IDENTIFICATION OF MISO WIENER AND HAMMERSTEIN SYSTEMS

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Abstract

This paper describes an unified new recursive identification method in the prediction error method and model scheme for three MISO Wiener and Hammerstein systems. It is also extension of our earlier work for SISO cases. With the estimation of intermediate variables by using the key term separation principle, a MISO Wiener and Hammerstein system can be approximately transformed into a pseudo-linear MISO dynamic system. Using the adaptive recursive pseudo-linear regressions (RPLR) for a linear MISO dynamic system and smoothing and filtering techniques for estimation of the intermediate variables, satisfied parameter estimates of the MISO Wiener and Hammerstein system can be obtained in the presence of a white or a coloured measurement noise without parameter redundancy. The performance of the developed method is both analysed theoretically and illustrated by means of simulation results.

1. Instruction

Identification of nonlinear dynamic systems are widely studied and applied in practice. A survey of the nonlinear system model structures and identification methods are given in [9]. A Wiener system or a Hammerstein system is adequately to describe a nonlinear dynamic system. A Wiener system is defined as a linear dynamic subsystem in cascade with a nonlinear static subsystem (Fig. 1). A Hammerstein system is defined as a nonlinear static subsystem followed by a linear dynamic subsystem (Fig. 2), The steady-state behaviour is determined completely by the static nonlinearity, while the dynamic behaviour is determined by both the static nonlinearity and the linear dynamic subsystem. Combinations of the SISO Wiener and Hammerstein systems construct MISO nonlinear dynamic systems (Figs. 3-5).

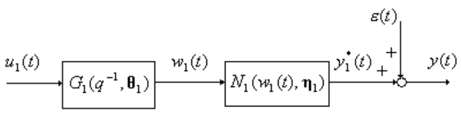

Figure 1 A Wiener system

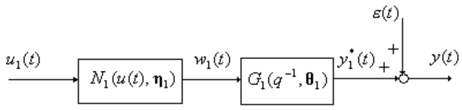

Figure 2 A Hammerstein system

Till now, the identification algorithms in the literatures are mainly for SISO Wiener and Hammerstein systems. An identification algorithm for a Hammerstein system was developed in [7] by estimating separately and sequentially the linear and the nonlinear subsystems. A Hammerstein system was identified in [3] by simplifying and transforming it into a linear MISO system. In [10] a Wiener system and a Hammerstein system were identified using a transfer function for the linear dynamic subsystem and a polynomial for the nonlinearity and with the estimation of internal variables. Our earlier work [2] introduced an unified recursive identification method for different SISO Wiener and Hammerstein systems. In some literatures, a strong assumption was made that the nonlinear block in a Wiener system was invertible or with the known nonlinear blocks, see e.g. [4, 8]. A SISO Wiener and Hammerstein system can be seen as a special case of a MISO Wiener and Hammerstein system. Some of the identification methods have been extended to identify a MISO Wiener and Hammerstein system. An extended method for a MISO Wiener and Hammerstein system was developed in [5] based on the recursive prediction error method. In [1] two recursive identification methods were extended to a MISO Hammerstein system along the lines of the basic Kalman filter.

To write out the description of the whole nonlinear system analytically, it presents usually nonlinearities or redundancy in parameters. And the relevant identification algorithms would be also complicated and inefficient. There is an implicit or explicit trade-off between the acceptable model complexity, the identification algorithm complexity and how well the model matches the data. It can be concluded from the literatures that the most important attempt is trying to reduce parameter redundancy by using special linear and nonlinear model structures. Some other attempts are to select a parameterisation and approximation or a relax algorithm to simplify the computation procedure to fit the individual nonlinear system to process data.

The purpose of this contribution is to develop an unified identification method and strategies in the prediction error method and model scheme to identify: a MISO Wiener system (Figure 3), a MISO Hammerstein system (Figure 4) and a mixed MISO Wiener and Hammerstein system (Figure 5). The rest of this paper is organised as follows: Section 2 introduces the system structures and identification strategies. Section 3 derives the identification algorithm to each system. Section 4 presents the simulation examples. Finally, section 5 gives a brief summary of the major results and conclusions.

2. System Statement

It is assumed that the MISO Wiener and Hammerstein systems are all asymptotically stable.

