DYNAMIC OPTIMIZATION FOR ACTIVATED SLUDGE INTEGRATED DESIGN

M. Francisco*, **P. Vega***, **O. Pérez****, **M. Poch**†

* Dpto. Informática y Automática. Escuela Superior de Ingeniería Industrial. Universidad de Salamanca.
Av. Fernando Ballesteros 2 , 37700 Béjar, Salamanca, (Spain). Fax: +34 923 408127
e-mail: mfs@usal.es , pvega@usal.es

** Dpto. de Procesos y Sistemas. Universidad Simón Bolívar, Caracas, 1080-A (Venezuela).
e-mail: operez@usb.ve

† Laboratori d'Enginyeria Química i Ambiental. Universitat de Girona.
Campus de Montilivi, 17071 Girona (Spain). e-mail: manel@lequia1.udg.es

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Abstract

This work presents an algorithmic approach to allow for the Integrated Design of processes and their control systems taking into account the controllability and the stability properties of the resulting system. The application of the proposed method has been carried out taking as an example model an alternative configuration to an activated sludge process belonging to a real wastewater treatment plant. In the Integrated Design, the process parameters are evaluated simultaneously with the parameters of the control system by solving a multiobjective constrained non-linear optimization problem. The considered cost function includes the investment and the operation costs and the constraints are selected to ensure that the values of some controllability are within specified ranges (disturbances sensitivity gains and the Integral Square Error norm), to ensure stability conditions using Lyapunov theory and also to guarantee bounds on the H_∞ performance and many others convex performance criteria. The methodology is posed in the Linear Matrix Inequality (LMIs) framework by means of two types of models, a set of dynamical non-linear equations of the plant and a set of linearized models, iteratively obtained during the numerical optimization process.

1. Introduction

The traditional mode of designing processes has been the use of heuristic knowledge concentrated on determining the economically optimal process configuration among many possible alternatives. After the configuration is selected the process parameters and a stationary working point are evaluated by means of stationary models of the process in order to satisfy the operational requirements and to reduce investment costs. In this procedure, there was no consideration about the operability and controllability of the processes under design. The results have been plants very difficult to control and, consequently, in practice, there are a lot of self-controlled and very inflexible plants. The traditional approach to process control has been, given a designed process, to find the best selection and pairing of controlled and manipulated variables and also to find the controller parameters with the best closed loop performance to work in a given operating point. The design and the control of processes were tasks performed sequentially; examination of controllability occurs only after the optimal process configuration and parameters are known.

Better solutions can be found in the area of Integrated Design that considers that changes in the process design might make the system more controllable [2],[3],[4],[5]. This methodology allows for the evaluation of the plant parameters and the control system at the same time. This problem is stated mathematically as a non-linear multiobjective optimization problem with non-linear constraints, including economic and control considerations.

In this paper, a systematic approach for the Integrated Design of activated sludge processes and their control system is presented. The proposed methodology combines the design of the plant and controller following a cost optimization procedure, in addition with the desired closed loop dynamic as constraints. The cost functions include the investment and the operation costs and dynamical indexes like the Integral Square Error (ISE). The constraints are selected to ensure that the values of some controllability parameters (the disturbance sensitivity gains of the plant) are within specified ranges, to ensure stability conditions using the Lyapunov theory and also to guarantee bounds on the H_∞ norm performance and many other performance criteria and convex constraints. The independent variable set includes the volumes of bioreactors, the settler cross-section area, the gain and integral time of a PI controller and the working point.

Our paper main contribution is to propose a general framework to design the activated sludge process together with a PI controller, and the optimal set point for this control. The optimization problem is subjected to a set of dynamical constraints expressed by means of a set of LMIs and other dynamical performance criteria. The optimization model definition requires two types of mathematical models: dynamical non-linear models for the plant representation and a set of linearized models, iteratively obtained during the numerical optimization process. The use of non-linear models allows us for the evaluation of any dynamical performance criteria (ISE, ITAE,...) that can be included in the cost function to be minimized, and the use of linear models allows us for the specification of many convex performance criteria and convex constraints, including robustness in the presence of uncertainties and load disturbances. In the present work asymptotic stability and disturbance rejection based on H_∞ norm have been considered.

The paper begins describing the basis of the activated sludge process and the control objectives. An alternative configuration to the original plant is also presented. The third section is devoted to the optimization problem formulation and controllability measures for design. The forth section explains the Integrated Design methodology to be followed to get the optimal solution, and presents the studied cases. Finally, in the last section, some results are commented in both the time and the frequency domain, to end up with some conclusions.

