A NOVEL WORK IN PROGRESS BASED PRODUCTION CONTROL SYSTEM

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Abstract

A stability and controllability analysis of a production control scheme based on the demand forecast, on finished product inventory level and work in progress feedback signal is presented. The controllability analysis helps to identify the causes of the production control inefficiency in recovering inventory offsets when an unpredictable event changes the work in progress or the completion rate values. A modification to the original scheme is also proposed. The modified scheme ensures the recover of inventory offset in all working conditions and it also presents reduced production fluctuations compared to the original scheme. Moreover the modified scheme is easy to implement in an operating plant.

1 Introduction

In make-to-stock manufacturing systems demand is met from a finished good inventory. The production facilities aim is to maintain a small positive inventory, to avoid stock outs, and constrained containing holding costs. For a single stage manufacturing system inventory costs can be minimized by an optimal safety stock and adopting a base-stock policy [1]: produce when inventory follows below the safety stock and idle otherwise.

The aim of a production control system is to determine the manufacturing rate (*i.e.*, the production orders to be placed on the pipeline in a time interval) which maintains the inventory close to the optimal level, while minimizing short terms fluctuation in production rate. The reason for this is that in many industries production costs increase when the production rate changes rapidly [2].

Many authors have approached the problem of designing an efficient production control system from an automatic control perspective. Starting from the pioneering work of Simon [3], different models have been developed by applying both 'classical' and 'modern' control theory (see [4] for a survey of the first models). In most of the models presented in literature the decision on the orders to be produced is based on one or more of three fundamental information flows, namely demand, finished product inventory level and work in progress (see Fig. 1). Simon [3] and Towill [5] have used the inventory level and market demand as feedback and feedforward signal respectively. Wiendahl and Breithaupt [6], John, Naim and Towill [7] and Grubbström and Wikner [8] have used work in progress, in different contexts, as an additional feedback signal to recover more rapidly inventory offsets, *i.e.* the difference between the desired inventory level and the actual inventory level. Despite the presence in manufacturing literature of production control systems based on the scheme of Fig. 1, the lack of any analysis of the system stability has been noticed [9].

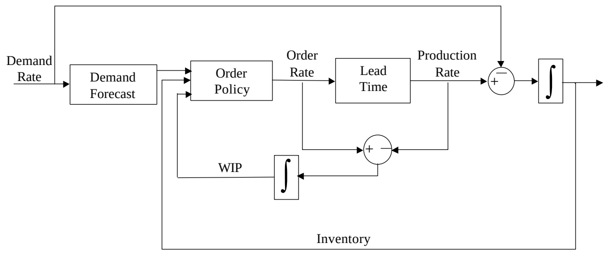

Figure 1: Basic production and inventory control scheme [8]

Here we present the stability and controllability analysis of production control models based on the scheme of Fig. 1 where the lead time is modelled using a first order lag. We will refer in particular to the automatic pipeline, inventory and order based production control system (APIOBPCS) developed by John, Naim and Towill [7]. We will demonstrate that APIOBPCS presents a non stabilizable mode which is responsible for the system inability to recover inventory offsets under some working conditions. We will present a modification of the scheme, named PICS for production and inventory control scheme, which addresses APIOBPCS inefficiency and is also easier to implement in an operating plant.
The modification we present is applied to APIOBPCS but the key idea has a general validity. It can be applied to all production control schemes where work in progress is evaluated as the difference between Order Rate and Production Rate (see Fig. 1) and it is fed back to the system.

The paper is organized as follows. In Section 2 the structure and the dynamics of APIOBPCS is analyzed as it constitutes the backbone of our model. In Section 3 we present the PICS. Finally in Section 4 we give some indications on possible future works.

2 Automatic Pipeline Inventory and Order Based Production Control System (APIOBPCS)

2.1 Model Description

The APIOBPCS is a production control model for a single stage, single product manufacturing system with continuous flow, unconstrained production capacity and infinite availability of raw material. Production is organized according to a "make-to-stock" scheme: the product is manufactured and placed in stock while external demand is met from the available stock of finished product inventory. Unsatisfied demand is backordered. As production runs continuously, the production control system issues continuously the level of the manufacturing rate to be achieved, on the basis of the information available regarding market demand, the inventory excess or deficiency and the production flowing into the pipeline.