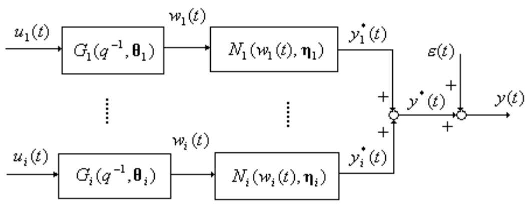

Figure 3 A MISO Wiener system

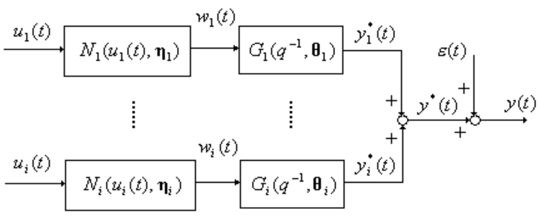

Figure 4 A MISO Hammerstein system

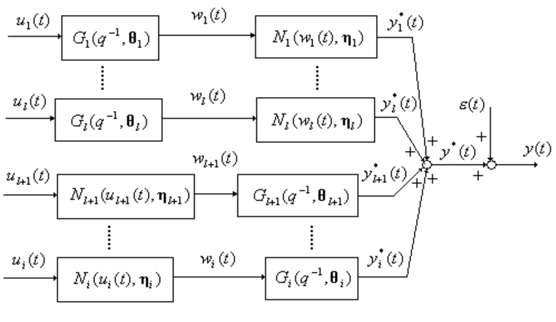

Figure 5 A mixed MISO Wiener and Hammerstein system

In above Figures, $u_1(t),\cdots,u_l(t),u_{l+1}(t),\cdots,u_i(t)$ are i system inputs. $y(t)$ is the whole system output. $y_1^*(t),\cdots,y_l^*(t)$, $y_{l+1}^*(t),\cdots,y_i^*(t)$ are i unmeasurable outputs of branches. And $y^*(t)$ is the summed system output without measurement noise. The intermediate variables, $w_1(t),\cdots,w_l(t)$, $w_{l+1}(t),\cdots,w_i(t)$, can not be measured. $\boldsymbol{\theta}=[\boldsymbol{\theta}_1,\cdots,\boldsymbol{\theta}_l,\boldsymbol{\theta}_{l+1},\cdots,\boldsymbol{\theta}_i]^{\mathbf{T}}$ is the parameter vector determining the linear dynamic subsystems, $G_1(q^{-1},\boldsymbol{\theta}_1),\cdots,G_l(q^{-1},\boldsymbol{\theta}_l),G_{l+1}(q^{-1},\boldsymbol{\theta}_{l+1}),\cdots,G_i(q^{-1},\boldsymbol{\theta}_i)$, respectively. q^{-1} denotes the discrete shift operator. $\boldsymbol{\eta}=[\boldsymbol{\eta}_1,\cdots,\boldsymbol{\eta}_l,\boldsymbol{\eta}_{l+1},\cdots,\boldsymbol{\eta}_i]^{\mathbf{T}}$ is the parameter vector determining the nonlinear static subsystems, $N_1(\bullet,\boldsymbol{\eta}_1)$, $\cdots,N_l(\bullet,\boldsymbol{\eta}_l)$, $N_{l+1}(\bullet,\boldsymbol{\eta}_{l+1}),\cdots,N_i(\bullet,\boldsymbol{\eta}_i)$, respectively. $\varepsilon(t)$ is a coloured measurement noise which is a white noise $e(t)$ through a linear filter $H(q^{-1},\boldsymbol{\xi})$,

$$H(q^{-1},\boldsymbol{\xi})=\frac{C(q^{-1})}{D(q^{-1})} \tag{1}$$

where, $C(q^{-1})=1+c_1q^{-1}+\cdots+c_{n_c}q^{-n_c}$ and $D(q^{-1})=1+d_1q^{-1}+\cdots+d_{n_d}q^{-n_d}$. $\boldsymbol{\xi}$ is the noise filter parameter vector.

