FAULT DETECTION AND ISOLATION ON FLEXIBLE-JOINT MANIPULATORS

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Abstract

In this paper, first we survey Fault Detection and Isolation (FDI) for nonlinear systems. A necessary condition for the problem to be solvable is derived in terms of an *unobservability* distribution, which is computable by means of suitable algorithms. Then we apply this method on m-link manipulators with flexible joints in order to detect actuator faults. The simulation result for the case of a single-link flexible joint manipulator reveals the effectiveness of approach.

1 Introduction

The problem of FDI in dynamical system is the problem of generating diagnostic signals sensitive to occurrence of faults. Regarding a fault as an input action on the system, a diagnostic signal must be able to detect its occurrence, as well as isolate this particular input from other inputs (e.g. disturbances and other faults) affecting the system behavior. One specific diagnostic signal (also called *residual*) must be generated per each fault to be detected. Each diagnostic signal is sensitive only to one particular fault. In terms of above definition, the problem of FDI may be viewed as a problem of designing a system which, processing all available information about the plant, yields a noninteractive map between faults (viewed as inputs) and residuals (viewed as output).

This problem has attracted a good deal of attention since its formulation by Beard [2] and Jones [9]. The original work of these authors addresses the problem in a fashion that corresponds to the solution of dual version of a problem of noninteracting control by means of a memory-less feedback. Later, Massoumnia *et al.* [12], [13] have shown that the problem can be addressed and successfully solved in a more general setting, which turns out to correspond to the solution of dual version of a problem of noninteracting control by means of dynamic feedback. In this way, a number of obstructions inherent in the Beard-Jones approach, namely the necessity of a *vector relative degree* and stability of certain *fixed modes* were removed.

The most important issue of FDI in the presence of noise-corrupted measurements has been thoroughly investigated, in a framework corresponding to that considered by Beard and Jones in the noise free case, by Speyer and coauthors in the series of recent paper [1].

Fault detection is important in many robotic applications. Failures of powerful robots, fast robots, or robots in hazardous environments are quite capable of causing significant and possibly irreparable havoc if they are not detected promptly and appropriate action is taken. Since robots are commonly used because of their various potentials with respect to those of human, fault detection is then a common and serious concern in the robotics arena [4,11,17, 18].

Focuses on a nonlinear version of an observability-based fault detection method known as analytical redundancy (AR) may be found in the work of Leuschen [11]. In addition, applications of NLAR in the field of robotics are presented in the work [12].

The paper is organized as follows. In section II, we introduce some notations and definitions. Two algorithms will then presented to estimate observable and unobservable codistributions. Then, we change the coordinate in the way our new model disparts the observability and unobservability equations. At last, in the new coordinate, we introduce the residual generator structure [14], [15].

By using the results of Section II, we design m residual generator to detect and isolate actuator faults for m-link manipulators with flexible-joint in the presence of disturbance signals. This comprises Section III.

The simulation results for the case of a single-link flexible joint manipulator will be given in Section IV and Section V is devoted to conclusion.

2 Problem Formulation

Before problem formulation, we describe what we mean by fault and disturbance and then describe the difference between them as follows.

From the point of view of how good a control system works, indeed, a fault may be as bad as disturbance. Also, as one might have seen, faults are modeled as extra inputs and hence treated truly as disturbances. From the appearance point of view, a fault is the same as a disturbance. The difference is that, when a fault is detected, usually it can be repaired. On the other hand, a disturbance is something against which no repair is generally possible.

We consider systems modeled by equation of the form

$$\dot{x} = g_0(x) + \sum_{i=1}^{m} g_i(x)u_i + l(x) f_1 + \sum_{i=1}^{d} p_i(x) w_i \qquad (1)$$
$$y = h(x)$$

with state *x* defined in a neighborhood of the origin in R^n, inputs

$$u = [u_1 \quad \cdots \quad u_m]^T \in R^m, \quad f_1 \in R, w \in R^d$$

and output $y \in R^q$, in which $g_i(x)$, $l(x)$ and $p_i(x)$ are smooth vector fields, *h(x)* is a smooth mapping and $g_0(0) = 0, \quad h(0) = 0$.

