CONTROL RECONFIGURATION DEMONSTRATED AT A TWO-DEGREES-OF-FREEDOM HELICOPTER MODEL

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Abstract

Any technical system is liable to the occurrence of faults. A major fault in the plant like the complete loss of an actuator or sensor breaks the control loop and renders the system inoperational, unless the control structure is changed in response to the fault. This paper shows how the LQG (linear quadratic Gaussian) optimal control problem can be posed and solved automatically for the faulty plant, resulting in a new controller that allows to stabilise the plant despite of an actuator fault. The same approach can be used to design a bank of observers (one per fault case) that can be used for sensor fault diagnosis and reconfiguration at the same time. The approach is experimentally verified at a two-degrees-of-freedom helicopter model.

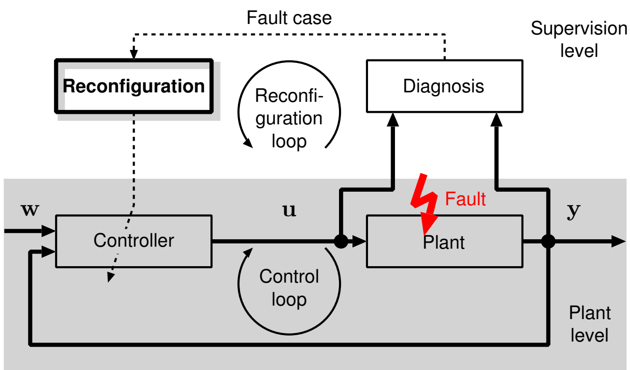

Figure 1: Controller Reconfiguration

1 Problem of Controller Reconfiguration

1.1 Introduction

Controller reconfiguration concerns the problem of changing the control structure and the control laws after a severe fault (like the loss of an actuator or sensor) has occurred in the plant. The aim is to keep the system operational despite of the fault, while possibly accepting lower control performance.

The reconfiguration problem has to be considered on two decision levels as shown in Figure 1. At the plant level, a feedback controller is used to ensure set-point following and to attenuate disturbances. The reconfiguration task is handled at the supervision level when a fault in the system is identified. The controller is “reconfigured” in the sense that the process of controller design is carried through again for the faulty plant. For the severe faults considered in this paper, it is not sufficient to change the controller parameters; instead the selection of the new control structure with new sensors and actors is required by the fault. The aim is to find a reconfigured control automatically from the model of the faulty plant.

1.2 Relevant Literature

There are three main approaches to control reconfiguration. The first kind is aimed at small faults like a degraded actuator performance or increased sensor noise, where no structural changes of the control loop are necessary. Adaptive methods have been used to handle a degradation by adjusting the controller parameters [6]. The pseudo-inverse method [5] can also be used with the advantage that the controller itself does not have to be changed.

The common approach is to treat the reconfiguration problem as a question of controller redesign. A framework for the manual design of fault tolerant control schemes has been developed in [2, 3]. Using a suitable control approach, it is possible to automate the redesign, as shown for predictive control in [9].

A third option is to extend the control loop by a reconfiguration block that specifically addresses the faulty part of the plant without replacing the nominal controller. This approach includes some early studies on the use of observers, and the pseudo-inverse method can be seen as a special case of this. A practical example is given in [7] and a systematic approach is published in [8].

This paper follows the second approach in that the controller is automatically redesigned. In addition, a performance measure is defined for comparing different reconfiguration approaches.

1.3 Approach

This paper focuses on the reconfiguration problem, where the diagnostic task is assumed to be already solved. Therefore, the model of the faulty plant is assumed to be known.

The idea is to design the nominal controller using a design algorithm that can be applied automatically. In case of a fault, the controller design can be repeated using the same algorithm and the same set of parameters, without manual intervention. While in theory every design algorithm could be used, the practical choice is limited to algorithms that respond in a sensible way to the fault in the system. For example pole placement is not suitable, because it cannot be expected that the same set of poles is suitable for all fault cases. Optimal controller design is at an advantage here, because it can automatically find a compromise between the original goal as formulated in the optimal control problem and the reduced availability of control inputs and outputs, as defined in the model of the faulty plant.

Because of its simplicity, the linear quadratic Gaussian (LQG) optimal control problem is used in this paper as defined for the nominal case in Section 2. The properties of the faulty plant and the corresponding optimal control are discussed in Sections 3 and 4. Practical implications of the reconfiguration are described in Section 5, and an application to an experimental flight model is given in Section 6.

