local direct control [7] within the multilayer control structure. This program is also used to simulate the no-control case, so as to illustrate the achievable improvements via application of an efficient central control strategy to the particular network. In the no-control case, the gates are assumed opened to 28%, 27%, 100%, 50%, and 100% of their maximum opening height for reservoirs 1, 2-6, 7, 8-10, and 11, respectively. The selection of the above percentages for the opening heights of the orifices, which have different geometric characteristics, was performed by doing many simulation investigations using different percentages so as to achieve acceptable fixed-control performance without overloading reservoir 7.

5 Results

5.1 No-control case

The simulation results of the no-control case are summarized in Table 1. In the scenario considered here which has a duration of 6 h, external inflows are stronger downstream of reservoir 7 than upstream of reservoir 7. Reservoirs 10, 8, and 9 receive very strong external inflows (d_{11}, d_8, d_{10}, respectively) and thus, large overflows appear in these reservoirs (Table 1, Figure 2a) when no control actions are taken. Reservoirs 1 and 2 are also overflowing, although they do not have very strong external inflows. This is due to the opening height of the gates of reservoirs 1 and 2. However, it should be noted that the selection of the percentages of the opening heights of the orifices leads to the avoidance of overloading of reservoir 7 for this particular scenario [3].

5.2 Multivariable regulator without feedforward terms

The main observations in the scenario presented here are summarized in the following remarks:

- The regulator manages to significantly reduce the total overflows in the network (Table 1) compared to the no-control case. During the critical period where overflows occur, the regulator closes reservoir 7 in order to equalize its relative storage with that of the other reservoirs and so, reservoirs 8, 9 and 10, that are strongly overflowing in the no-control case, can have high outflows which leads to a significant reduction of their overflows (Figure 2b). At the same time, the overflows of reservoirs 1, 2 are completely avoided (Table 1).
- The treatment plant is fed with its maximum capacity, and so the network is emptied as soon as possible in order to have free storage space for a possible future rainfall.

5.3 Multivariable regulator with feedforward terms

The multivariable regulator with additional feedforward terms anticipates to some extent the impact of future inflows. For the various scenarios of external inflows used in [3] the results obtained using the regulator with feedforward terms are equally efficient or slightly superior to the control results obtained using the regulator without feedforward terms, depending upon the particular inflow event.

When accurate inflow predictions are assumed available for the whole simulation horizon, the results of Table 1 are obtained for the scenario considered here. The regulator with feedforward terms results in slightly less overflows (Table 1) than the regulator without feedforward terms. This is because the regulator with feedforward terms, knowing about the large inflow peaks that will reach reservoirs 8, 9 and

