MODELLING AND CONTROL OF A SINGLE DEGREE-OF-FREEDOM DYNAMIC WIND TUNNEL RIG

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Abstract

Development of flight control laws for modern highly-manoeuvrable aircraft generally takes many years. Controller design is often only started after a detailed mathematical model of the aircraft exists, which may well already be many years into the development programme. This problem is being exacerbated by the need for modern aircraft to be controllable at extremely high angles of attack (up to $\approx 80°$), which increases control system complexity and requires more time to be spent in development and testing. A possible method for reducing the time spent in both aerodynamic and control system design phases is being developed at the University of Bristol. By using a novel, multi-axis dynamic wind tunnel rig it is hoped that control system design can be conducted in parallel with aerodynamic development and mathematical modelling to reduce the overall aircraft procurement time. This paper aims to give a brief outline of the pilot rig at the University of Bristol and present modelling and experimental control results obtained on the rig in a single degree-of-freedom configuration.

1 Introduction

There have been relatively few attempts to design control laws for highly-manoeuvrable aircraft using actively controlled models 'flying' in multiple degrees of freedom in wind tunnels. A single degree-of-freedom (DOF) rig (pitch only) with actuated control surfaces was used at the National Aerospace Laboratories in India to model time-dependent effects on highly swept delta wings [9]. A 2 DOF rig (roll and yaw) using active control surfaces augmented with compressed-air blowing was developed at Cambridge University and successfully used to develop lateral-directional controllers for the HHIRM model using H_∞ methods [8]. In [5] a 4 DOF rig was used to develop gust alleviation controllers for a small, high-wing turboprop aircraft. The model was free to roll, pitch and yaw about a central gimbal within the model, and could slide up and down a vertical wire mounted in the tunnel. Torque-motors were used to actuate elevators, ailerons, and trailing edge flaps. A similar 4 DOF rig was developed at Cranfield Institute of Technology [1] to extract aerodynamic models, develop control systems and perform wind tunnel simulations of dynamic motions.

A full 6 DOF free-flight rig was developed by NASA for the 30' by 60' Langley full-scale tunnel (e.g. [4]). The model to be tested was free to fly within the tunnel working section, with electrical power, compressed air (for propulsion) and control and feedback signals being fed from the top of the tunnel via a slack umbilical chord. Controllers were implemented using computers outside the tunnel, with three 'pilots' providing control inputs and handling quality feedback. The cost of this type of rig is prohibitive in most cases due to the size of the wind tunnel required, complexity of the model and number of operators needed.

The Pendulum Support Rig (PSR) [3, 6, 7] was proposed by TsAGI in Russia, and hopes to address some of the problems associated with obtaining aerodynamic derivatives and designing control laws, most notably time and cost. The rig is multi-purpose in nature; it can be used for aerodynamic modelling (where periodic and arbitrary motions can be generated), control system design and evaluation using active control surfaces. A 5 DOF pilot rig has been built and is currently being tested at the University of Bristol, however the results presented here are for single degree of freedom tests carried out during development of the rig.

The rest of the paper is arranged as follows. In section 2 the wind tunnel rig at the University of Bristol is described in more detail. Section 3 outlines the non-linear behaviour of the rig, and describes the novel mathematical modelling technique developed to capture the observed experimental dynamics. Example model validation results are also presented in Section 3. In Section 4 some initial control system comparisons are presented and discussed, before conclusions and recommendations are given in Section 5.