2 The Nominal Control Loop

The nominal control loop consists of the nominal plant and the optimal controller, which can be divided into a state observer and a state feedback controller. The nominal plant is given by:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{B}_v\mathbf{v} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}_w\mathbf{w} \\ \mathbf{x}(0) &= \mathbf{x}_0 \end{aligned}$$

where $\mathbf{u} \in \mathbb{R}^{n_u}$, $\mathbf{x} \in \mathbb{R}^{n_x}$ and $\mathbf{y} \in \mathbb{R}^{n_y}$ are the plant input, state and output. $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are the system matrices with the corresponding dimensions, and $\mathbf{B}_v\mathbf{v}$ and $\mathbf{D}_w\mathbf{w}$ are state and output disturbances. The weighting matrices $\mathbf{B}_v$ and $\mathbf{D}_w$ are assumed to be square, and the vectors $\mathbf{v}$ and $\mathbf{w}$ represent uncorrelated, normalised white noise

$$E\left\{\begin{pmatrix}\mathbf{v}(t)\\ \mathbf{w}(t)\end{pmatrix}\begin{pmatrix}\mathbf{v}(t)\\ \mathbf{w}(t)\end{pmatrix}^T\right\} = \mathbf{I}\sigma(t-\tau) \quad .$$

The control problem is to minimise the objective function, which is the expected norm of the weighted plant state and input

$$J = \lim_{t\to\infty} E\left\{|\mathbf{C}_J\mathbf{x}|^2 + |\mathbf{D}_J\mathbf{u}|^2\right\} \tag{1}$$

where $\mathbf{C}_J$ and $\mathbf{D}_J$ are square weighting matrices.

The solution to the optimal control problem is calculated by solving the following matrix Riccati equations for $\mathbf{P}$ and $\mathbf{Q}$:

$$\begin{aligned} \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{C}_J\mathbf{C}_J^T - \mathbf{P}\mathbf{B}(\mathbf{D}_J\mathbf{D}_J^T)^{-1}\mathbf{B}^T\mathbf{P} &= \mathbf{O} \\ \mathbf{A}\mathbf{Q} + \mathbf{Q}\mathbf{A}^T + \mathbf{B}_v\mathbf{B}_v^T - \mathbf{Q}\mathbf{C}^T(\mathbf{D}_w\mathbf{D}_w^T)^{-1}\mathbf{C}\mathbf{Q} &= \mathbf{O} \ . \end{aligned}$$

The optimal controller is a state observer/state feedback controller (see Fig. 2) defined by

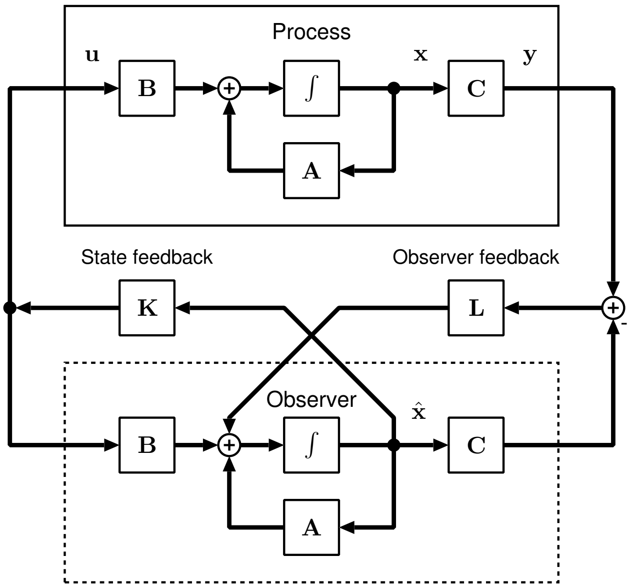

Figure 2: The nominal control loop

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{L}(\mathbf{C}\hat{\mathbf{x}} - \mathbf{y}) \tag{2a}$$

$$\mathbf{u} = \mathbf{K}\hat{\mathbf{x}} \tag{2b}$$

with

$$\begin{aligned} \mathbf{K} &= (\mathbf{D}_J\mathbf{D}_J^T)^{-1}\mathbf{B}^T\mathbf{P} \\ \mathbf{L} &= \mathbf{Q}\mathbf{C}^T(\mathbf{D}_w\mathbf{D}_w^T)^{-1} \quad . \end{aligned}$$

The optimal value of the objective function is known to be

$$\begin{aligned} J^* &= \text{trace}(\mathbf{B}_v^T\mathbf{P}\mathbf{B}_v) \\ &+ \text{trace}((\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{C}_J\mathbf{C}_J^T)\mathbf{Q}\} \quad . \end{aligned} \tag{3}$$

More details on this quadratic optimal control problem can be found in [1].