The three sets of components *u*, f_1 and *w* of the input of (1) correspond, respectively, to an input channel *u* to be used for control purposes, to a *fault* signal f_1 and *w* whose occurrence has to be detected, and to a *disturbance* signal *w* as well as other fault signals from which the specific fault f_1 has to be isolated. The main problem addressed in this paper is the design of a residual generator, modeled by the equations of the form

$$\dot{\hat{x}} = \hat{g}_0(\hat{x}, y) + \sum_{i=1}^{m} \hat{g}_i(\hat{x}, y)u_i \qquad (2)$$
$$r = \hat{h}(\hat{x}, y)$$

with the state $\hat{x}$ define in the a neighborhood of the origin in $R^{\hat{n}}$, inputs *u* and *y*, and output $r \in R^{\hat{q}}$ with $\hat{q} \le q$, in which $\hat{g}_0(\hat{x}, y)$ and $\hat{g}_i(\hat{x}, y) \quad (i = 1, \cdots, m)$ are smooth vector fields, $\hat{h}(\hat{x}, y)$ is a smooth mapping, and $\hat{f}(0,0) = 0, \quad \hat{h}(0,0) = 0$, such that the response *r*(*t*) of the augmented system

$$\dot{x}^e = \frac{d}{dt}\begin{pmatrix} x \\ \hat{x} \end{pmatrix} = \begin{pmatrix} g_0(x) \\ \hat{g}_0(\hat{x}, h(x)) \end{pmatrix} + \sum_{i=1}^{m} \begin{pmatrix} g_i(x) \\ \hat{g}_i(\hat{x}, h(x)) \end{pmatrix} u_i + \begin{pmatrix} l(x) \\ 0 \end{pmatrix} f_1 + \sum_{i=1}^{d} \begin{pmatrix} p_i(x) \\ 0 \end{pmatrix} w_i \qquad (3)$$
$$r = \hat{h}(\hat{x}, h(x))$$

depends nontrivially on (i.e., is affected by) the input f_1, depends trivially on (i.e., is decoupled from) the input *w* and asymptotically converges to zero whenever f_1 is identically zero, no matter what *u* is.

For finding the conditions in designing the residual generator (3), we state the following theorem [7].

Consider the system:

$$\dot{x} = g_0(x) + \sum_{i=1}^{m} g_i(x)u_i \qquad (4)$$
$$y = h(x)$$

and let Ω_O denotes the smallest codistribution invariant under $g_0(x), \cdots, g_m(x)$ which contains $span\{dh\}$.

Theorem: In (4) if $g_i(x) \in \Omega_O{}^{\perp}$, then the output *y* is decoupled from input u_i and if $g_i(x) \notin \Omega_O{}^{\perp}$ then *y* is affected from u_i. ◁

Now, consider the system (3) and let Ω_O^e denote the smallest codistribution, which contains $span\{dr\}$ and is invariant under $g_i^e(x) \quad (i = 1, \cdots, m)$ and $p_i^e(x) \quad (i = 1, \cdots, d)$, where

$$g_i^e(x) = \begin{pmatrix} g_i(x) \\ \hat{g}_i(\hat{x}, h(x)) \end{pmatrix}, \quad p_i^e(x) = \begin{pmatrix} p_i(x) \\ 0 \end{pmatrix}$$

In view of the above theorem, it can be asserted that the output *r*(*t*) of (3) depends nontrivially on input f_1 if

$$l^e(x) \notin \Omega_O^{e\,\perp}$$

and when $f_1 = 0$, depends nontrivially on the input w_i if

$$p_i^e(x) \in \Omega_O^{e\,\perp} \qquad i = 1, \cdots, d.$$

Now we obtain Ω_O^e by an algorithm quoted in [14-15]. Consider again system (3). Let Θ be a fixed coditribution and define the following nondecreasing sequence of coditribution:

$$Q_0 = \Theta \cap span\{dh\}$$
$$Q_{k+1} = \Theta \cap \left(\sum_{i=0}^{m} L_{g_i} Q_k + span\{dh\} \right). \qquad (5)$$

Suppose that all coditribution of this sequence are nonsingular, so that there is an integer $k^* \le n-1$ such that $Q_k = Q_{k^*}$ for all $k > k^*$ and set $\Omega^* = Q_{k^*}$. It is convenient to use the notation $\Omega^* = o.c.a.(\Theta)$ to stress the dependence on Θ of the codistribution $\Omega^* = Q_{k^*}$ at which the algorithm(5) has stopped. Clearly if Ω^* stands for Θ in (5) then $o.c.a.(\Omega^*) = \Omega^*$.