2. Description of the activated sludge process

For applying Integrated Design methodology, we have selected the wastewater treatment structure represented in Figure 1, which is an alternative to a real plant located in Spain. It consists of two aeration tanks working in series and one secondary settler. The basis of the process lies in maintaining a microbial population (biomass) into each bioreactor, transforming the biodegradable pollution (substrate) with dissolved oxygen supplied through aeration turbines. Water coming out of each reactor goes to the settler, where the activated sludge is separated from the clean water and recycled to both bioreactors.

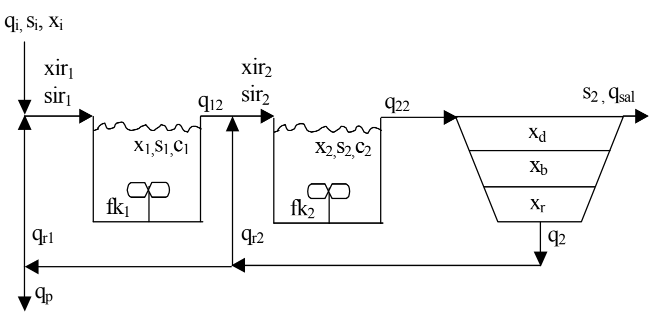

Figure 1: Selected plant for Integrated Design

The control aim is to keep the substrate at the output, s_2, bellow a certain legal value despite the large variations of the flow rate and the substrate concentration of the incoming water (q_i and s_i). The whole set of variables is presented in Figure 1. It can be noted that generically "x" is used for the biomass concentrations (mg/l), "s" for the organic substrate concentrations (mg/l), "c" for the oxygen concentrations (mg/l), and "q" for flow rates (m^3/h).

A first principle model of the system is obtained by considering the mass balances of oxygen, biomass and organic substrate in the whole plant, together with the equilibrium equations for the flows of water and sludge. Note that three layers of different and increasing biomass concentration are considered in the settler [3],[9].

3. Formulation of the optimization problem

The Integrated Design problem for our plant consists of determining simultaneously the plant and controller parameters, and a stationary working point, while the investment and operation costs are minimized. Mathematically it is stated as a non-linear optimization problem:

$$\min_{x} \quad f(x) \tag{1}$$

subject to

$$lb \le x \le ub \tag{2}$$

$$g(x) \le 0 \tag{3}$$

where

x = plant dimensions, flows and working point parameters.
lb = lower bounds for optimization variables.
ub = upper bounds for optimization variables.
g = nonlinear function that represents the physical, process and controllability constraints.

3.1. Cost function

For this Integrated Design problem, a typical quadratic cost function has been used:

$$f(x) = w_1 \cdot v_1^2 + w_2 \cdot v_2^2 + w_3 \cdot A_d^2 + w_4 \cdot fk_1^2 + w_5 \cdot fk_2^2 + w_6 \cdot q_2^2 + w_7 \cdot ISE \tag{4}$$

where w_i ($i = 1,\ldots,7$) are the corresponding weights; v_1, v_2, A_d are volume of the reactors and the area of the settler; and fk_1, fk_2 are the aeration factors for the two reactors.

The first three terms represent the construction costs, terms from fourth to sixth represent operation costs (aeration turbines and pumps), and the last one is the ISE norm, that is included to minimize errors around a substrate reference . The ISE norm can be evaluated as:

$$ISE = \int_{t=0}^{T\max} (s_{2r} - s_2)^2 \cdot dt \tag{5}$$

where T_{max} is the simulation time, s_2 is the output substrate and s_{2r} is a steady state (or a set point for the controller).

3.2. Process constraints

- *Residence times* and *mass loads* in the aeration tanks:

$$2.5 \le \frac{v_1}{q_{12}} \le 5\,;\ 0.001 \le \frac{q_i s_i + q_{r1} s_2}{v_1 x_1} \le 0.06 \qquad (6)$$

- Limits in *hydraulic capacity* and *sludge age* in the settler, and limits in the relationship between the input, recycled and purge flow rates:

$$\frac{q_{22}}{A_d} \le 1.5\,;\ 3 \le \frac{v_1 x_1 + v_2 x_2 + A_d l_r x_r}{q_p x_r 24} \le 10 \qquad (7)$$

$$0.03 \le \frac{q_p}{q_2} \le 0.07\,; 0.5 \le \frac{q_2}{q_i} \le 0.9 \qquad (8)$$