It shows that the formulation of the optimisation problem is slightly unfortunate in that the optimal objective value is not symmetric and not separable (considering the two dual parts of the problem). The problem will get relevant later on, but no solution is proposed in this paper.

3 The Reconfiguration Problem

A severe fault like the unavailability of an actuator or sensor breaks the nominal control loop and renders the system inoperational. However, under certain assumptions the controller can be reconfigured in such a way that the faulty plant is stabilised by a reconfigured controller. This is called control reconfiguration.

In this paper, the reconfiguration problem will be solved by an automatic partial redesign of the controller. Therefore, the reconfiguration problem is posed in such a way that a simple and unique solution exists.

It is assumed that the fault has been identified and, therefore, the model of the faulty plant is known to be

$$\begin{aligned} \dot{\mathbf{x}}_f &= \mathbf{A}_f \mathbf{x}_f + \mathbf{B}_f \mathbf{u}_f + \mathbf{B}_v \mathbf{v} && (4a)\\ \mathbf{y}_f &= \mathbf{C}_f \mathbf{x}_f + \mathbf{D}_w \mathbf{w} && (4b)\\ \mathbf{x}_f(0) &= \mathbf{x}_0 && (4c) \end{aligned}$$

where the index f denotes a symbol of the faulty control loop. It is assumed that the number of inputs, outputs and states has not changed, though some of them may have lost their function. Moreover, it will be assumed that the plant is fully observable and stabilisable. This requires that every unstable mode is controllable via the remaining sensors and actuators.

In case of an actuator fault, only the matrix $\mathbf{B_F}$ changes, while the other system matrices are identical in the nominal and the faulty plant. Similarly, in case of sensor faults only the matrix $\mathbf{C}_\mathrm{F}$ changes.

The control objective (1) remains unchanged from the original problem, it is to minimise

$$J_f = E\left\{ |\mathbf{C}_J \mathbf{x}_f|^2 + |\mathbf{D}_J \mathbf{u}_f|^2 \right\} \quad . \tag{5}$$

4 Solution by Redesign

As the control problem for the faulty plant is formulated as an LQG problem, it can be solved automatically using the same technique as in the nominal controller design. Due to the separation principle, it is sufficient to redesign the state feedback matrix in case of actuator faults and the observer feedback matrix in case of sensors faults. The new controller has the same structure as the nominal one:

$$\begin{aligned} \dot{\hat{\mathbf{x}}}_f &= \mathbf{A}_f \hat{\mathbf{x}}_f + \mathbf{B}_f \mathbf{u}_f + \mathbf{L}_f(\mathbf{y}_f - \mathbf{C}_f \hat{\mathbf{x}}_f) && (6a)\\ \mathbf{u}_f &= \mathbf{K}_f \hat{\mathbf{x}}_f \quad . && (6b) \end{aligned}$$

4.1 Actuator Faults

Because $\mathbf{A}$ and $\mathbf{C}$ are not affected by the fault, the nominal observer feedback remains unchanged:

$$\mathbf{L}_f = \mathbf{L} \quad .$$

The state feedback controller is redesigned by solving the matrix Riccatti equation

$$\begin{aligned} \mathbf{A}^T \mathbf{P}_f + \mathbf{P}_f \mathbf{A} + \mathbf{C}_J \mathbf{C}_J^T & \\ -\mathbf{P}_f \mathbf{B}_f (\mathbf{D}_J \mathbf{D}_J^T)^{-1} \mathbf{B}_f^T \mathbf{P}_f &= \mathbf{O} \quad . \end{aligned}$$

The controller is then given by

$$\mathbf{K}_f = (\mathbf{D}_J \mathbf{D}_J^T)^{-1} \mathbf{B}_f^T \mathbf{P}_f \tag{7}$$

and the achieved objective value is

$$\begin{aligned} J_f^* &= \mathrm{trace}(\mathbf{B}_v^T \mathbf{P}_f \mathbf{B}_v) && (8)\\ &+ \mathrm{trace}((\mathbf{A}^T \mathbf{P}_f + \mathbf{P}_f \mathbf{A} + \mathbf{C}_J \mathbf{C}_J^T) \mathbf{Q}) \quad . \end{aligned}$$

Note that both terms have changed, although the problem is separable and only the controller part had to be updated. Therefore it is difficult to use this formulation of the objective function to compare the control performance of the nominal and the reconfigured loop.