We say that a distribution Ω is an observability codistribution for (3) if

$$L_{g_i}\Omega \subset \Omega + span\{dh\}, \quad i = 0, \cdots, m$$
$$o.c.a.(\Omega) = \Omega. \qquad (6)$$

We may say that Δ is an unobservability distribution if $\Omega = \Delta^{\perp}$ is an observability codistribution.

Let $o.c.a.(\Sigma^P)^{\perp}$ be the largest codistribution. If the algorithm (5) is initialized at $(\Sigma^P)^{\perp}$ then $o.c.a.(\Sigma^P)^{\perp}$ is an observability coditribution contained in $P^{\perp}$. We construct (Σ^P) as follow:

Let $P = span\{p_1, \cdots, p_d\}$ and consider the nondecreasing sequence of distribution defined as follows:

$$S_0 = \overline{P}$$
$$S_{k+1} = \overline{S}_k + \sum_{i=0}^{m} \left[g_i, \overline{S}_k \cap \ker\{dh\} \right] \qquad (7)$$

where $\overline{S}$ denote the involutive closure of *S*. Suppose there exits an integer k^* such that $S_{k^*+1} = \overline{S}_{k^*}$ and set $\Sigma^p = \overline{S}_{k^*}$. Then Σ^p is involutive, contains *P* and is conditionally invariant.

After finding Σ^p and Ω one may do a coordinate transformation based on the properties of the observability codisribution algorithm, which is quite useful in addressing the problem of designing residual generator.

Proposition 1: Consider system (3). Let Ω be an observability codistribution. Let n_1 denote the dimension of Ω. Suppose that Ω is locally spanned by exact differentials. Suppose $span\{dh\}$ is nonsingular. Let $p - n_2$ denote the dimension of $\Omega \cap span\{dh\}$ and suppose there exists a smooth surjection $\psi_1 : R^p \to R^{p-n_2}$ such that

$$\Omega \cap span\{dh\} = span\{d(\psi_1 \circ h)\}.$$

Fix $x^\circ \in X$ and let $y^\circ = h(x^\circ)$. Then, there exists a selection matrix *H* (i.e, a matrix in which any row has only one nonzero element, which is equal to 1) such that

$$\psi(y) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \psi_1(y) \\ Hy \end{pmatrix} \quad (8)$$

is local diffeomorphism at y° in R^p .Chose a neighborhood U° of x° and a function $\Phi_1 : U^\circ \to R^{n_2}$ such that

$\Omega = span\{d\Phi_1\}$

at any point of U°. Then there exists a function $\Phi_3 : U^\circ \to R^{n-n_2-n_2}$ such that

$$\Phi(x) = [x_1, x_2, x_3]^T = [\Phi_1(x), Hh(x), \Phi_2(x)]^T \quad (9)$$

is a local difeomorphism at $x^\circ \in X$.

In the new coordinates defined by (8) and (9), system (4) may be described by equations of the from

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2) + g_1(x_1, x_2)u \\ \dot{x}_2 &= f_2(x_1, x_2, x_3) + g_2(x_1, x_2, x_3)u \\ \dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3)u \\ y_1 &= h(x_1) \\ y_2 &= x_2 \end{aligned} \quad (10)$$

If $p(x)$ is a vector field in the null space of Ω, then, trivially in the new coordinate this vector field is expressed in the form $(0 \quad p_2^T(x_1, x_2, x_3) \quad p_3^T(x_1, x_2, x_3))$.

After finding Ω, we describe how to design residual generator in the case of nonlinear system. For the sake of simplicity, we consider the case in which y_1 is one-dimensional. Extension to the case in which y_1 is a vector requires appropriate modification of the following proposition.

Motivated by the previous discussion, we focus our attention on the system

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, y_2) + g_1(x_1, y_2)u \\ y_1 &= h(x_1) \end{aligned} \quad (11)$$

viewed as a system with inputs (u, y_2) and output y_1. Now address how to design an asymptotic observer for x_1. We have already shown that, under mild hypotheses, such a system is guaranteed to be locally weakly observable construction; however we also know that this property may be insufficient for the purposes of designing an asymptotic observer. To this end, a stronger observability property, introduced in [6], has to be assumed.