The structure of the system is represented in Fig. 2. The control system reference is the optimal safety stock, or desired inventory (DINV), assumed to be constant, here in particular equal to zero. The system output is the actual inventory level of finished goods (AINV). The difference (positive or negative) between DINV and AINV is the inventory error (EINV) which is fed back to the system. The customer orders per unit of time (CONS) is the external load of the system; its smoothed average (AVCON) is used as a feedforward signal for the control system. The work in progress on the pipeline is indicated with WIP while the desired level of work in progress (DWIP) is evaluated, according to Little's law [2, page 5], on the basis of the average of the external demand (AVCON) and of the estimated lead time $\hat{T}_p$. The difference between the desired work in progress DWIP and the WIP on the production pipeline is indicated with EWIP and is fed back to the system. Finally, the completion rate signal (COMRATE) represents the actual production rate per unit of time and the order rate signal (ORATE) is the planned new production per unit of time issued by the controller according to the inventory, demand and pipeline policies.

A first order lag is used to model the time required to adjust the production rate to the level issued by the controller. That is the actual production rate is equal to

$$\text{COMRATE}(t) = \int_0^t \frac{1}{T_p} e^{-\frac{1}{T_p}\tau} \text{ORATE}(t-\tau) d\tau.$$

Since

$$\int_0^\infty \frac{1}{T_p} e^{-\frac{1}{T_p}\tau} d\tau = 1$$

the unit lag can be regarded as a probability function [3]. It represents the probability the lag in producing a scheduled item will be equal to τ. For large values of τ we expect this probability to be very small. The mean lag, given by

$$\int_0^\infty \tau \frac{1}{T_p} e^{-\frac{1}{T_p}\tau} d\tau = T_p,$$

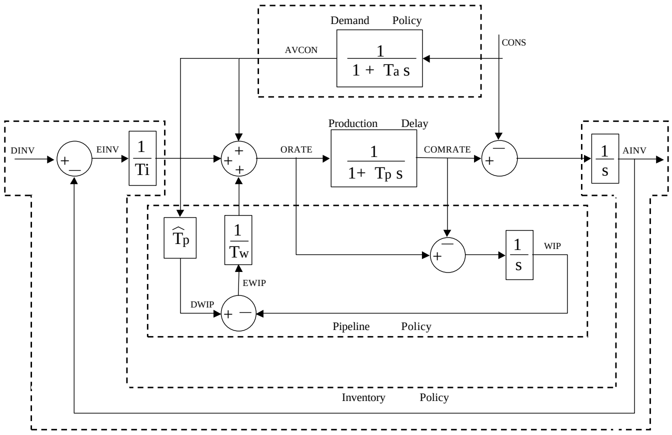

Figure 2: The Automatic Pipeline Inventory and Order Based Production Control System (APIOBPCS); the orders issued are determined on the inventory, demand, and WIP information.

represents the pipeline lead time, *i.e.*, the average amount of time a row product spends in the system ([2], page 5). Then T_p is a measure of the mean production delay for the order rate (ORATE) issued by the controller to be completed. It is difficult to predict as it changes continuously due to frequent modifications of the production plans on the shop floor. Its estimate, $\hat{T}_p$, can be evaluated by designing a measurement system that is continuously updated with production data.

The desired WIP level (DWIP) is proportional to $\hat{T}_p$, and its value depends on the accuracy of the lead time estimation. Due to the difficulties in correctly measuring $\hat{T}_p$, DWIP can be frequently wrong and this affects also the error on the WIP (EWIP) fed back to the system.

Finally, the dynamic equations of the APIOBPCS are:

$$\dot{x}_{AINV} = x_{COMRATE} - w_{CONS}, \tag{1a}$$

$$\dot{x}_{AVCON} = -\frac{1}{T_a} x_{AVCON} + \frac{1}{T_a} w_{CONS}, \tag{1b}$$

$$\dot{x}_{COMRATE} = -\frac{1}{T_p} x_{COMRATE} - \frac{1}{T_p T_w} x_{WIP} - \frac{1}{T_p T_i} x_{AINV} + \frac{T_w + \hat{T}_p}{T_p T_w} x_{AVCON} + \frac{1}{T_p T_i} u_{DINV}, \tag{1c}$$

$$\dot{x}_{WIP} = -x_{COMRATE} - \frac{1}{T_w} x_{WIP} - \frac{1}{T_i} x_{AINV} + \frac{T_w + \hat{T}_p}{T_w} x_{AVCON} + \frac{1}{T_i} u_{DINV}, \tag{1d}$$

$$y = x_{AINV}, \tag{1e}$$

where we indicated by x the state variables, by u the reference input , by w the load input and by y the output of the control system.