4.2 Sensor Faults

Similarly to the actuator faults, in case of sensors faults the state feedback matrix remains unchanged

$$\mathbf{K}_f = \mathbf{K}$$

and the observer is redesigned according to

$$\begin{aligned} \mathbf{A}\mathbf{Q}_f + \mathbf{Q}_f \mathbf{A}^T + \mathbf{B}_v \mathbf{B}_v^T & \\ -\mathbf{Q}_f \mathbf{C}_f^T (\mathbf{D}_w \mathbf{D}_w^T)^{-1} \mathbf{C}_f \mathbf{Q}_f &= \mathbf{O} \end{aligned}$$

$$\mathbf{L}_f = \mathbf{Q}_f \mathbf{C}_f^T (\mathbf{D}_w \mathbf{D}_w^T)^{-1} \quad . \tag{9}$$

which leads to an objective value of

$$\begin{aligned} J_f^* &= \mathrm{trace}(\mathbf{B}_v^T \mathbf{P} \mathbf{B}_v) && (10)\\ &+ \mathrm{trace}((\mathbf{A}^T \mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{C}_J \mathbf{C}_J^T) \mathbf{Q}_f\} \quad . \end{aligned}$$

This is the expected result: only the second term changed due to the fault.

5 Control Reconfiguration

While the idea of the controller redesign is straightforward, there are some general problems with respect to the practical application that deserve mentioning.

5.1 Solution of the Rank Deficiency Problem

The first problem is that the commonly used definition of the LQG problem requires that $\mathbf{B}$ and $\mathbf{C}$ have full rank and that the plant is fully observable and controllable. Neither of this is guaranteed for the faulty plant, leading to numerical singularities in common design algorithm implementations. To achieve compliance, the faulty plant model has to be reduced, a controller is then calculated for the reduced problem, which is finally expanded to apply to the actual plant. This approach will be described in detail for the case of actuator faults. A corresponding reduction is possible for the treatment of sensors faults.

In the first step, a matrix $\mathbf{T}_x$ is determined that maps the controllable subspace of the state space onto a reduced state space. To find it, the singular value decomposition of the controllablity matrix is calculated

$$\mathbf{U}\mathbf{W}\mathbf{V}^T = \begin{pmatrix} \mathbf{B}_f & \mathbf{A}\mathbf{B}_f & \cdots & \mathbf{A}^{n_x-1}\mathbf{B}_f \end{pmatrix}$$

where $\mathbf{U}$ is a square and $\mathbf{V}$ a rectangulare orthogonal matrix, while $\mathbf{W}$ is a diagonal matrix with ordered non-negative diagonal elements. If n' elements are nonzero, then the first n'

columns vectors of $\mathbf{U}$ span the the controllable subspace of the state space. Therefore, these vectors can be used to define the mapping matrix $\mathbf{T}_x$:

$$\mathbf{T}_x^T = \mathbf{U}(1 \dots n_x, 1 \dots n')$$

The right side inverse of $\mathbf{T}_x$ is calculated using

$$\mathbf{T}_x^* = \mathbf{T}_x^T(\mathbf{T}_x\mathbf{T}_x^T)^{-1} \quad ,$$

giving the inverse transformation from the reduced to the original state space. As long as $\mathbf{T}_x$ is normalised and orthogonal, the second factor disappears and $\mathbf{T}_x^* = \mathbf{T}_x^T$.

A second set of matrices $\mathbf{T}_u$ and $\mathbf{T}_u^*$ with $\mathbf{T}_u\mathbf{T}_u^* = \mathbf{I}$ is determined for the reduction of the input space, such that $\mathbf{T}_u$ maps the effective subspace of the input space onto a reduced input space.[1] Because $\mathbf{T}_u$ does not typically have full rank, $\mathbf{T}_u^*$ is not uniquely defined. Depending on the application the remaining degrees of freedom can be used to distribute the control effort between several redundant actuators, or to achieve minimal control energy by using

$$\mathbf{T}_u^* = \mathbf{T}_u^T(\mathbf{T}_u\mathbf{T}_u^T)^{-1} \quad . \tag{11}$$

It will be assumed that $\mathbf{T}_u^*$ does not utilise broken actuators.