Proposition 2. The maps f_1, g_1, h_1 in (11) are analytic and globally defined. Moreover, there exists a globally defined analytic change of coordinate $H : x_1 \to \xi$ that transforms system (11) into a system of the form

$$\begin{aligned} \dot{\xi}_1 &= \phi_1(\xi_1, \xi_2, y_2, u) \\ \dot{\xi}_2 &= \phi_2(\xi_1, \xi_2, \xi_3, y_2, u) \\ &\cdots \\ \dot{\xi}_{n_1} &= \phi_{n_1}(\xi_1, \xi_2, \cdots, \xi_{n_1}, y_2, u) \\ y_1 &= h_0(\xi_1) \end{aligned}$$

and

$$\frac{\partial h_0}{\partial \xi_1} \neq 0, \quad \frac{\partial \varphi_1}{\partial \xi_2} \neq 0, \quad \cdots, \quad \frac{\partial \varphi_{n_1}}{\partial \xi_{n_1}} \neq 0$$

for all ξ, y_2, u.

We consider a residual generator of the form

$$\begin{aligned} \dot{\hat{x}}_1 &= \varphi_1(\hat{x}_1, \hat{x}_2, y_2, u) + Gk_1(y_1 - \gamma_0(\hat{x}_1)) \\ \dot{\hat{x}}_2 &= \varphi_2(\hat{x}_1, \hat{x}_2, \hat{x}_3, y_2, u) + G^2 k_2(y_1 - \gamma_0(\hat{x}_1)) \\ &\cdots \\ \dot{\hat{x}}_{n_1} &= \varphi_{n_1}(\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_{n_1}, y_2, u) \\ &\quad + G^{n_1} k_{n_1}(y_1 - \gamma_0(\hat{x}_1)) \\ r &= y_1 - \gamma_0(\hat{x}_1) \end{aligned}$$

in which *G* is a (sufficiently large) positive number and $k_1, \cdots, k_{n_1}$ are numbers depending on the parameters α, β where

$$\alpha \leq \left| \frac{\partial \gamma_0}{\partial \xi_1} \right| \leq \beta \;, \quad \alpha \leq \left| \frac{\partial \varphi_i}{\partial \xi_{i+1}} \right| \leq \beta \,.$$

In last part, we study systems with some disturbance input and one fault input as extra inputs. But if some concurrent faults occur, we can change system (1) model to

$$\begin{aligned} \dot{x} &= g_0(x) + \sum_{i=1}^{m} g_i(x) u_i + l_i f_i + \tilde{p}_i(x) \tilde{w}_i \\ y &= h(x) \end{aligned}$$

where

$$\begin{aligned} \tilde{p}_i &= (l_1 \cdots l_{i-1} l_{i+1} \cdots l_s \, p) \\ \tilde{w}_i &= \mathrm{col}(f_1, \cdots, f_{i-1}, f_{i+1}, \cdots, f_s, w) \end{aligned} \quad (12)$$

for each fault and design separate residual generator for each fault.

3 Robot Fault Detection

Robotic systems play an essential role in our society. Their presence and our dependence on them are increasingly growing. Manufacturing industry has been able to make tremendous leaps only due to the advances in robot technology. Robotic systems are the best and most of the times the only replacement to human beings in applications where human presence is either not possible or harmful.

The internal environment in robotic system is very unstable as well, and it can exert even larger dynamic faults. Friction, noise, vibration, and etc, are regular guests in many robotic systems. Among these faults, actuator fault is one of the worst cases of faults and its effects on the system are obviously devastating. These types of faults may occur because of heat, brush friction, core faults, stray-load, vibration and short circuit in electric motor, etc [3-10].

Multiple-axis robot manipulators are widely used in industrial and space application. The significance of accurate positioning of these robot manipulators has motivated the researchers in the design and control of robot. Normally, what industries need is rigid manipulators. However, by the inception of harmonic drives and their wide usage in the design of many electrically driven robots, the rigidity of robots is affected greatly. Due to this flexibility there exists an error dynamics between rotation of the robot drive to that of the robot link.

Spong [17], has derived a nonlinear model for flexible joint manipulators, in which the slow states are the positions and velocities of the joints and the fast states are the forces and their derivatives. In order to model an m-link manipulator, let $q_i : i = 1,\cdots,n$ denote the position of i link and $q_i : i = n+1,\cdots,2n$ denote the position of i actuator scaled by the actuator gear ratio.