In the following APIOBPCS simulation we will use, without loss of generality, the parameter choice indicated by Disney, Naim and Towill in [10] and reported in Table 1. This choice is recommended by the authors as it guarantees a recovery of inventory offsets with a relatively short production fluctuation. Note that it is assumed $\hat{T}_p$ equal to T_p.

Parameter	Used Value
T_p	4
T_i	0.875*T_p
T_w	5.125*T_p
T_a	2.125*T_p
$\hat{T}_p$	=T_p

Table 1: An optimal choice of T_p, T_i, T_w, T_a,$\hat{T}_p$ [10].

2.2 System Dynamics

We will now discuss the dynamics of the APIOBPCS. We will first analyze the behaviour of the system at the steady state when a constant load is applied; then we will analyze the system stability. In both cases we will show the system does not exhibit a desired behaviour in the sense that we wish the actual inventory level to be close to the desired inventory level, and the work in progress level to be consistent with the product flowing on the pipeline.

The system transfer function G_{Aw} relating the actual inventory level x_{AINV} and the consumption rate w can be derived from equations (1):

$$G_{Aw}(s)=-T_i\left[\frac{\frac{(T_p-\hat{T}_p)}{T_w}+(T_a+T_p+\frac{T_aT_p}{T_w})s+T_aT_ps^2}{(1+T_as)\left[1+(1+\frac{T_p}{T_w})T_is+T_pT_is^2\right]}\right], \quad (2)$$

and the corresponding static gain μ is equal to:

$$\mu=\lim_{s\to 0}G_{Aw}(s)=-\frac{T_i(T_p-\hat{T}_p)}{T_w}\,. \quad (3)$$

Hence from (3) we see that for a constant load (*i.e.*, the system is subject to a constant consumption rate), the steady state inventory level AINV is zero if the lead time estimate $\hat{T}_p$ is equal to the actual pipeline lead time T_p. As previously argued the estimate is always affected by errors due to the unpredictable changes of the production lead time.

From equations (1) we can also evaluate the system eigenvalues as:

$$\begin{aligned}\lambda_1 &= -\frac{1}{T_a}\\ \lambda_{2,3} &= \frac{1}{2}\frac{-K_1-K_2\pm\sqrt{{K_1}^2+2K_1K_2+{K_2}^2-4\,{T_w}^2\,T_i\,T_p}}{T_w\,T_i\,T_p}\\ \lambda_4 &= 0,\end{aligned} \quad (4)$$

where

$$K_1 = T_i\,T_p \quad (5)$$
$$K_2 = T_i\,T_w \quad (6)$$

and deduce the system is marginally stable as the eigenvalue λ_4 is equal to zero. Then for some initial conditions the system free response does not converge to zero. It also holds:

$$w_4^TB=\left[\begin{array}{cccc}0 & 0 & -T_p & 1\end{array}\right]\left[\begin{array}{cc}0 & -1\\ 0 & \frac{1}{T_a}\\ \frac{1}{T_pT_i} & 0\\ \frac{1}{T_i} & 0\end{array}\right]=\left[\begin{array}{cc}0 & 0\end{array}\right] \quad (7)$$

where w_4^T is the left eigenvector corresponding to the eigenvalue λ_4 and B is the input matrix associated with the input vector $[u_{DINV}\ \ w_{CONS}]$. Then the marginally stable mode corresponding to λ_4 is uncontrollable (see, for example, [11], page 82).

In Appendix one can find details of a Kalman's decomposition of system 1 into controllable and uncontrollable subsystems [12, page 130]. The decomposition reveals that the uncontrollable component $z_{\overline{c}}$ is equal to

$$z_{\overline{c}}=-T_p\ \ x_{COMRATE}+x_{WIP}$$

which in turn means that initial condition on WIP and COMRATE such that $z_{\overline{c}}\ |_{t=0}\neq\ 0$ cannot be driven to zero, hence the inventory level AINV does not stabilize on a value different from zero, the desired value of inventory. Fig. 3 shows the free response of APIOBPCS system for an initial condition assigned to the completion rate variable (COMRATE). The inventory level (AINV) does not recover the offsets and stabilizes on a value different the desired inventory level (DINV). Note that WIP is different from what we would have expected. As we are considering system free response, there is no more product flowing on the pipeline after the transient, while we observe that WIP value is different from zero at the steady state.