The plant model is then reduced according to the following equations:

$$\mathbf{A}'_f = \mathbf{T}_x\mathbf{A}_f\mathbf{T}_x^* \tag{12a}$$
$$\mathbf{B}'_f = \mathbf{T}_x\mathbf{B}_f\mathbf{T}_u^* \tag{12b}$$

The cost matrices $\mathbf{C}_J$ and $\mathbf{D}_J$ are reduced accordingly:

$$\mathbf{C}'_J = \mathbf{C}_J\mathbf{T}_x^* \tag{13a}$$
$$\mathbf{D}'_J = \mathbf{D}_J\mathbf{T}_u^* \tag{13b}$$

The reduced plant model is completely controllable, and $\mathbf{B}'_f$ has full rank. Therefore, standard LQG design implementations can be applied. The obtained feedback matrix $\mathbf{K}'_f$ is then expanded to fit the actual plant dimensions:

$$\mathbf{K}_f = \mathbf{T}_u^*\mathbf{K}'_f\mathbf{T}_x \quad . \tag{14}$$

It should be noted that the objective value $\mathbf{J}'^*_f$ of the reduced problem should is identical to the value $\mathbf{J}^*_f$ achieved after the expansion, as long as the noncontrollable subspace of the state space is assumed to be zero. Due to the disturbance, this cannot be expected. Therefore, the two objective values are not directly comparable.

[1] In most cases it is sufficient to remove the broken actuators from the reduced problem. $\mathbf{T}_u$ is then derived from a unity matrix by deleting the columns belonging to actuators without an effect. The general approach to making $\mathbf{B}'_F$ a full rank matrix starts with the singular value decomposition of $\mathbf{B}_f$ and follows the same steps as the reduction of the state space.

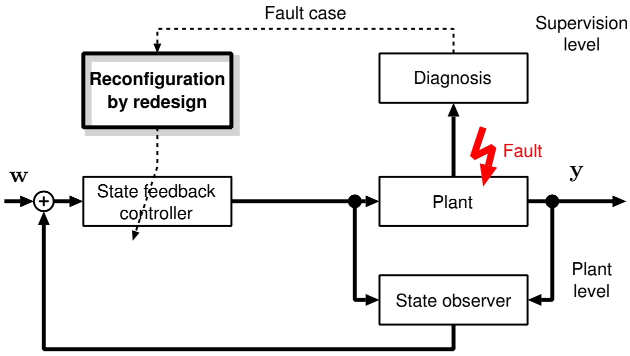

Figure 3: Reconfiguration Structur for Actuator Faults by Controller Redesign

5.2 Reconfiguration Algorithm for Actuator Faults

The update of the state feedback matrix $\mathbf{K}_f$ after an actuator fault can be performed online (while the system is operating) without serious problems. The sudden change of the feed back matrix can lead to a jump in the control input, but there should be no further difficulties. In theory, the state of the observer could be affected by the fault of the input during the time between the occurrence and the detection of the fault, but this problem has not been seen in the practical application. Therefore, there is no need to change the observer state at the time of the reconfiguration. In the formalism presented here, $\hat{\mathbf{x}}_f = \hat{\mathbf{x}}$ is assumed at the instant of reconfiguration.

To implement the reconfiguration, the following algorithm has been successfully used. All steps are performed on-line:

Given: linear model of the faulty plant (4), design objective J

1. The plant model and the LQG optimisation problem is reduced according to (12) and (13).

2. A new state feedback controller for the reduced plant is design according to (7).

3. The controller matrix is expanded to the system dimensions according to (14).

4. The controller is updated with the new state feedback matrix $\mathbf{K}_f$.

Result: an optimal controller for the faulty plant

In the specific case of a loss of an actuator it is not necessary to update the plant model in the observer, because the corresponding actuator input is zero due to the choice of $\mathbf{T}_u^*$ in equation (11). In cases where faulty actuators may receive a nonzero input signal, it is necessary to update the matrix $\mathbf{B}_\mathrm{F}$ in the observer to make it consistent with the faulty plant.

5.3 Reconfiguration Algorithm for Sensors Faults

The change of the observer feedback matrix $\mathbf{L}_F$ during the reconfiguration algorithm is more problematic. Because of the faulty sensor value, the nominal observer will contain an invalid state by the time the fault is detected. The effect can be reduced if reconfiguration is made faster, or it can be dealt with explicitely by a re-initialisation of the observer.