For flexible joints, the equation can be written in the following form using Euler-Lagrange formulation

$$M(q_1)\ddot{q}_1 + h(q_1,\dot{q}_1) + \zeta = K(q_2 - q_1)$$
$$J\ddot{q}_2 = K(q_1 - q_2) + u + \tau$$

where q_1 and q_2 are m-dimensional vectors of the generalized coordinates, $M(q_1)$ and J are symmetric nonsingular inertia matrices, and u is m input. The term $h(q_1,\dot{q}_1)$ accounts for centrifugal, Coriolis, and gravity forces, and K is a diagonal matrix of joint spring constants, t is the vector of faults acting on each actuator or is the vector of bounded unmodeled dynamics on actuator side and ζ is the vector of disturbance acting on each joint or is the vector indicating the effect of the bounded unmodeled dynamics [11],[17]. We can choose state variables

$$x_1 = q_1, x_2 = \dot{q}_1, x_3 = q_2, x_4 = \dot{q}_2$$

in R^m and write the state equations as follows:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= M^{-1}(x_1)[-h(x_1,x_2) - K(x_1 - x_3) - \zeta]\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= J^{-1}[K(x_1 - x_3) + u + \tau].\end{aligned} \quad (13)$$

In (13), τ and ζ are extra imposed inputs and both are treated faults. By using (12), we consider two cases for the system (13): t be a fault and ζ be a disturbance and vice versa [8].

Case1: τ is the fault, ζ is a disturbance

By comparing (1) and (13), we find $p(x), l(x), g_i(x) \quad i = 0,\cdots,m$:

$$p(x) = \left[0, -M^{-1}(x_1), 0, 0\right]^T, l(x) = g_1(x) = \left[0,0,0,J^{-1}\right]^T$$

$$g_0(x) = \begin{bmatrix} x_2 \\ M^{-1}(x_1)[-h(x_1,x_2) - K(x_1 - x_3)] \\ x_4 \\ J^{-1}[K(x_1 - x_3)] \end{bmatrix}$$

where 0 denotes the $m \times m$ zero matrix. Let

$$M^{-1}(x_1) = [t_1(x_1) \quad \cdots \quad t_m(x_1)]$$

where

$$t_j(x_1) = [t_{ij}(x_1)]_{i=1\cdots m} \; (j = 1,\cdots,m)$$

is an $m \times 1$ column vector. Also we can write $p(x)$ in the following form

$$p_j(x) = \left[0, -t_j(x_1), 0, 0\right]^T.$$

Hence $P = \operatorname{span}\{p_1(x_1),\cdots,p_m(x_1)\}$ and $\overline{P} = P$.

Now by using algorithm (7), we obtain

$$\begin{cases} S_0 = \overline{P} \\ \overline{P} \cap \ker\{dh\} = \{0\} \end{cases} \Rightarrow S_1 = \overline{P}.$$

As a result $S_1 = S_0$ and the algorithm is stopped. Therefore $\Sigma^P = \overline{P}$ and Σ^P is smooth, nonsingular and involutive. Hence, $(\Sigma^P)^\perp$ obtain

$$(\Sigma^P)^\perp = \operatorname{span}\left\{\begin{pmatrix} I \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ I \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ I \end{pmatrix}\right\}. \quad (14)$$

We can now proceed to the computation of the maximal observability codistribution, which is locally spanned by exact differentials and contained in $P^\perp$ via the algorithm (5). To this end, it is enough to note that $(\Sigma^{\mathbf{P}})^\perp \subset span\{dh\}$, to conclude that $o.c.a.((\Sigma^P)^\perp) = (\Sigma^P)^\perp$.

Now we can change coordinate

$$v_1 = [v_{11}, v_{21}, v_{31}]^T = [x_1, x_1, x_1]^T, v_2 = x_2$$

and rewrite (13) in the following form

$$\dot{v}_{11} = v_2$$
$$\dot{v}_{21} = J^{-1} v_{31}$$
$$\dot{v}_{31} = K(v_{11} - v_{21}) + u + \tau$$
$$v_2 = -M^{-1}(v_{11})[h(v_{11}, v_2) + K(v_{11} - v_{21}) + \zeta]$$
$$y_1 = \begin{pmatrix} y_{11} & y_{21} & y_{31} \end{pmatrix}^T = v_1$$
$$y_2 = v_2$$

(compare with (10)).