Towill, Evans and Cheema [13] have used a PI controller in the inventory error loop instead of a proportional controller. This solution stabilizes AINV free evolution but it does not affect WIP as depicted in Fig. 4.

Free response onse is equivalent to the response we would get from an impulsive change in one or more state variables. A sudden change in COMRATE is realistic in an operating plant. If a break down in the pipeline occurs, or finite products are acquired from other sources, the completion rate variable suddenly changes and a production and inventory control system built on the APIOBPCS model would not be able to recover inventory offsets with a WIP signal correspondent to the production flowing on the pipeline.

3 The Production Inventory Control Scheme (PICS)

We will now introduce the production control scheme we have developed to address APIOBPCS inefficiency. The key idea is

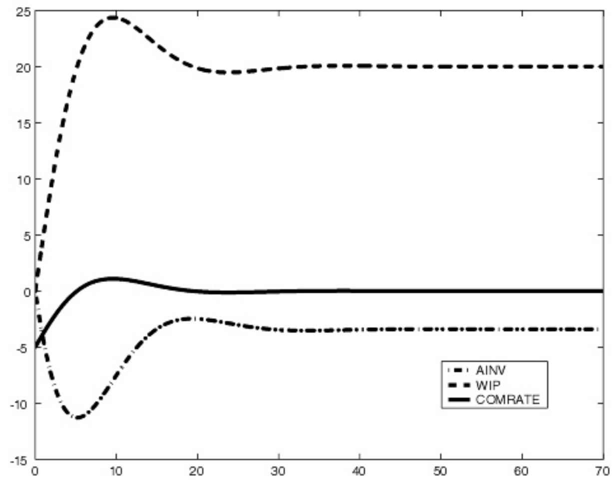

Figure 3: APIOBPCS free response for an initial condition equal to -5 assigned to the variable COMRATE

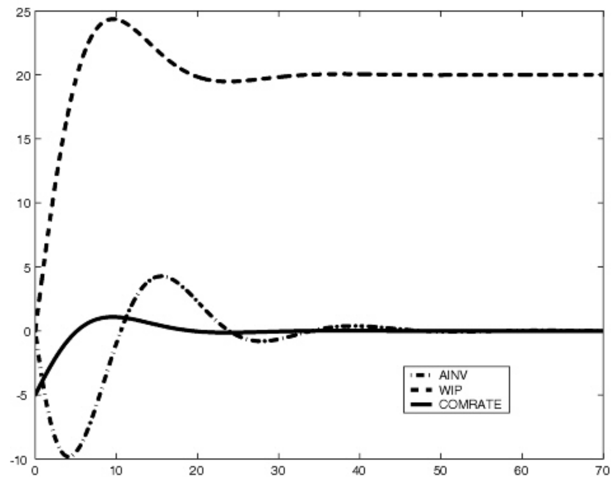

Figure 4: Free response of APIOBPCS with the addition of an integral controller to the inventory error loop. An initial condition equal to -5 has been assigned to the variable COMRATE

to replace the WIP signal in the pipeline loop with a signal proportional to the pipeline output. The resulting production control scheme has the same forced evolution of the APIOBPCS but is controllable and is able to recover inventory offsets in any working condition. We will present the modifications using the block diagram representation and analyze the dynamic properties of the resulting production control scheme.

Fig. 5(a) shows the subsystems formed by the production delay block and the WIP signal in the APIOBPCS. The subsystem transfer function relating WIP and the production order rate (ORATE) is equal to

$$\frac{\text{WIP}(s)}{\text{ORATE}(s)} = \frac{T_p}{T_p\, s + 1}, \tag{8}$$

and the cancellation of the pole in the origin determines the uncontrollablity. Hence we replace the scheme in Fig. 5(a) with that one in Fig. 5(b) which has the same transfer function.