Considering the $j-th$ branch for $j=1,\cdots,i$ in a MISO Wiener and Hammerstein system. The nonlinear static subsystem $N_j(\chi_j(t),\boldsymbol{\eta}_j)$ is assumed to be a m_j order polynomial function and $\chi_j(t)$ is the input of the static nonlinear block,

$$N_j(\chi_j(t),\boldsymbol{\eta}_j)=\sum_{k=1}^{m_j}\beta_{jk}\chi_j^k(t) \tag{2}$$

The linear dynamic subsystem $G_j(q^{-1},\boldsymbol{\theta}_j)$ with order n_j can be written as,

$$G_j(q^{-1},\boldsymbol{\theta}_j)=\frac{B_j(q^{-1})}{F_j(q^{-1})} \tag{3}$$

where, $B_j(q^{-1})=b_{j1}q^{-1}+b_{j2}q^{-2}+\cdots+b_{jn_j}q^{-n_j}$ and $F_j(q^{-1})=1+f_{j1}q^{-1}+\cdots+f_{jn_j}q^{-n_j}$. $B_j(q^{-1})$ and $F_j(q^{-1})$ are coprime for $j=1,\cdots,i$. Without losing generality, it is assumed that the delay of the linear block is one. Eq. (3) can be rewritten as,

$$\begin{aligned}G_j(q^{-1},\boldsymbol{\theta}_j)&=b_{j1}q^{-1}+\frac{B_j^*(q^{-1})}{F_j(q^{-1})}\\&=b_{j1}q^{-1}+\frac{b_{j2}^*q^{-2}+\cdots+b_{j(n_j+1)}^*q^{-n_j-1}}{1+f_{j1}q^{-1}+\cdots+f_{jn_j}q^{-n_j}}\end{aligned} \tag{4}$$

where, $B_j^*(q^{-1})=b_{j2}^*q^{-2}+\cdots+b_{j(n_j+1)}^*q^{-n_j-1}$, in which, $b_{jk+1}^*=b_{jk+1}-b_{j1}f_{jk}$ for $k=1,2,\cdots,n_j$.

We consider the identification problem in a prediction error method and model scheme. The parameter vector of an i-

inputs MISO Wiener and Hammerstein system is defined as $\vartheta = [\boldsymbol{\theta}, \boldsymbol{\eta}, \boldsymbol{\xi}]^{\mathbf{T}}$. To identify the parameters, we compare the predicted system output $\hat{y}(t/\vartheta)$ with the measured system output $y(t)$ in the following prediction error criterion,

$$J(\vartheta) = \frac{1}{2}[(y(t) - \hat{y}(t/\vartheta))^{\mathbf{T}}(y(t) - \hat{y}(t/\vartheta))] \tag{5a}$$

$$\vartheta = \arg\min_{\vartheta} J(\vartheta) \tag{5b}$$

A possible prediction model is $\hat{y}(t/\vartheta) = \boldsymbol{\varphi}^{\mathbf{T}}(t,\vartheta)\vartheta$. The so called pseudo-regression vector $\boldsymbol{\varphi}(t,\vartheta)$ contains relevant past data, partly reconstructed using the current model. The parameter-dependent reconstructed elements are determined in some recursive fashion and arrive at the recursive pseudo-linear regressions (RPLR) estimates. As pointed out in [6]: no matter how $\boldsymbol{\varphi}(t,\vartheta)$ is formed, it is the known data at time t. It can contain arbitrary transformations of measured data. We could rewrite $\hat{y}(t/\vartheta) = \boldsymbol{\varphi}^{\mathbf{T}}(t,\vartheta)\vartheta$ as,

$$\begin{aligned} \hat{y}(t/\vartheta) &= \boldsymbol{\varphi}_1^{\mathbf{T}}(t,\vartheta_1)\vartheta_1 + \boldsymbol{\varphi}_2^{\mathbf{T}}(t,\vartheta_2)\vartheta_2 \\ &+ \cdots + \boldsymbol{\varphi}_s^{\mathbf{T}}(t,\vartheta_s)\vartheta_s \end{aligned} \tag{6}$$

with arbitrary functions $\boldsymbol{\varphi}_j(t,\vartheta_j)$ of past data for $j = 1 \cdots s$, where $s > i$. Eq. (6) could be regarded as a finite-dimensional parameterisation of a general, unknown nonlinear predictor. The key is how to choose the functions $\boldsymbol{\varphi}_j(t,\vartheta_j)$ for $j = 1 \cdots s$, and this is where physical insight into the system is required. It can also be seen as a transformed result from a s pseudo linear and multiple inputs system,

$$\begin{aligned} y(t) &= G_1(q^{-1},\vartheta_1)\cdot u_1(t) + G_2(q^{-1},\vartheta_2)\cdot u_2(t) \\ &+ \cdots + G_s(q^{-1},\vartheta_s)\cdot u_s(t) + \varepsilon(t) \end{aligned} \tag{7}$$

To identify a MISO Wiener and Hammerstein system, one problem is that a constant gain can be distributed arbitrarily among the linear and nonlinear subsystems in each branch. In order to get a unique solution, the gain of one subsystem must be fixed. Without losing generality, a simpler solution is just to fix one of the parameters of the linear or nonlinear subsystem in each branch and let it be constant during the minimisation.