2 Wind Tunnel Model

The model being used for the pilot rig is an approximate 1/16th scale BAe Hawk, constructed mainly of fibreglass covered wood (Figure 1). The dimensions of the model are: wing span 0.612m, length 0.655m, wing surface area 0.078m^2 and mean aerodynamic chord 0.135m. When balanced about the rota-

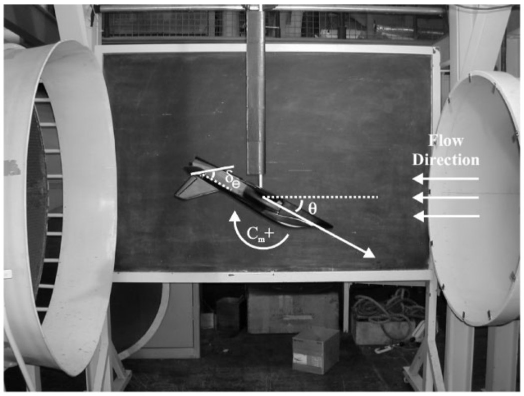

Figure 1: Wind tunnel rig in the Department 7'×5' open-jet tunnel.

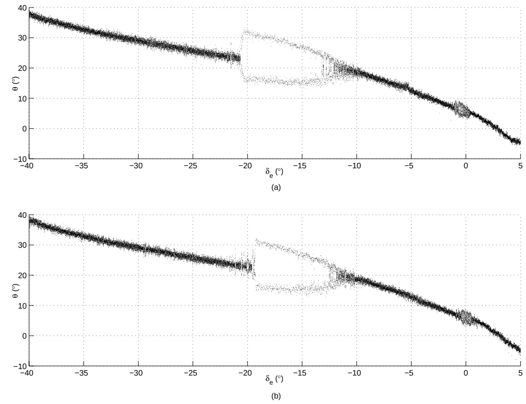

Figure 2: Experimental bifurcation diagrams for the single degree-of-freedom wind tunnel model with decreasing (a) and increasing (b) tailplane deflection.

tion point the weight of the model without gimbal is 1.8kg. Aluminium gimbals are used to achieve rotation in up to five degrees of freedom (three degrees in the model (roll, pitch and yaw) and two at the tunnel mounting point to give (constrained) heave and lateral motion), using precision ball bearings to minimise friction. Precision carbon-film potentiometers are used for angular position feedback (accurate to $\pm 0.05\,°$), and solid-state rate gyros for angular velocity ($\pm 0.25\,°$/s accuracy). For the results presented here, the model is mounted inverted in a single DOF only (pitch). Moment of inertia in pitch is 0.0343kgm^2 when balanced about the pivot point. The model has all-moving tailplanes which are directly driven by miniature model aircraft servos. A dSPACE DS1103 real-time control system is used for data acquisition and control, using Matlab/Simulink and Real-Time Workshop for rapid control system prototyping. For all results presented here, the sampling rate was 100Hz.

The wind tunnel used for testing is a 1.1m diameter open-jet tunnel with a maximum speed of 40m/s, and a turbulence level of approximately 1.5% at 20m/s. All tests were performed at 20m/s (corresponding to Re $= 0.2 \times 10^6$ based on model wing chord).

3 Open-Loop Experimental Results and Modelling

To aid the development of controllers for the rig it was necessary to first develop a mathematical model of the system. Even in a single DOF the model exhibits some interesting non-linear behaviour in the form of stable and unstable limit cycle regions, with both sub- and supercritical Hopf bifurcations. Experimental bifurcation diagrams were derived to analyse the dynamics (Figure 2). These were constructed by slowly varying the input (symmetric tailplane deflection) from one deflection limit to the other over a one hour period. Tests were performed using both increasing and decreasing tailplane angle, and revealed a region of hysteresis associated with the bifurcations at $\delta_e \approx 20°$.

The bifurcation diagrams were constructed by only plotting points at which the absolute value of pitch rate, $|q|$, was less than $2\,°/\text{s}$[1]. Figure 2 shows some interesting non-linear behaviour in the form of a large amplitude limit cycle ($-20° < \delta_e < -10°$), a small amplitude limit cycle at low angle-of-attack ($-1° < \delta_e < 1°$), and a region of hysteresis at $-21° < \delta_e < -19°$.