Note that the work in progress is evaluated differently in the two subsystems. In Fig. 5 (b) $\overline{\text{WIP}}$ is a fraction of the production completion rate (COMRATE) and then it is a fraction of the pipeline output. Differently in the APIOBPCS subsystem (Fig. 5 (a))the WIP signal is a state variable equal to the difference between the completion rate (COMRATE) and the production order rate (ORATE). The WIP and $\overline{\text{WIP}}$ signals are identical when considering the forced evolution as the two subsystems have the same transfer function. Instead they have different values when considering the free response.
We believe $\overline{\text{WIP}}$ is a more reliable signal to be used in the pipeline loop as it is based on what is actually manufactured. The WIP signal in APIOBPCS depends on the orders issued by the controller (ORATE); its measure being based on the hypothesis that the orders issued would be actually produced. This is a restrictive hypothesis: production plans are often changed due to unplanned events (e.g. breakdowns, specials orders, strikes, etc.). Then, if some of the issued orders are not placed on the production line, WIP is erroneously evaluated and this affects system dynamics.

We replace the WIP in the APIOBPCS (Fig. 2) with the $\overline{\text{WIP}}$ signal and the resulting PICS scheme is represented in Fig. 6. Note that we have also replaced the lead time T_p with its measure $\hat{T}_p$ as lead time can only be estimated. Then the $\overline{\text{WIP}}$ in PICS (Fig. 6) is proportional to COMRATE through the factor $\hat{T}_p$. Both $\overline{\text{WIP}}$ and the average consumption (AVCON) are proportional to the estimated lead time (see Fig. 6), then we have included $\hat{T}_p$ in the control parameter.

Due to the modification introduced the production policy turns to be a control on the pipeline output; as the completion rate (COMRATE) is compared to the averaged demand (AVCON) and their difference is fed back to the controller.
In Fig. 7 a comparison between WIP and $\overline{\text{WIP}}$ is presented. A step is applied at $t = 5$, and a negative pulse is applied at $t = 20$ on the COMRATE signal. When considering the forced evolution the two signals are identical while their evolution is different once the impulse is applied.

Note that $\overline{\text{WIP}}$ depends on the lead time estimation and is wrong if T_p is different from $\hat{T}_p$. In PICS we have not eliminated the WIP signal (although we do not use it as a feedback signal) as it can be used to monitor the operative perturbations and to have an estimate of the production lead time. If $\overline{\text{WIP}}$ and the WIP signal differ, than some further investigation can be done in order to understand if the difference is due to the er-

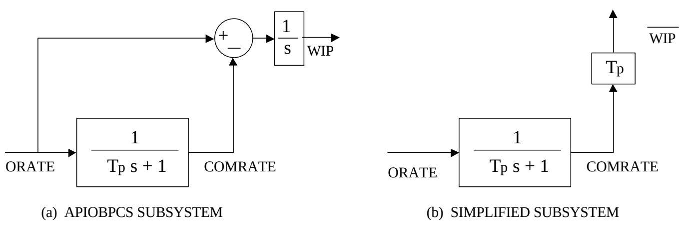

Figure 5: Block Diagram Representation of the Production Delay and WIP: a) from APIOBPCS scheme, b) a simplified representation.

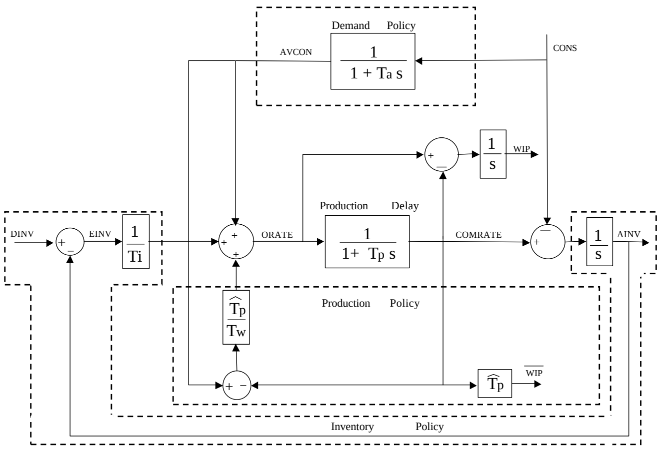

Figure 6: The PICS: we have substituted the WIP signal with $\overline{\text{WIP}}$ of figure 5. The production policy is a pipeline's output control.