Intermediate variables are unmeasurable variables among the linear and nonlinear blocks in a MISO Wiener and Hammerstein system. Generally, the unmeasurable intermediate variables could be cancelled in the whole analytical system, but it presents usually nonlinearities or redundancy in parameters. According to the key term separation and half-substitution principle, see e.g. [10], the unmeasurable intermediate variable of each branch can be individually separated as the key term which will be substituted with its front expression. The non-key terms will be calculated recursively from the corresponding subsystem with the estimated parameters from last time instant. Therefore, smoothing and filtering techniques in parameter estimation are necessary, in order to mitigate the estimate errors of intermediate variables and to avoid the possible oscillations to achieve better convergence. These intermediate variables could be seen as pseudo multiple inputs to formulate a pseudo-linear MISO dynamic system. Here, the pseudo multiple inputs become double-meaning: they are pseudo multiple inputs of a nonlinear dynamic system and they illustrate the nonlinear couplings between the pseudo multiple inputs. With the system input, output data and the estimated intermediate variables, every subsystem can be reconstructed in other structure type or in plot curve. In order to accelerate the convergence, we use the forgetting factor approach with a variant forgetting factor, $\lambda(t) = \lambda(t-1) + (1-\lambda(t-1))\cdot\Delta\lambda$ to adaptation and to identify the parameters.

Therefore, the system identification process and assumptions are as follows:

1. Each MISO Wiener and Hammerstein system (Figs. 3-5) is asymptotically stable and the orders of all subsystems m_j, n_j for $j = 1,\cdots,i$ and n_c, n_d are known a priori.
2. Defining the intermediate variables. Using the key term principle to extract the key terms and half-substitute the key terms. Then transforming the MISO Wiener and Hammerstein system approximately into a pseudo-linear MISO system.
3. The system inputs are persistent. The all multiple pseudo-inputs and the interference signal $\varepsilon(t)$ are assumed to be independent. And the linear and nonlinear blocks in one branch have no direct influence to the blocks of different branches.
4. Fixing some parameters to obtain an unique parameterisation. With the smoothing and filtering techniques to estimate the intermediate variables recursively and using adaptive recursive pseudo-linear regressions (RPLR) to identify the parameters of the transformed pseudo-linear MISO system.

3. Algorithms derivation

3.1. A MISO Wiener system

Considering the $j-th$ branch, $j = 1,\cdots,i$, of a MISO Wiener system which consists of i single Wiener system branches (Figure 3). The nonlinear static and the linear dynamic subsystems are defined as eq. (2) and eq. (3). Each branch output is,

$$y_j^*(t) = N_j(w_j(t), \boldsymbol{\eta}_j) = \sum_{k=1}^{m_j} \beta_{jk} w_j^k(t) \tag{8}$$

Without losing generality, let $\beta_{j1} = 1$. And the first corresponding term $w_j(t)$ is the key term in the $j-th$ branch. Half-substituting eq. (3) into the key term $w_j(t)$ in eq. (8), the whole system output is,

$$\begin{aligned} y(t) &= \sum_{j=1}^{i} y_j^*(t) + \varepsilon(t) \\ &= \sum_{j=1}^{i} [\frac{B_j(q^{-1})}{F_j(q^{-1})} u_j(t) + \sum_{k=2}^{m_j} \beta_{jk} w_j^k(t)] + \frac{C(q^{-1})}{D(q^{-1})} e(t) \end{aligned} \tag{9}$$

It is seen, a MISO Wiener system has been transformed into a pseudo-linear MISO system which has $\sum_{j=1}^{i} m_j$ pseudo-inputs: $u_j(t)$, $w_j^2(t), w_j^3(t), \cdots, w_j^{m_j}(t)$ for $j = 1, \cdots, i$. All the unknown parameters of the system are explicitly given. Using the adaptive linear MISO system recursive identification method, supplemented with parameters smoothing and filtering techniques and with eq. (3) estimated unmeasurable variables $w_j(t)$ for $j = 1, \cdots, i$, the unknown parameters in eq. (9) can be identified without parameter redundancy.