- Constraints on the non-linear differential equations of the plant model to obtain a solution close to a steady state (ε close to zero). For example, the constraint for substrate in the first reactor is:

$$\left|\frac{ds_1}{dt}\right| = \left|-\mu\frac{s_1 x_1}{k_s + s_1} + f_{kd} k_d \frac{x_1^2}{s_1} + f_{kd} k_c x_1 + \frac{q_{12}}{v_1}(sir_1 - s_1)\right| \le \varepsilon \qquad (9)$$

3.3. Control law for the closed loop Integrated Design

In the closed loop design, a PI controller is considered in the plant. The controlled variable is the substrate concentration at the output (s_2), the set point is a stationary point (s_{2r}), and the control signal is the flow rate of recycled sludge to the first reactor, q_{r1} [3]. The control law is this:

$$q_{r1}(t) = K_p((s_{2r} - s_2(t)) + \frac{1}{T_i}\int_0^t (s_{2r} - s_2(\tau))d\tau) \qquad (10)$$

3.4. Controllability measures for design

The desired plant characteristics are good rejection of the two main disturbances (qi, si) at the output variable (s_2), ensuring the stability of the resulting system. Mathematically the effect of disturbances can be expressed by means of several magnitudes (see [1],[2],[3],[7] for further details) like:

- Disturbance sensitivity gains at the dominant frequencies. By analysing time series of signals, taken from the real system, in the frequency domain it can be observed that ω_1=0.5 rad/h and ω_2=0.1 rad/h. are the dominant frequencies. In this work we consider the following definition for the disturbance sensitivity gains, where G_d is the disturbance transfer function and d is the disturbance vector in the worst case:

$$Ds(w) = \frac{\|G_d(jw)\cdot d\|_2}{\|d\|_2} \qquad (11)$$

- Asymptotic stability:

For the autonomous system $\dot{x} = Ax$, the asymptotic stability condition can be expressed as a Linear Matrix Inequality, where P is a parameters matrix:

$$\begin{pmatrix} P & 0 \\ 0 & -PA^T - AP \end{pmatrix} > 0 \qquad (12)$$

- H_∞ norm of the disturbance transfer function:

$$\|G_d\|_\infty = \max_w \overline{\sigma}(w) \qquad (13)$$

where $\overline{\sigma}$ is the maximum singular value. An LTI system presents a disturbance rejection given by $\|G_d\|_\infty \le \gamma$, if for one fixed value γ < 1, there exists one matrix P such as the following Linear Matrix Inequality constraint is feasible:

$$\begin{pmatrix} AP + PA^T + \gamma^{-2} B_p B_p^T & PC^T \\ CP & -I \end{pmatrix} < 0 \qquad (14)$$

In order to be able to calculate the controllability measures and incorporate them in the design as constraints, we need to linearize the model around the current point.

For the open loop plant we get a linearized model like this:

$$\begin{cases} \dot{x} = Ax + Bu + B_p d \\ y = Cx \end{cases} \qquad (15)$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control input, d is the disturbance and $y \in R^p$ is the measurable output. A, B, B_p, C are constant matrices with appropriate dimensions.

For closed loop design, the plant includes a PI with parameters K_p and T_i, so the state space representation of the closed loop system is this:

$$\begin{cases} \dot{x}_a = \begin{pmatrix} A - BK_pC & B\frac{K_p}{T_i} \\ -C & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ x_r \end{pmatrix} + \begin{pmatrix} BK_p \\ 1 \end{pmatrix} \cdot r + \begin{pmatrix} B_p \\ 0 \end{pmatrix} \cdot d \\ y = (C \quad 0) \cdot x_a \end{cases} \qquad (16)$$

where x_r is the additional state due to the PI, and x_a is the new state vector $x_a = (x \quad x_r) \in R^{n+1}$.

3.5. Disturbances

Two sets of disturbances have been considered for designing the plants. The first one (Figure 2) has been taken out from a real wastewater treatment plant, and it has been used as system inputs in dynamic simulations to calculate the ISE norm. The second one is a random set of disturbances, and it has been used only for validating results. The two main disturbances are q_i and s_i.

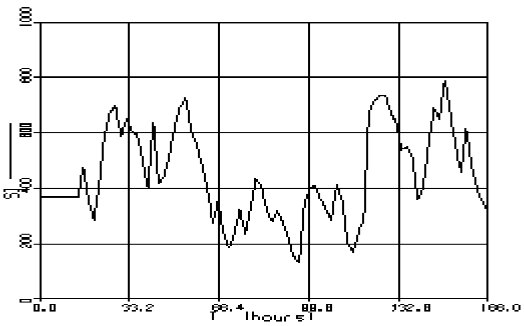

Figure 2: Real disturbance s_i (substrate at the plant input).