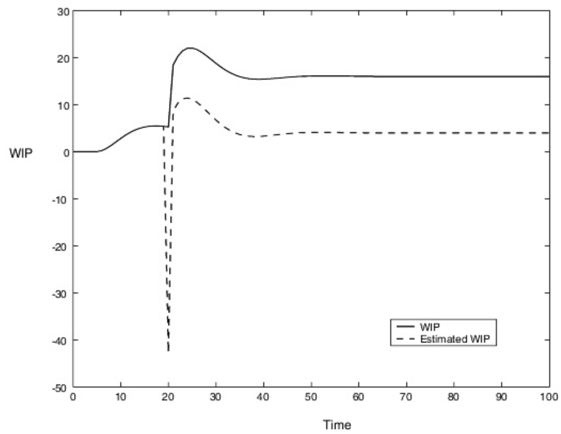

Figure 7: WIP and $\overline{\text{WIP}}$: the two signals are equal for forced evolution, while they differ when an input is applied on COMRATE signal

roneous lead time estimation, and, in this case, an appropriate correction can be applied.

We think the solution proposed is easy to implement as the measure of the completion rate can be obtained using an automatic system of measure applied to the pipeline (e.g. bar code, infrared light reading, etc.).

We conclude this section presenting the dynamic equations of the modified system. They are

$$\dot{x}_{COMRATE} = -\frac{T_w + \hat{T}_p}{T_p T_w}\, x_{COMRATE} - \frac{1}{T_p T_i}\, x_{AINV} + \frac{T_w + \hat{T}_p}{T_p T_w}\, x_{AVCON} + \frac{1}{T_p T_i}\, u_{DINV} \quad (9a)$$

$$\dot{x}_{AINV} = x_{COMRATE} - w_{CONS} \quad (9b)$$

$$\dot{x}_{AVCON} = -\frac{1}{T_a}\, x_{AVCON} + \frac{1}{T_a}\, w_{CONS} \quad (9c)$$

$$y = x_{AINV}. \quad (9d)$$

As a consequence of the WIP simplification, the PICS is of third order while APIOBPCS is a fourth order system.

3.1 Dynamics of PICS

The transfer function $\overline{G}_{Aw}$ relating the actual inventory level (AINV) and the consumption rate w_{CONS} is equal to

$$\overline{G}_{Aw} = -\frac{(K_3 + \frac{T_a K_1}{T_w} + K_1)s + K_2 s^2}{1 + \left(T_a + T_i + \frac{K_1}{T_w}\right)s + \left(K_3 + \frac{T_a K_1}{T_w} + T_p T_i\right)s^2 + K_2 s^3} \quad (10)$$

where

$$K_1 = T_i \hat{T}_p, \quad K_2 = T_p T_i T_a, \quad K_3 = T_a T_i.$$

Since

$$\lim_{s \to 0} \overline{G}_{Aw} = 0, \quad (11)$$

the static gain is always equal to zero, independently from the lead time estimation. Then for a constant consumption rate the PICS is always able to recover inventory offsets.

The eigenvalues of PICS are equal to

$$\lambda_1 = -\frac{1}{T_a}$$

$$\lambda_2 = \frac{1}{2}\frac{-K_1 - K_2 + \sqrt{{K_1}^2 + 2K_1 K_2 + {K_2}^2 - 4\,T_i T_p {T_w}^2}}{T_i T_p T_w}$$

$$\lambda_3 = -\frac{1}{T_a}$$

$$\lambda_2 = \frac{1}{2}\frac{-K_1 - K_2 - \sqrt{{K_1}^2 + 2K_1 K_2 + {K_2}^2 - 4\,T_i T_p {T_w}^2}}{T_i T_p T_w} \quad (12)$$

$$K_1 = T_i T_w, \quad K_2 = T_i \hat{T}_p$$

and it can be shown the system to be asymptotically stable. It is also easy to show that the controllability matrix has full rank and then the system is controllable.

In Fig. 8 the free response of the PICS variables is depicted. An initial condition equal to -5 is assigned to the variable COMRATE. The PICS free response is different from APIOBPCS (see Fig. 3 where the same initial condition was assigned to COMRATE). At the steady state the inventory level is equal to the desired level and therefore the system recovers the offsets, as desired.