3.2. A MISO Hammerstein system

Considering the $j-th$ branch for $j = 1, \cdots, i$ of a MISO Hammerstein system (Figure 4) which consists of i single Hammerstein system branches. The nonlinear static and the linear dynamic subsystems are defined as eq. (2) and eq. (4). Each branch output is,

$$\begin{aligned} y_j^*(t) &= G_j(q^{-1}, \boldsymbol{\theta}_j) w_j(t) \\ &= b_{j1} q^{-1} w_j(t) + \frac{b_{j2}^* q^{-2} + \cdots + b_{j(n_j+1)}^* q^{-n_j-1}}{1 + f_{j1} q^{-1} + \cdots + f_{jn_j} q^{-n_j}} w_j(t) \end{aligned} \tag{10}$$

Without losing generality, we let $b_{j1} = 1$. Then the first term $w_j(t)$ in eq. (10) is the key term in the $j-th$ branch. Half-substituting eq. (2) into the key term $w_j(t)$, the whole system output of a MISO Hammerstein system can be written as,

$$\begin{aligned} y(t) &= \sum_{j=1}^{i} y_j^*(t) + \varepsilon(t) \\ &= \sum_{j=1}^{i} [\sum_{k=1}^{m_j} \beta_{jk} q^{-1} u_j^k(t) + \frac{B_j^*(q^{-1})}{F_j(q^{-1})} w_j(t)] + \frac{C(q^{-1})}{D(q^{-1})} e(t) \end{aligned} \tag{11}$$

It is seen, a MISO Hammerstein system has been transformed into a pseudo-linear MISO system which has $\sum_{j=1}^{i} (m_j + 1)$ pseudo-inputs: $u_j(t), u_j^2(t), \cdots, u_j^{m_j}(t)$ and $w_j(t)$ for $j = 1, \cdots, i$. All the unknown parameters of the system are explicitly given. Using the adaptive linear MISO system recursive identification method, supplemented with parameters smoothing and filtering techniques and with eq. (2) estimated unmeasurable variables $w_j(t)$ for $j = 1, \cdots, i$, the unknown parameters in eq. (11) can also be identified without parameter redundancy.

3.3. A mixed MISO system

Considering a i-inputs MISO mixed Wiener and Hammerstein system (Figure 5) which consists of a l-inputs MISO Wiener subsystem and a p-inputs MISO Hammerstein subsystem, $l + p = i$. In the $j-th$ branch of the MISO Wiener subsystem for $j = 1, \cdots, l$, we use the identification analysis of a MISO Wiener system in section 3.1. And in the $j-th$ branch of the MISO Hammerstein subsystem for $j = l+1, \cdots, i$, we use the identification analysis of a MISO Hammerstein system in section 3.2. Defining and half-substituting the corresponding key terms, the whole system output of a MISO mixed Wiener and Hammerstein system can be written as,

$$\begin{aligned} y(t) &= \sum_{j=1}^{i} y_j^*(t) + \varepsilon(t) \\ &= \sum_{j=1}^{l} [\frac{B_j(q^{-1})}{F_j(q^{-1})} u_j(t) + \sum_{k=2}^{m_j} \beta_{jk} w_j^k(t)] \\ &+ \sum_{j=l+1}^{i} [\sum_{k=1}^{m_j} \beta_{jk} u_j^k(t) + \frac{B_j^*(q^{-1})}{F_j(q^{-1})} w_j(t)] + \frac{C(q^{-1})}{D(q^{-1})} e(t) \end{aligned} \tag{12}$$

It is seen, a MISO mixed Wiener and Hammerstein system has been transformed approximately into a pseudo-linear MISO system which has $\sum_{j=1}^{l} m_j + \sum_{j=l+1}^{i} (m_j + 1)$ pseudo-inputs: $u_j(t)$, $w_j^2(t), w_j^3(t), \cdots, w_j^{m_j}(t)$ for $j = 1, \cdots, l$ and $u_j(t), u_j^2(t), \cdots, u_j^{m_j}(t)$, $w_j(t)$ for $j = l+1, \cdots, i$. All the parameters of the system are explicitly given. Using the adaptive linear MISO system recursive identification method, supplemented with parameters smoothing and filtering techniques, with eq. (3) estimated unmeasurable variables $w_j(t)$, for $j = 1, \cdots, l$ and with eq. (2) estimated unmeasurable variables $w_j(t)$, for $j = l+1, \cdots, i$, the unknown parameters in eq. (12) can be identified without parameter redundancy.