An innovative method for accurately modelling the observed experimental dynamics was developed, making explicit use of the experimental bifurcation diagram, as well as time histories of the motion (for the sake of brevity, the reader is referred to [2] for a detailed description of the identification procedure). The model returns pitch acceleration ($\ddot{\theta}$) as a function of its states, the pitch angle θ and the pitch rate $\dot{\theta}$ (also labelled q), and the control input corresponding to the tailplane deflection δ_e. The limit cycle amplitudes, stability and frequencies observed in the experimental results were used to identify the model parameters as described in [2]. Figure 3 shows the bifurcation diagram for the mathematical model of the rig which can be seen to capture well the results of the experiments. A second order rate and position limited actuator model was developed from frequency-sweep tests on the model servos. The actuator model includes a large deadband ($\pm 0.7°$) and stiction, giving significant hysteresis.

3.1 Model Validation

Open-loop validation of the model is presented in [2]. An example comparison between experimental time history and numerical simulation is given in Figure 4, for a fixed tailplane deflection of $-15°$. The model is released from rest at a point off the limit cycle, and the oscillatory decay shows very good agreement between experimental results and the mathematical model.

Leaving to [2] further details on the model derivation and its validation, focus is now turned to its use for the synthesis of ap-

[1] due to experimental noise it is not possible to plot purely points at which q is zero, as one would wish.

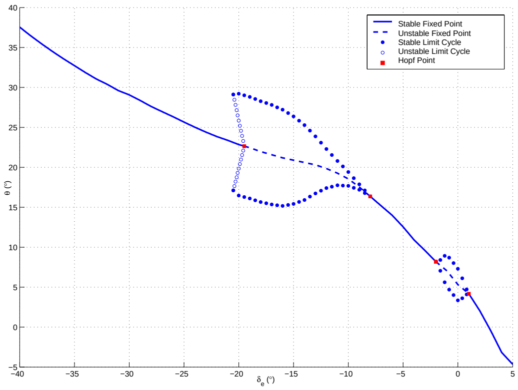

Figure 3: Bifurcation diagram for the mathematical model.

propriate control laws for the rig. After testing the performance of different control laws numerically on the derived model, their experimental application to the rig is presented with the aim of (i) validating the use of the model for control design and (ii) assessing the performance of different controllers on the rig.

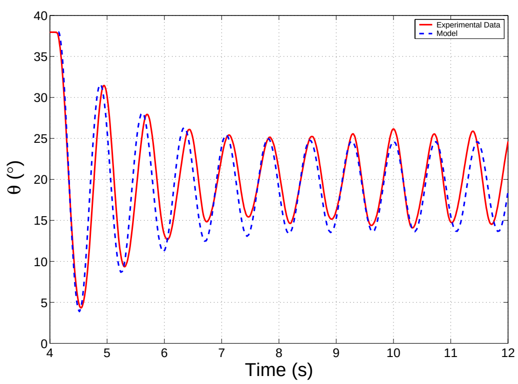

Figure 4: Comparison of mathematical model with experimental results; model released from rest at $t \approx 4.2$s ($\delta_e = -15°$).

4 Control System Design

4.1 Open-Loop θ Demand with Pitch Rate Compensator

As a starting point, an open-loop controller was considered consisting of just a proportional feed-forward gain K_{ff}. The reference signal is the demand on the pitch angle θ, which is simply multiplied by the constant feed-forward gain to produce the control input u corresponding to the tailplane deflection demand δ_e. Figure 5 shows the simulated and experimental response of the state variables θ and q under the action of the open-loop controller ($0 < t < 30$). Note the good agreement between the control performance predicted by the model numerical simulations (Fig. 5(b)-(d)-(f)) and the actual experimental results (Figure 5 (a)-(c)-(e)). No turbulence model was used in the numerical model.