Fig. 9 represents APIOBPCS free response when an integral controller is added in the inventory error loop. In the same figure the free response of the PICS is represented. In both cases an initial condition equal to -40 is applied to the variable

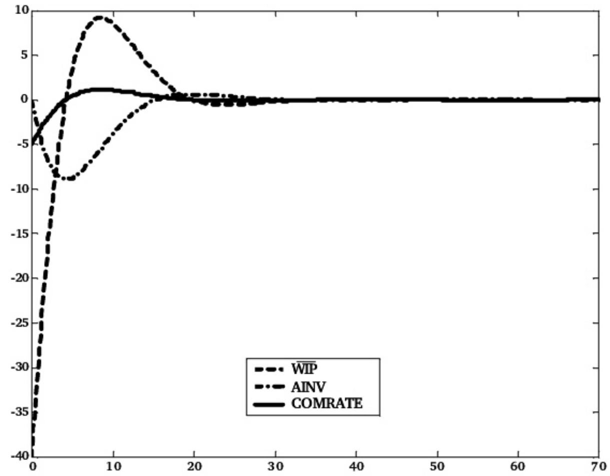

Figure 8: The PICS free response

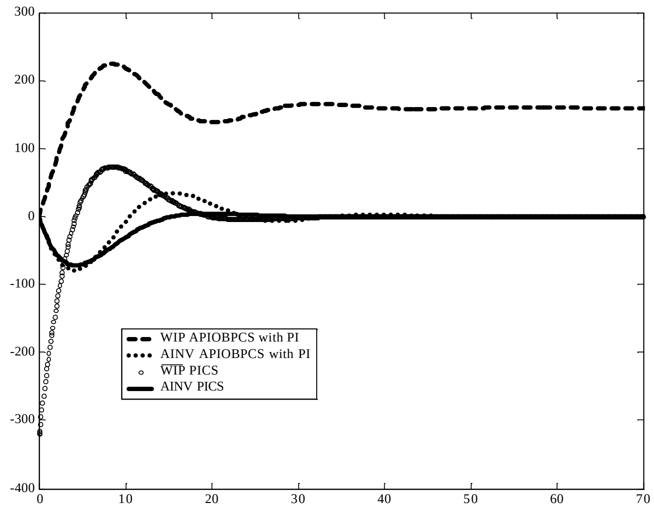

Figure 9: A comparison between APIOBPCS free responses with the addition of an integral controller to the inventory error loop, and of PICS free response

COMRATE. Note that AINV in the APIOBPCS with the PI controller reaches the desired inventory value, but with larger oscillation compared AINV evolution in PICS. Moreover, the WIP signal of the APIOBPCS is non zero at steady state, confirming its uncontrollability as discussed in section 2.2.

4 Conclusions

We have studied the structural properties of APIOBPCS and showed it presents a marginally stable and uncontrollable mode. The system has been modified by replacing the WIP error feedback signal with a signal proportional to the pipeline output. The resulting system named PICS is asymptotically stable and controllable, and can be easily implemented using an automatic measure system of the production output. We believe the modification introduced to APIOBPCS can be extended to other production control systems based on the same control scheme where the WIP error control can be replaced by a control on the production output.

We have analyzed a control scheme for a single stage manufacturing system. A complex manufacturing system is generally composed by many sequential and parallel sub-process or work units with possibly buffers which keep the production continuity if the line is interrupted. Then, if an initial operation is stopped or is slowed down by an unpredictable event, the pipeline output will be affected at some other time, since the WIP on the pipeline will continue to be processed.

Further investigations should be done on a multilevel and multistage production system in order to test the applicability of the solution proposed also to a more articulated production process. This is the subject of ongoing researches.

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5 Appendix

We apply a system transformation such that the controllable and uncontrollable state variables in system (1) can be clearly identified. We follow the procedure in [12] (see page 130).

We modify PICS system using the variables transformation:

$$z(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -Tp & 1 \end{bmatrix} x(t).$$

The $\bar{A}$, $\bar{B}$ and $\bar{C}$ matrices of the new PICS realization are equal to

$$\bar{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{T_a} & 0 & 0 \\ -\frac{1}{T_p\ T_i} & \frac{T_w + \hat{T_p}}{T_w\ T_p} & -\frac{T_w + T_p}{T_w\ T_p} & -\frac{1}{T_w\ T_p} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (13)$$

$$\bar{B} = \begin{bmatrix} 0 & -1 \\ 0 & \frac{1}{T_a} \\ \frac{1}{T_p\ T_i} & 0 \\ 0 & 0 \end{bmatrix}, \bar{C} = [\, 1\ 0\ 0\ 0 \,].$$

Looking to $\bar{A}$ matrix we deduce that the uncontrollable variable $z_{\bar{c}}$ is equal to the fourth component of the state vector z. It is straight forward to see:

$$z_{\overline{c}} = -T_p\ \ x_{COMRATE} + x_{WIP}.$$