4. Simulation examples

Two random numbers of zero mean are used as system inputs, $u_1(t)$ and $u_2(t)$. Another independent random numbers as white measurement noise $e(t)$. The noise filter for a coloured measurement noise is,

$$H(q^{-1},\xi)=\frac{1+0.2q^{-1}+0.1q^{-2}}{1-0.9q^{-1}+0.85q^{-2}}. \quad (13)$$

An average smoother using a moving window with fixed length Mov will be used to filter the estimated parameters to calculate the unmeasurable intermediate variables, $w_1(t)$ and $w_2(t)$. The Noise-Signal ratio is defined as,

$$Noise/Signal=\frac{Var[y(t)]_{Signal=0}}{Var[y(t)]_{Noise=0}} \quad (14)$$

We consider each MISO Wiener and Hammerstein system with non-noisy, with a N./S.=5% white measurement noise (5% W. N.) and with a N./S.=10% coloured measurement noise (10% C. N.), respectively. 2000 data points are collected for each case. Applying the standard recursive prediction errors method (RPEM) with forgetting factor algorithms for linear MISO system in MATLAB. The algorithm variable settings are $\lambda(0)=0.7$, $\Delta\lambda=0.01$ and $Mov=4$. The initial estimates of the unknown parameters are taken as zero.

4.1. The MISO Wiener system

A "standard" example of a MISO Wiener system is from [5] which is composed of two branches,

$$\begin{cases} w_1(t)=\dfrac{0.04308q^{-1}+0.0315q^{-2}}{1-1.3139q^{-1}+0.3886q^{-2}}u_1(t) \\ y_1^*(t)=w_1(t)+4w_1^2(t)+2w_1^3(t) \end{cases} \quad (15)$$

and

$$\begin{cases} w_2(t)=\dfrac{0.0305q^{-1}+0.0254q^{-2}}{1-1.5218q^{-1}+0.5778q^{-2}}u_2(t) \\ y_2^*(t)=w_2(t)+3w_2^2(t)+2w_2^3(t) \end{cases} \quad (16)$$

Satisfied identification results are shown in Table 1. At the same identification signals and noise level (N./S.=5% white measurement noise), the identification process is convergent quickly and the identification result is also better than that in [5].

2000	b_{11}	b_{12}	f_{11}	f_{12}
10% C. N.	0.0450	0.0237	-1.3725	0.4420
5% W. N.	0.0450	0.0249	-1.3614	0.4321
N. N.	0.0431	0.0315	-1.3139	0.3886
Real	0.0431	0.0315	-1.3139	0.3886
2000	β_{11}	β_{12}	β_{21}	β_{22}
10% C. N.	4.3116	2.2480	2.8777	2.1427
5% W. N.	4.3113	2.1100	2.9061	2.0621
N. N.	4.0000	1.9999	3.0000	2.0000
Real	4.0000	2.0000	3.0000	2.0000
2000	b_{21}	b_{22}	f_{21}	f_{22}
10% C. N.	0.0256	0.0305	-1.5228	0.5789
5% W. N.	0.0316	0.0222	-1.5419	0.5952
N. N.	0.0305	0.0254	-1.5218	0.5778
Real	0.0305	0.0254	-1.5218	0.5778
2000	c_1	c_2	d_1	d_2
10% C. N.	0.2568	0.1943	-0.8703	0.8237
Real	0.2000	0.1000	-0.9000	0.8500

Table 1: Identification results of a MISO Wiener system

4.2. The MISO Hammerstein system

As an example, a MISO Hammerstein system consists of the following two branches,

$$\begin{cases} w_1(t)=u_1(t)+4u_1^2(t)+1.5u_1^3(t) \\ y_1^*(t)=\dfrac{0.1333q^{-1}+0.0667q^{-2}}{1-1.5q^{-1}+0.7q^{-2}}w_1(t) \end{cases} \quad (17)$$

and

$$\begin{cases} w_2(t)=u_2(t)+3u_2^2(t)+2u_2^3(t) \\ y_2^*(t)=\dfrac{0.4q^{-1}+0.3q^{-2}}{1-0.9q^{-1}+0.6q^{-2}}w_2(t) \end{cases} \quad (18)$$

Note that the coefficients $b_{11}\neq 1$ and $b_{21}\neq 1$. Some transformations are necessarily made in the corresponding simulations. Satisfied identification results are shown in Table 2.