A different approach is taken here: a bank of observers running all the time is used, one for every fault case. When a sensor fails, the output of the observer not relying on this sensor is selected for control. Because this observer does produce a valid observation both with and without the sensor working properly, its state is valid at all times.

This approach eliminates the problem of the invalid observer state, at the expense of a significantly higher implementation cost. The automatic design algorithm is still useful, because it greatly facilitates the design of the observers.

The following algorithm has been used to implement a reconfigurable controller for sensors faults. The design in steps 1 and 2 is performed during the design phase, while only steps 3 and 4 are performed on-line:

Given: plant models for every fault case, control objective J

1. The nominal observer is designed by solving the LQG problem (1).
2. An observer is designed and implemented for every sensor fault case according to (9). A reduction of the faulty plant model may be necessary (cf. Section 5.1).
3. During the operation of the system, a sensor fault is detected and identified.
4. The corresponding observer is selected to feed the state estimate to the state feedback controller.

Result: a reconfigurable controller for sensor faults

5.4 Fault Diagnosis using the Bank of Observer

Since the sensor fault reconfiguration is realised with a bank of observers, it seems reasonable to put this bank to a second use for fault detection and identification (FDI). Since one observer has been designed for ever fault case, the bank of observers represents a "generalised observer scheme" which can be used for sensor fault isolation [4]. The update vectors of the observers can be used as residuals, since the update vector of the observer corresponding to the current fault case should be zero, while all other observers should have non-zero update vectors.

Such a system has been design and implemented successfully. For every observer, the update term $\mathbf{C}\hat{\mathbf{x}} - \mathbf{y}$ is low-pass filtered, the norm of the filtered vector is calculated, and this norm is filtered again. A threshold on the norm of the nominal observer

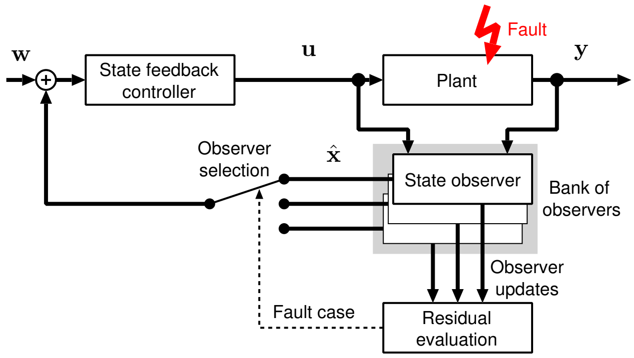

Figure 4: Reconfiguration Structur for Sensor Faults using a Bank of Observers

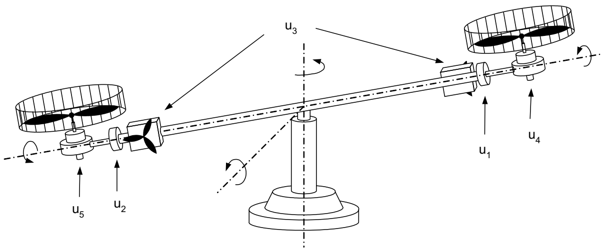

Figure 5: Flight model

is used to detect the fault. The update norms of the fault case observers are compared to identify the fault. The observer with the smallest norm is assumed to correspond to the fault case and it is selected to deliver the state estimate for the reconfigured control loop.

6 Application Example

6.1 Plant Description

The plant selected for experimental verification is a flight model shown in Figure 5. This system is well suited because it contains redundancies of different kinds that can be explored.

The flight model has two degrees of freedom: the horizontal axis with the tilt angle α and the vertical axis with the position angle β. The system is driven by two main rotors each of which can be turned individually around the third axis, which is measured by the angles γ_1 and γ_2. Two lateral rotors provide an additional control input that can be utilised during the nominal case or reserved for a fault case. The speed of the two main rotors is controlled by an independent controller based on the value of α. As there is only a second order influence on the part of the model considered here, the dynamics of the main rotors and of α are not modelled, and all three variables are assumed to take nominal values. So the control problem consists in stabilising β and the other 6 states of the model.

The system inputs are the voltages applied to the motors: the two servo motors for γ_1 and γ_2 are called u_1 and u_2, and the lateral rotors are named u_3. All angles β, γ_1 and γ_2 can be measured. Among all the control inputs used for stabilisation, β is the only one that cannot be replaced. Therefore the system is ideal for reconfiguration studies.