The fixed feed-forward gain value was chosen to give good cor-

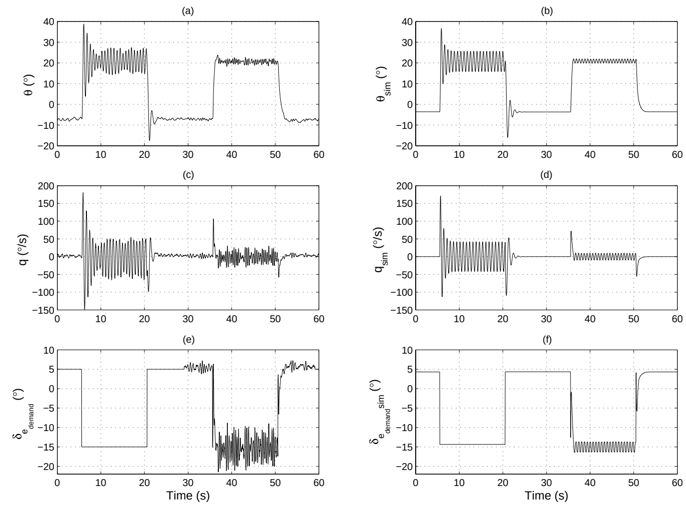

Figure 5: Experimental evolution (a),(c),(e) and numerical simulations (b),(d),(f) of the system response to a step reference input with an open-loop feed-forward controller with ($t > 30$s) and without ($t < 30$s) the pitch rate q feedback compensation. Here K_{ff}= - 0.75, $K_q = 0.2$.

relation between the reference and output for the step responses in Figure 5, however, when a reference signal of a different amplitude is used the tracking response is significantly worse given the open-loop nature of the controller and the fact that the rig is a highly non-linear system. Moreover, the open-loop response shown in Figure 5 for $t < 30s$ also confirms the presence of highly oscillatory modes at high angles-of-attack (α). Therefore, pitch damping was added by including a proportional fixed-gain pitch rate feedback as shown in Figure 6.

The resulting performance of the open-loop feed-forward control in the presence of the feedback pitch rate compensator is shown in Figure 5 for $t > 30s$. A feedback gain of $K_q = 0.2$ was found to limit RMS amplitude in the limit cycle regions without introducing self-excited oscillations. In fact, a consistent reduction of the oscillatory modes is observed, which reduces the amplitude of the high α limit cycle from approximately $5°$ to $1°$. As can be seen from Figure 5 (b)-(d)-(f), again the mathematical model shows good qualitative agreement with the experimental results but for a slight discrepancy in absolute values. A possible reason for this is sensitivity of the rig to centre of gravity location of the model, causing a

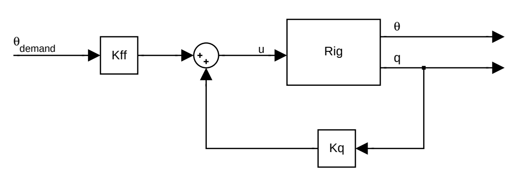

Figure 6: Open-loop pitch angle controller with pitch rate compensator.

fixed offset in θ.

4.2 Non-linear Feed-Forward with Pitch Rate Feedback

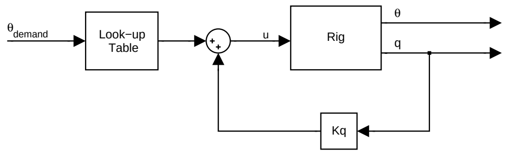

Figure 7: Non-linear inverse trim curve feed-forward with pitch rate compensator.

To improve the performance of the controller presented in the previous section, the fixed-gain forward path is replaced with a non-linear inverse trim curve derived from the rig model in order to have an improved correlation between the reference input (pitch angle demand) and the state being controlled (pitch angle). The look-up table gives tailplane deflection as a function of pitch angle and was derived by carrying out an extensive analysis of the model presented in [2], briefly described in Sec. 3.

The resulting control scheme is shown in Figure 7. Experimental time histories of the controlled rig to sinusoidal and step reference inputs are depicted in Figure 8. In both cases, note the presence of a non-zero steady-state error in the system response. This is to be expected because of the mainly open-loop nature of the control on θ. In order to eliminate this unwanted residual error, the synthesis of an appropriate feedback loop on θ is now investigated.