4.3. The mixed MISO Wiener and Hammerstein system

The example is composed of the following two branches,

$$\begin{cases} w_1(t)=\dfrac{0.4q^{-1}+0.3q^{-2}}{1-0.9q^{-1}+0.6q^{-2}}u_1(t) \\ y_1^*(t)=w_1(t)+4w_1^2(t)+2w_1^3(t) \end{cases} \quad (19)$$

and

$$\begin{cases} w_2(t) = u_2(t) + 3u_2^2(t) + 1.5u_2^3(t) \\ y_2^*(t) = \dfrac{1 + 0.1333q^{-1} + 0.0667q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}} w_2(t) \end{cases} \quad (20)$$

Satisfied identification results are shown in Table 3.

2000	b_{11}	b_{12}	f_{11}	f_{12}
10% C. N.	0.1144	0.0690	-1.4991	0.7010
5% W. N.	0.1418	0.0760	-1.5014	0.7028
N. N.	0.1321	0.0671	-1.5001	0.7000
Real	0.1333	0.0667	-1.5000	0.7000
β_{11}	β_{12}	β_{13}	β_{21}	β_{22}
0.7637	3.1767	1.8290	0.4765	4.0301
1.1187	3.1384	1.2280	0.8803	4.0327
1.0005	3.0005	1.4995	0.9996	4.0006
1.0000	3.0000	1.5000	1.0000	4.0000
β_{23}	b_{21}	b_{22}	f_{21}	f_{22}
3.2280	0.3677	0.2911	-0.9058	0.6028
2.0403	0.4013	0.2781	-0.9052	0.6077
2.0011	0.4002	0.2999	-0.9000	0.6000
2.0000	0.4000	0.3000	-0.9000	0.6000
2000	c_1	c_2	d_1	d_2
10% C. N.	0.2153	0.0901	-0.8484	0.0035
Real	0.2000	0.1000	-0.9000	0.8500

Table 2: Identification results of a MISO Hammerstein system

2000	b_{11}	b_{12}	f_{11}	f_{12}
10% C. N.	0.3688	0.3642	-0.8977	0.5340
5% W. N.	0.3731	0.3710	-0.8505	0.5731
N. N.	0.4054	0.2927	-0.9082	0.6023
Real	0.4000	0.3000	-0.9000	0.6000
β_{12}	β_{13}	β_{21}	β_{22}	β_{23}
3.6810	1.2669	1.1164	2.6320	1.3696
4.2533	2.1117	0.8998	3.1046	1.5872
3.9122	1.9183	1.0063	3.0335	1.5238
4.0000	2.0000	1.0000	3.0000	1.5000
2000	b_{21}	b_{22}	f_{21}	f_{22}
10% C. N.	0.2478	0.1195	-1.4665	0.6675
5% W. N.	0.0489	0.0708	-1.5165	0.7117
N. N.	0.0874	0.0926	-1.4995	0.7000
Real	0.1333	0.0667	-1.5000	0.7000
2000	c_1	c_2	d_1	d_2
10% C. N.	0.2883	0.0841	-0.8763	0.6439
Real	0.2000	0.1000	-0.9000	0.8500

Table 3: Identification results of a mixed MISO Wiener and Hammerstein system

5. Conclusions

In the prediction error method scheme and under some common assumptions, an unified new recursive identification method for three MISO Wiener and Hammerstein systems is proposed. After defining and estimating the intermediate variables recursively, a MISO Wiener and Hammerstein system can be approximately transformed into a pseudo-linear MISO system. With the recursive pseudo-linear regressions (RPLR), the system parameters can be identified in the presence of a white or a coloured measurement noise without parameter redundancy. From the derivations and simulations, it is seen that the new identification method and strategies are clear and efficient. They can be extended easily to identify other block-oriented MISO nonlinear dynamic systems.

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