Now consider a candidate residual generator of the form

$$\begin{aligned} \dot{\xi} &= [K(y_{11} - y_{21}) + u] + A(y_{31} - \xi) \\ r &= A(y_{31} - \xi) \end{aligned} \tag{15}$$

where K and A are diagonal matrices $K = diag\{k_1, \cdots, k_m\}$ and $A = diag\{a_1, \cdots, a_m\}$, and $r, \xi, u \in R^m$.

Simply we can write the above system in the form

$$\dot{r}_i = a_i(-r_i + \tau_i) \quad (i = 1, \cdots, m).$$

Hence, $r_i(t)$ is the response of a linear filter with transfer function $\dfrac{a_i}{s + a_i}$ to the input $\tau_i(t)$.

Case2: ζ is the fault, τ is a disturbance

Conversely, let τ be a disturbance signal and ζ be a fault. The system (13) can be changed to (1) if we define

$$p(x) = \left[0, 0, 0, J^{-1}\right]^T, \; l(x) = \left[0, -M^{-1}(x_1), 0, 0\right]^T.$$

By using (7) and (5), we have

$$\Sigma^P = span\left\{\begin{pmatrix} 0 & 0 & 0 & I \end{pmatrix}^T\right\}$$
$$(\Sigma^P)^{\perp} = span\left\{\begin{pmatrix} I \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ I \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ I \\ 0 \end{pmatrix}\right\}$$

where I is an $m \times m$ identity matrix. In order to state (13) in the new coordinates, one defines

$$v_1 = \left[x_1, -M(x_1)x_2, x_3\right]^T, v_2 = Jx_4.$$

Consequently, (13) is rewritten as

$$\dot{v}_1 = \begin{pmatrix} \dot{v}_{11} \\ \dot{v}_{21} \\ \dot{v}_{31} \end{pmatrix} = \begin{pmatrix} x_2 \\ -\dot{M}(x_1)x_2 - M(x_1)\dot{x}_2 \\ x_4 \end{pmatrix}$$
$$\dot{v}_2 = J\dot{x}_4 = K(v_{11} - v_{31}) + u + \tau$$
$$y_1 = v_1$$
$$y_2 = v_2$$

Hence, residual generator structure is

$$\begin{aligned} \dot{\xi} &= [-\dot{M}(y_{11})M^{-1}(y_{11})y_{21} + h(y_{11}, M^{-1}(y_{11})y_{21}) \\ &\quad + K(y_{11} - y_{31})] + A(\xi - y_{21}) \\ r &= A(\xi - y_{21}) \end{aligned} \tag{16}$$

Therefore, $\dot{r}_i = a_i(-r_i + \zeta_i) \quad (i = 1, \cdots, m)$.

Clearly $r_i(t)$ is the response of a linear filter in the form $\dfrac{a_i}{s + a_i}$ with input ζ_i.

4 Example

As an example of the results obtained in last section we may design a residual generator for detecting actuator fault and disturbance in a single-link flexible joint manipulator.

The model in question is provided by the following set of differential equations [17]

$$\begin{aligned} I\ddot{q}_1 + Mgl\sin q_1 + k(q_1 - q_2) + \zeta &= 0 \\ J\ddot{q}_2 - k(q_1 - q_2) &= u + \tau \end{aligned}.$$

where q_1, q_2 are angular positions, I and J are moments of inertia, k is a spring constant, M is the total mass, l is a distance, u is the torque input, τ is the actuator fault and ζ is disturbance signal. As before we choose these state variables and rewrite the state equations

$$x_1 = q_1, x_2 = \dot{q}_1, x_3 = q_2, x_4 = \dot{q}_2$$
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{1}{I}[Mgl\sin x_1 + k(x_1 - x_3) + \zeta] \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{1}{J}[k(x_1 - x_3) + u + \tau] \end{cases}. \tag{17}$$

Case1. Detecting Actuator Fault

In the first case, if $y = x$, the residual generator design is as follows:

$$\begin{aligned} \dot{\xi} &= [k(y_{11} - y_{21}) + u] + a(y_{31} - \xi) \\ r &= a(y_{31} - \xi) \end{aligned}.$$

Figures 1and 4 show simulation corresponding to the following scenario: the actuator u was supposed to provide constant trust equal to $u=2$. At time $t=5s$, a fault of actuator as failure of torque occurs [Fig.1(___)]. Moreover, a disturbance signal ζ is presented [Fig.2]. Observed variables are shown at [Fig.3: (___) joint angle, Fig.3: (....) actuator angle, Fig.4: (___)velocity of joint and Fig.4: (....):velocity of actuator]. The output of the residual generator [Fig.1 (....)] clearly converges to the actual value of fault τ.