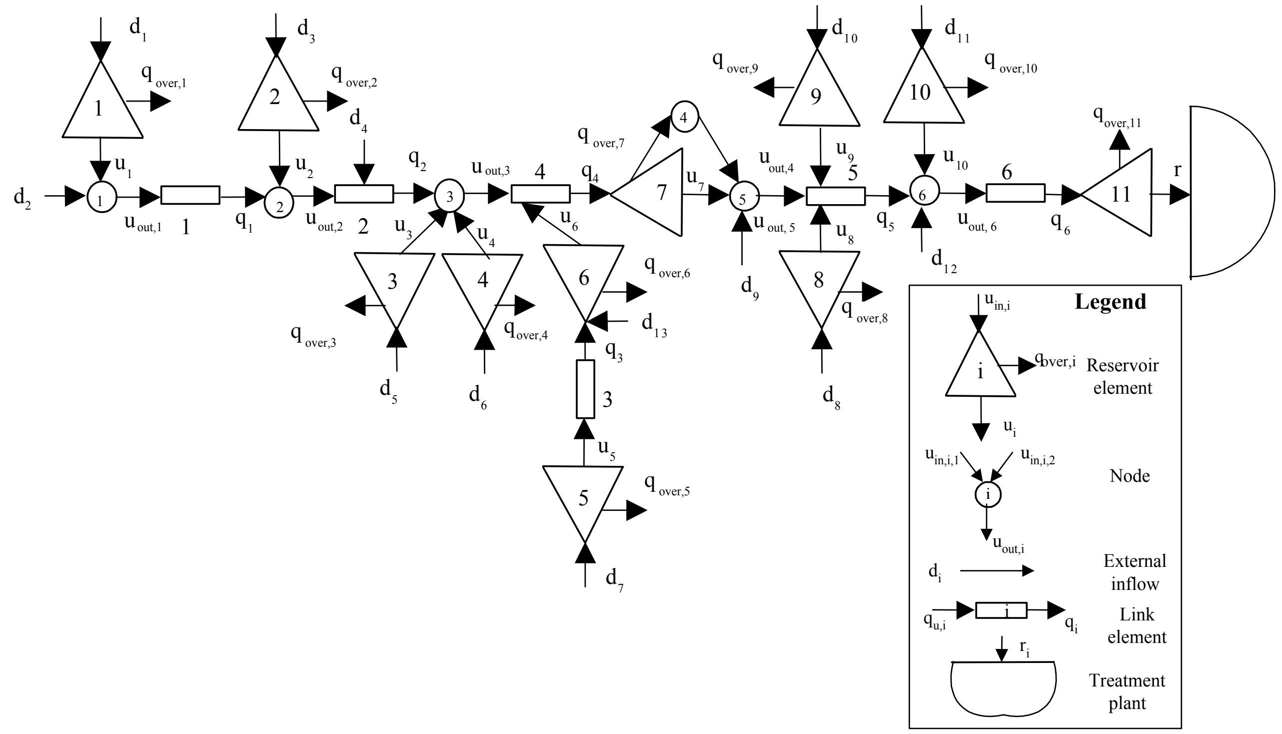
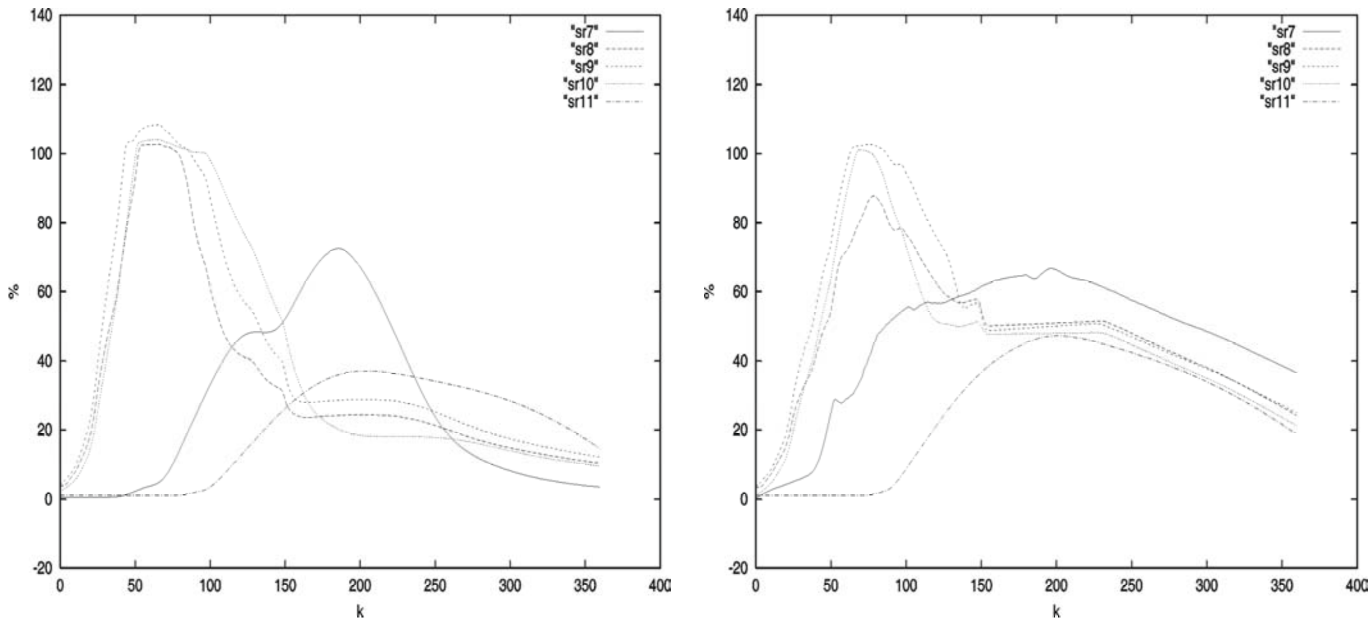


Figure 1: Application network.

10, retains more water in reservoir 7, during a certain critical period, and thus, reservoirs 9 and 10 can have greater outflows and smaller overflows than the ones in the case without feedforward terms [3].

The impact of inaccurate inflow predictions on the regulator's behavior is also investigated. Thus, the regulator with feedforward terms is applied when accurate inflow predictions are only available for 60 min (K_p =20), 30 min (K_p =10) or when there are no available predictions (K_p = 0, that is we have only extrapolation of current and past inflow values). The results are summarized in Table 1 and they can be seen to be very similar to the ones obtained with accurate inflow prediction. However, when only past values



(a) (b)

Figure 2: Relative reservoir storages $(V_i(k)/V_{i,max})100\%$ for i=7,…11, for the no-control case (a) and for the regulator without feedforward terms (b).

Rese-rvoir	No control	Regulator without feedforward terms	Regulator with feedforward terms K_p=K	Regulator with feedforward terms K_p=20	Regulator with feedforward terms K_p=10	Regulator with feedforward terms K_p=0
1	541	0	0	0	0	0
2	72	0	0	0	0	0
3-6	0	0	0	0	0	0
8	471	0	0	0	0	0
9	1150	252	217	217	218	246
10	2047	98	83	83	83	98
11	0	0	0	0	0	0
Total	4281	350	300	300	301	344
7	0	0	0	0	0	0

Table 1: Reservoir overflows and overload of reservoir 7 in [m^3].

are used for the prediction (K_p=0) and, thus, an underestimation or overestimation of the future inflow values is more likely, the results obtained may not always be as good as the ones obtained with accurate inflow predictions. Indeed for K_p=0, results are slightly inferior to the ones of the regulator with feedforward terms and accurate inflow predictions but are quite similar to the ones of the regulator without feedforward terms.

6 Conclusions

A generic problem for central sewer network control has been outlined. The developed methods, multivariable feedback regulator with and without feedforward terms, have been applied to a large real-life sewer network. A realistic simulation model has been used as a representation of the real network for the assessment of the control results. The results obtained were very satisfactory and were significantly better than the ones obtained when no control actions were taken. These results indicate that the main goals required by a control system for combined sewer networks are met by these control methods.

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