The linearised plant model is given by

$$\dot{\mathbf{x}} = \begin{pmatrix} -8.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3750 & -8.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3750 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0.6 & 0.6 & 73.6 & -0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} u_1 & u_2 & 0 & 0 & u_3 & 0 & 0 \end{pmatrix}^T \quad (15)$$

with

$$\begin{aligned} \mathbf{u} &= \begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix}^T \\ \mathbf{x} &= \begin{pmatrix} \dot{\gamma}_1 & \dot{\gamma}_2 & \gamma_1 & \gamma_2 & \omega & \dot{\beta} & \beta \end{pmatrix}^T \\ \mathbf{y} &= \begin{pmatrix} \beta & \gamma_1 & \gamma_2 \end{pmatrix}^T \end{aligned}$$

The following weighting matrices are used for the design:

$$\begin{aligned} \mathbf{C}_J = \mathbf{I} \qquad \mathbf{D}_J = 10\mathbf{I} \\ \mathbf{B}_v = \mathbf{I} \qquad \mathbf{D}_w = \mathbf{I} \quad . \end{aligned}$$

6.2 Nominal Case

The following graphs document the reconfiguration experiments performed at the flight model. A reference step from $\beta = 0°$ to $\beta = 45°$ is shown because this exercises all parts of the system. All other reference values are set to zero.

The nominal closed-loop system response with respect to all available controls is shown in the Figure 6. A theoretical objective value of

$$\mathbf{J}^* = 427 + 275$$

is achieved. The practical performance is acceptable: there is little overshoot, and the new reference is reached within 4 seconds. A small steady state error is caused by friction that is not modelled. The angles of the main rotors γ_1 and γ_2 are small enough to make the nonlinear effects negligible (higher values of γ could render the system unstable). The momentum necessary to turn the flight model is distributed roughly equally between the main rotors and the lateral rotor.

6.3 Actuator Fault

The loss of the actuator of input u_1 is considered here. Figure 7 shows the behaviour after the reconfiguration according to Section 5.2 has been carried out. The resulting objective value is

$$\mathbf{J}_{\mathrm{F}}^* = 223 + 151 \quad .$$

Note that this is the value for the reduced problem, which is not directly comparable with the original value.

The fault is assumed to be known immediately. The system remains operational and the performance is reduced very slightly.

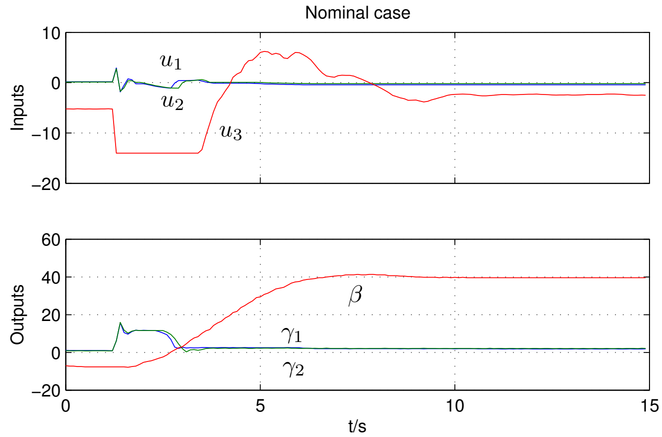

Figure 6: Nominal Case

The overshoot is again very small, and the time to reach to new reference is just above 5 seconds. The remaining actuators are working slightly harder than in the nominal case, but within reasonable limits.

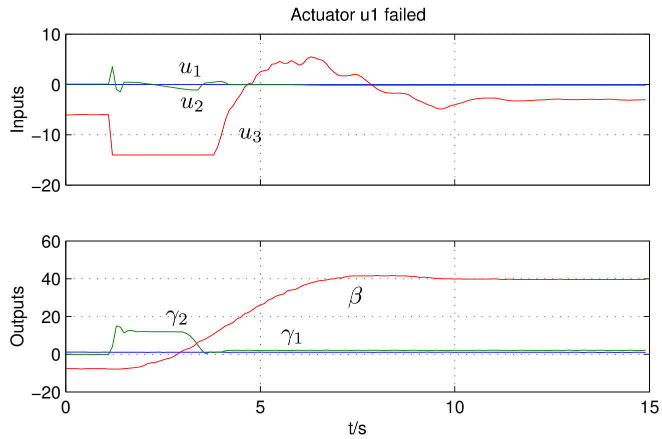

Figure 7: Reconfigured system after u_1 failed

6.4 Sensor Fault

The results of a fault in sensor γ_1 are shown in Figure 8, where the algorithm from Section 5.3 is used. The objective value is

$$\mathbf{J}_{\mathrm{F}}^* = 427 + 7520 \quad .$$

As the high objective value shows, the system is difficult to observe and therefore the control performance will deteriorate significantly due to the fault. The reason is that the influence between the variable γ_1 that is to be observed and the available measurement β is both small and slow.