In the last case we need to measure all states, but if x_2 cannot be measured, we can define the output as follows:

$$y = h(x) = \left[x_1, x_3, x_4\right]^T.$$

To change (17) into the form (3), we define

$$g_0(x) = \begin{bmatrix} x_2 \\ \dfrac{1}{I}[-Mgl\sin x_1 - k(x_1 - k_3)] \\ x_4 \\ \dfrac{1}{J}[k(x_1 - k_3)] \end{bmatrix},$$

$$g_1(x) = l(x) = \left[0,0,0,\frac{1}{J}\right]^T, \; p(x) = \left[0,0,\frac{1}{I},0\right]^T.$$

By using the algorithm (7), we have:
step1:

$$\bar{S}_0 = S_0 = \bar{P} = P = span\{(0 \quad 1 \quad 0 \quad 0)^T\}$$

step2:

$$S_1 = S_0 + \sum_{i=0}^{1}[g_i, S_0 \cap \ker\{dh\}]$$

$$S_0 \cap \ker\{dh\} = span\left\{(0 \quad 1 \quad 0 \quad 0)^T\right\} = P$$

$$S_1 = span\{(0 \quad 1 \quad 0 \quad 0)^T \quad (1 \quad 0 \quad 0 \quad 0)^T\}$$

step3:

$$\begin{cases} S_2 = S_1 + \sum_{i=0}^{1}[g_i, S_1 \cap \ker\{dh\}] \\ S_1 \cap \ker\{dh\} = P \subset S_1 \end{cases} \Rightarrow S_2 = S_1$$

and the algorithm is stopped.
Therefore, the minimal conditioned invariant distribution containing span$\{p\}$ is found to be

$$\Sigma^P = S_1 = span\{(0 \quad 1 \quad 0 \quad 0)^T, \quad (1 \quad 0 \quad 0 \quad 0)^T\}$$

where Σ^P is smooth, nonsingular and involutive. $(\Sigma^P)^\perp$ can be expressed as

$$(\Sigma^P)^\perp = span\left\{[0,0,1,0]^T, [0,0,0,J]^T\right\} = span\{dx_3, d(Jx_4)\}.$$

The maximal observability codistribution, which is locally spanned by exact differentials and is contained in $P^\perp$ is obtained via the (5). To this end, it is enough to note that $(\Sigma^{\mathbf{P}})^\perp \subset span\{dh\}$ to conclude that $o.c.a.((\Sigma^P)^\perp) = (\Sigma^P)^\perp$.
The change of coordinate in the state space induced by $o.c.a.((\Sigma^P)^\perp)$ is given by

$$v_1 = [v_{11}, v_{21}]^T = [x_3, Jx_4]^T, v_2 = x_1, v_3 = x_2$$

and in the output space

$$y_1 = v_1 = [y_{11}, y_{11}]^T, y_2 = v_2.$$

In the new coordinates, the system is rewritten as:

$$\begin{pmatrix} \dot{v}_{11} \\ \dot{v}_{21} \end{pmatrix} = \begin{pmatrix} v_{21} \\ k(v_2 - v_{21}) + u + \tau \end{pmatrix}$$
$$\dot{v}_2 = v_3 .$$
$$\dot{v}_3 = -\frac{1}{I}[Mgl \sin v_2 + k(v_2 - v_{21}) + \zeta]$$

Consider now a candidate residual generator of the form

$$\begin{aligned} \dot{\xi} &= k(y_2 - y_{21}) + u + a(y_{21} - \xi) \\ r &= a(y_{21} - \xi) \end{aligned} . \qquad (18)$$

Let $e = (y_{21} - \xi)$ then (15) is rewritten as

$$\begin{aligned} \dot{e} &= \tau + ae \\ r &= ae \end{aligned} .$$

Hence, $r(t)$ is the response of a linear filter with transfer function $\frac{a}{s+a}$ to the input t. Thus, $r(t)$ converges to the actual value of t and the rate of convergence depends on parameter a.