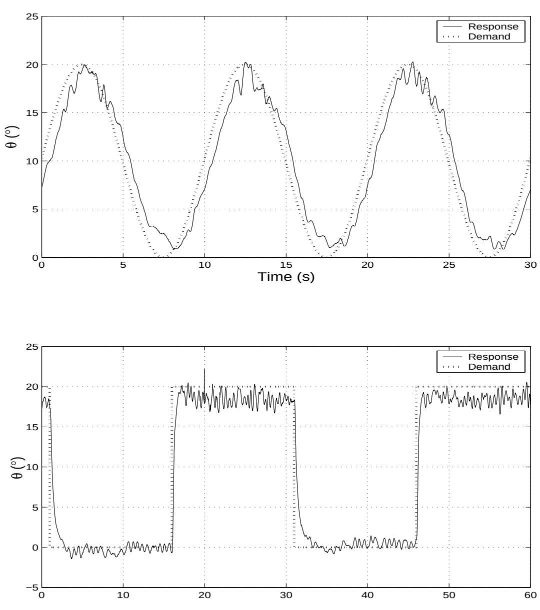

Figure 8: Response of experimental system to step and sine wave reference using non-linear feed-forward control.

4.3 Non-linear Feed-Forward with PID Pitch Angle Feedback

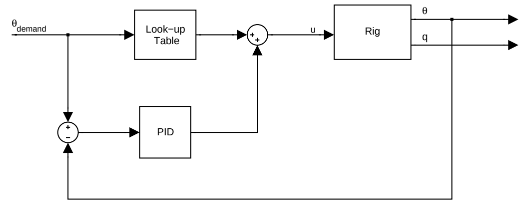

Figure 9: Feed-forward and pitch angle feedback control.

To compensate for the steady-state error present in the response shown in Fig. 8, a PID controller on θ is added, as shown in Figure 9. The presence of a derivative action in the PID controller makes the pitch rate compensation redundant, while adding a ‘predictive’ element by feeding forward the rate-of-change of the reference signal.

The gains of the PID controller were tuned using different methodologies, with the best performance being obtained by trial-and-error. Namely, a small proportional gain was used ($K_p = 0.1$) in order not to cause unwanted oscillatory behaviour. The integral gain, K_i was set to -1 to give a good compromise between response and stability, while the derivative gain, K_d, was set to 0.2 (i.e. to the same values of the feedback gain K_q used in the pitch rate compensator presented in Section 4.1). Responses for this control scheme are presented in Figures 10 and 11, where the experimental behaviour of the controlled system is compared with the prediction of the numerical simulation of the model.

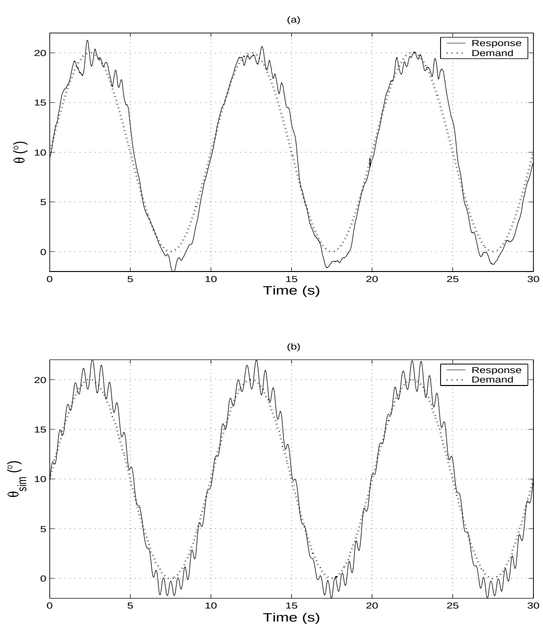

Figure 10: Comparison of experimental (a) and numerical simulation (b) results for large amplitude sine wave reference (feed-forward plus PID).