A threshold based fault detection logic (cf. Section 5.4) is used, which means that the fault is detected with a delay of 0.3 seconds. While the detection is correct, the depicted observation of γ_1 shows a significant offset due to unmodeled friction. This

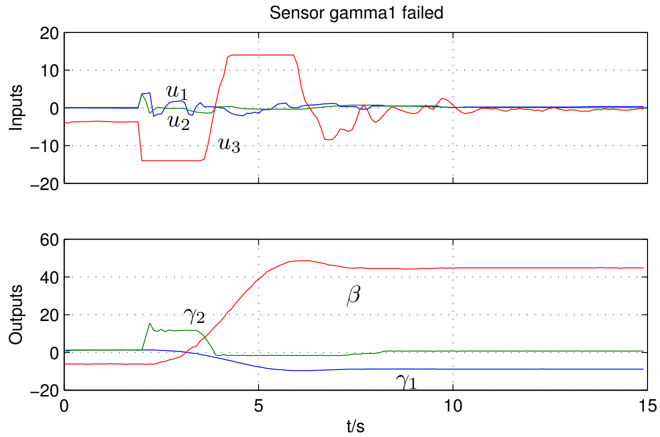

Figure 8: Reconfigured system after γ_1 failed

breaks the symmetry of the plant so that γ_1 and γ_2 are no longer identical. The steady state error and the overshoot both result from the poor observation.

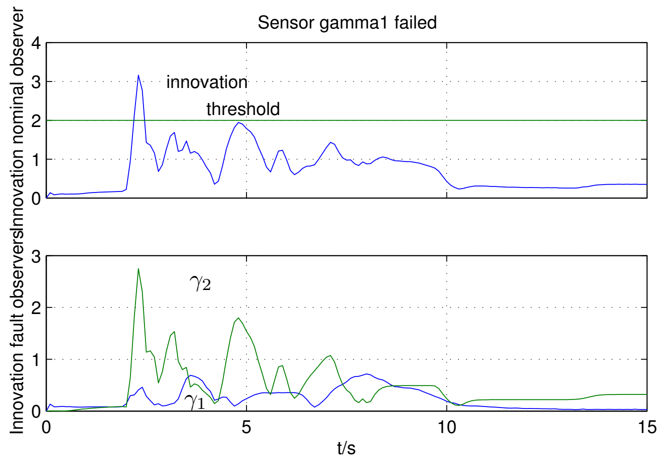

Figure 9: Filtered observer update norms used for fault detection

This experiment uses the observer updates for fault detection as described in Section 5.4. The filtered norms are shown in Figure 9. The norm of the nominal observer clearly violates the threshold of normal operation shortly after the fault. By comparing the innovation of the two fault case observer (fault in γ_1 and fault in γ_2), the fault case can be easily identified: the observer without γ_2 shows a high update signal just after the occurance of the fault, while the signal of observer 1 without γ_1 (the "correct" observer) is moderate.

On the other hand, after the successful reconfiguration and after the new steady state is reached, the plotted values do not allow to identify the fault. This demonstrates that a fault can only be detected when the system is in motion. A stationary situation does not deliver enough information for fault detection or identification. Therefore the fault detection is switched off, once a fault has been identified. The detection of a changing fault scenario would required some sort of re-initialisation that is beyond the scope of this paper.

7 Conclusion

It has been demonstrated that an LQG controller can be automatically redesigned in case of the fault in the plant, thus keeping the system operational. The algorithm is efficient and automatic. No manual intervention is necessary.

It has been shown that in case of sensors faults the observer part of the controller has to be re-designed whereas in case of an actuator fault the state feedback part is to be changed. Rank deficiency problems that occur during the re-design steps have been solved.

The objective value of the LQG problem is proposed as a performance measure. It can be applied to different solutions for the reconfiguration task, allowing to compare different approaches according to their achieved control performance. Of course the solution presented here is by definition the optimal one, so all other approaches are bound to be inferior according to this measure.

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