Simulation results are as similar to the previous case. However, in the latter there is no need to use the angular velocity of joint for designing the residual generator.

Case2: Detecting Disturbance

As before, for detecting disturbance from outputs, we design a residual generator. By using (16) for a single-link manipulator, we have

$$\dot{\xi} = [Mgl \sin y_{11} + k(y_{11} - y_{31})] + a(y_{21} - \xi)$$
$$r = a(\xi - y_{21})$$

Fig 2. (___) shows the actual value of z and (.....) shows the output of the residual generator $r(t)$.

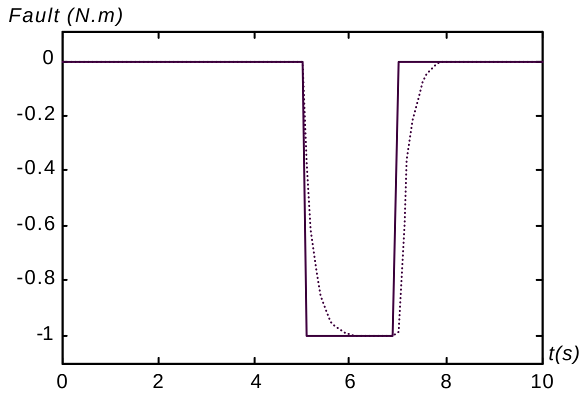

Figure1: (___) Fault and (....) Fault estimation

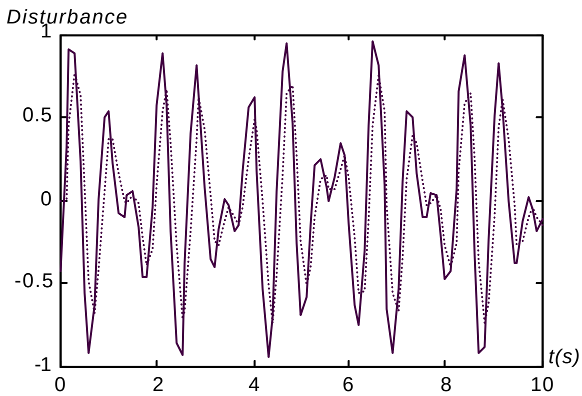

Figure2: (___) disturbance and (....) disturbance estimation

5 Conclusion

In this paper, we have studied the general case of faults in flexible- joint manipulators and have modeled them by the system of differential equations. Then we have applied the method proposed in [14] to detect the faults in the above system. In order to detect all faults, it is necessary to measure all variable states of system correctly. In this method the estimates of faults are more exact than the linearization methods, which have been realized since yet, because of the structures of residual generators. Furthermore, by suitable choosing of diagonal matrix of parameters, *A*, convergence rate of estimate of faults may be adjusted.

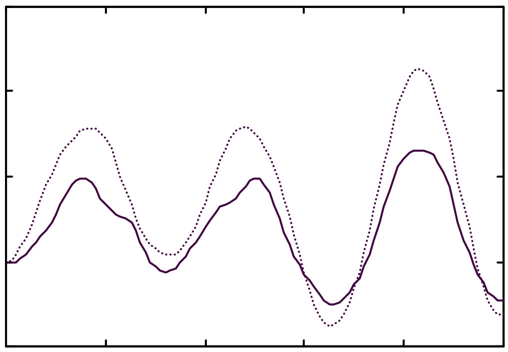

Figure3: (___) Joint angle and (....) Actuator angle

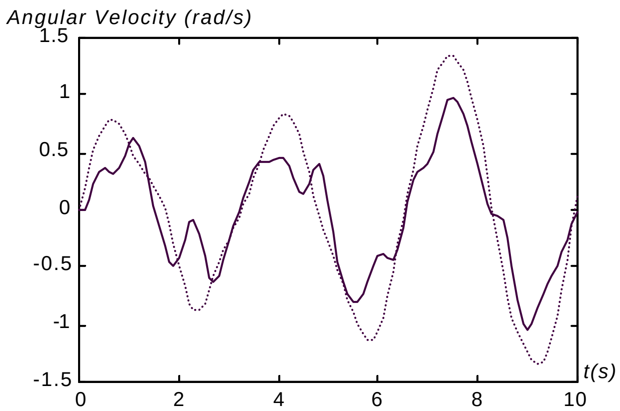

Figure4: (___) Velocity of joint and (....) Velocity of actuator

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