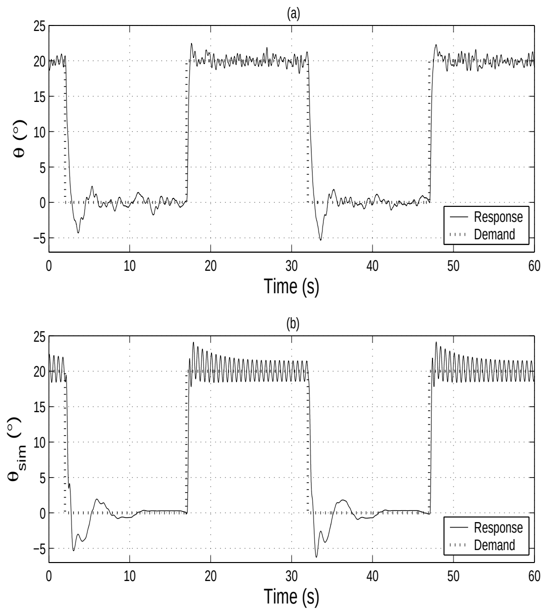

Figure 11: Large-amplitude step response for (a) experimental rig and (b) numerical simulation (feed-forward plus PID).

Again, a very good agreement between experiments and numerics can be observed, further confirming the suitability of the model presented in [2] for control design applications. The control responses show good sine wave tracking performance, with deviations from the demand occurring mainly in the high and low angle-of-attack limit cycle regions. The step response, shown in Fig. 11, is also good with rapid transients and, when compared with Fig. 8, showing the absence of any residual steady-state error. A higher overshoot is observed in the downward direction.

4.4 PID Control

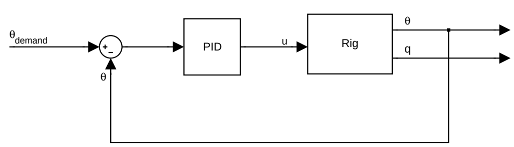

Figure 12: PID pitch angle controller.

For the purposes of comparison, the feed-forward non-linear term was removed to assess the performance of just a PID feedback controller on θ (Figure 12). The controller gains were tuned via numerical simulations on the model and then tested on the rig. To implement this controller on the experimental rig, the pitch angle reference signal was differentiated and used to give a pitch rate error signal which is then fed through the derivative gain. This is equivalent to differentiating the pitch angle error, but avoids noise problems associated with differentiating the pitch angle signal on the experimental rig.

A non-linear unconstrained optimisation function (Matlab function *fminsearch*) was used to tune the PID controller gains. A series of 30 second simulations were run with a 20° amplitude frequency-swept sine wave pitch angle demand, and a cost function calculated as the integral of pitch angle error over the simulation. The optimisation proved to be local, depending heavily on the values passed to the function, hence the gains were tuned initially by trial-and-error to get a good initial 'guess' for the optimisation routine. The gain values found were $K_p = 0.33$, $K_i = 10.24$ and $K_d = 0.25$. (Note the relatively large integral action needed to compensate for the non-zero steady-state error due to the non-linear perturbation acting on the system.)

An example response of the system using these feedback gains is shown in Figures 13 and 14. It is observed that, in the absence of the non-linear feed-forward action, the PID controller results in a highly oscillatory response and therefore an overall poorer tracking performance. This increase in oscillation is possibly due to the larger pitch rate feedback gain used by the PID controller (K_d) introducing self-excited oscillations, as mentioned in Section 4.1. The step response in Fig. 14 also shows a large amount of oscillation in the low angle-of-attack region. This might be overcome by re-tuning the PID gains which were originally optimised using a sinusoidal reference signal.

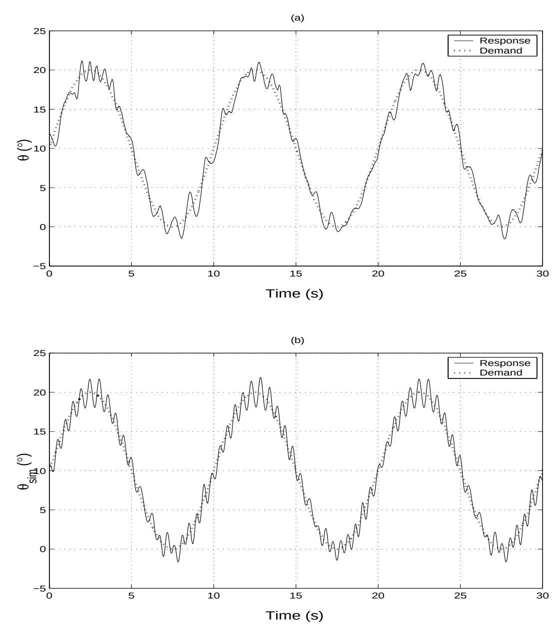

Figure 13: Comparison of experimental (a) and numerical simulation (b) results for large amplitude sine wave reference (PID controller).

5 Conclusions and Future Work

The control of a novel dynamic wind tunnel configuration has been discussed. In particular, after presenting the experimental bifurcation diagrams of the rig, the derivation and valida-

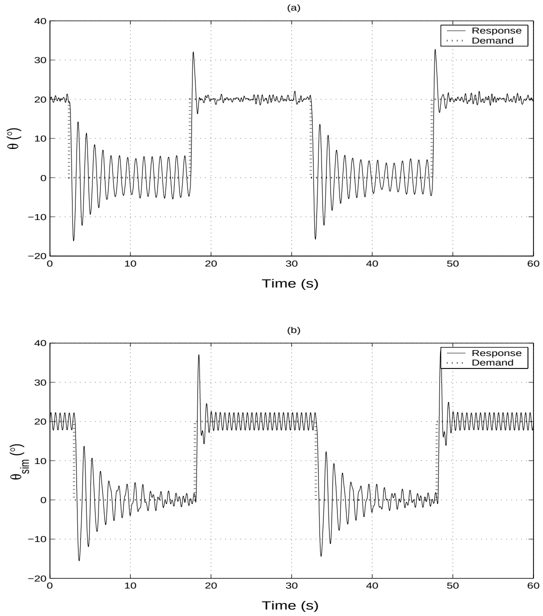

Figure 14: Large-amplitude step response for (a) experimental rig and (b) numerical simulations (PID controller).

tion of an appropriate model based on such diagrams has been briefly discussed. The model was then used to design different controllers which have been experimentally implemented and tested. The system response in the presence of different control laws has been presented and evaluated. It was shown that the best performance can be obtained by using a feed-forward non-linear action coupled with a feedback PID controller which guarantees good tracking properties characterised by zero steady-state error and relatively low oscillatory modes. Both the feed-forward inverse look-up table and the tuning of the PID gains were achieved through the numerical analysis of the model, confirming its suitability for control system design.

Ongoing work is addressing the design of more sophisticated control laws including gain scheduling, adaptive and robust controllers in order to address typical aircraft-related issues such as handling qualities.

While carrying out the experimental work described in the paper the use of the rig for control system design has been demonstrated, however, problems and limitations have also been identified. One such limitation is imposed by the actuators used (off-the-shelf miniature radio-controlled model type servos); they have a relatively poor resolution and a large position dead-band which limits the accuracy of results. Using a larger model would allow higher specification actuators to be used. A further limitation is tunnel turbulence, as shown in the time histories presented. Thus, a representative turbulence model is to be included in the mathematical model at a later stage.

Finally, a full 5 DOF rig has recently been demonstrated at the University of Bristol and it is hoped will allow the comparison of multivariable control schemes on a multi degree-of-freedom experimental simulator. The rig is expected to provide a method for rapid prototyping of control schemes and generation of aerodynamic data, especially applicable to the development of small Unmanned Aerial Vehicles (UAVs).

Acknowledgements

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