4. Integrated Design problem

In order to apply the Integrated Design techniques, we have studied various kinds of design.

4.1. Open loop design

In the open loop design, only the plant is considered, without any controller, and the aim is to design an optimum one that rejects the two main load disturbances at the output variable, s_2. This is achieved including in the optimization problem some measures presented in the previous point as new constraints.

More specifically, we have considered three open loop design cases, with three sets of constraints, and the unconstrained case, that corresponds to a plant designed without taking into account controllability issues.

Case 1:
Design without controllability constrains, just with process and operation constraints. The cost function weights are: w_1=0.01,w_2=0.1,w_3=0.001,w_4=2,w_5=2,w_6=0.001,w_7=0.

Case 2:
Design including as constraints upper values for the disturbance sensitivity gains for the two reactors at the dominant frequencies:

$$Ds1(0.5) < 0.4; Ds2(0.5) < 0.4$$
$$Ds1(0.1) < 0.4; Ds2(0.1) < 0.4$$

The cost function weights are: w_1=2, w_2=2, w_3=1, w_4=2, w_5=2, w_6=10, w_7=0.

Case 3:
Design including the ISE norm in the cost function. The weights are: w_1=0.2, w_2=0.4, w_3=0.2, w_4=2, w_5=2, w_6=0.1, w_7=10000.

Case 4:
Design including asymptotic stability condition and the following constraint: $\|G_p\|_\infty \leq 0.07$, both using the LMI approach. The cost function weights are: w_1=0.01, w_2=0.1, w_3=0.001, w_4=2, w_5=2, w_6=0.001, w_7=0.

4.2. Closed loop design

For closed loop design, the desired characteristics are a proper tuning of the PI controller and good rejection of the two main disturbances ensuring the stability of the resulting system. The optimum plant and controller parameters are the solution to the optimization problem. Mathematically, the problem is stated as in the open loop case, with the only difference that in dynamic simulations the PI equations are included, to calculate the ISE norm the controller set point is included, and two additional optimization variables are considered: Kp and Ti. Note that the controller set point is one of the optimization variables (s_2), so it changes at every iteration, and that the ISE is always introduced in the cost function in order to tune the PI controller.

For closed loop design, the following constraints have been considered: asymptotic stability condition and constraints over the H_∞ norm of the disturbance transfer function. The design cases presented are:

Case 1:
Design considering asymptotic stability condition, and this constraint $\|G_p\|_\infty \leq 0.1$ over the G_p transfer function. The cost function weights are: w_1=0.1, w_2=0.1, w_3=0.2, w_4=2, w_5=2, w_6=0.01, w_7=10.

Case 2:
Design considering asymptotic stability condition, and this constraint $\|G_p\|_\infty \leq 0.08$ over the G_p transfer function. The cost function weights are: w_1=0.2, w_2=0.2, w_3=0.4, w_4=2, w_5=2, w_6=0.1, w_7=0.1.

4.3. Algorithm implementation

The main steps to solve the optimization problem are the following:

- The plant design problem is defined, considering the model in one operation point, the upper and lower bounds of the optimization variables, and the linear and non-linear constraints.
- The problem is solved using an iterative algorithm (SQP method from Matlab Optimization Toolbox). This is what we call the main optimization loop.
- After every iteration, for the design cases that need it, the model is linearized around the optimum values at that moment. With the linearized system (A,B,B_p,C) the disturbance sensitivity gains are calculated, or a LMI problem is solved, depending on the specific design case.

- When the LMI problem is solved, one index indicating if the LMI problem is feasible or not, is transferred to the main optimization loop as a new constraint. With this technique, the SQP algorithm will evolve so as to get candidate solutions satisfying also the LMI constraints.

This process is done iteratively until an optimum solution is found, satisfying all constraints proposed. The software tools that have been used are the Matlab and the Advanced Continuous Simulation Language (ACSL) for dynamic simulations when needed.

5. Results

5.1. Open Loop results

Some results for each design case explained in the previous point are shown in Table 1 (plant sizes and the numerical values of controllability measures). Obviously, the complete solution includes also a set of steady state operating conditions (aeration factors and all concentrations and flows in the plant).

In the open loop design, what we want is to obtain the most economic plant with good disturbance rejection. Comparing the values of the different controllability measures in the table 1, it can be seen that the best plant obtained is in case 3, when the design has been performed including the ISE in the cost function. In the figure 4, the dynamic behaviour of s_2 for this plant is shown. As one could expect, cases 2 and 4 present also a better disturbance rejection that the unconstrained case.

	Case 1	Case 2	Case 3	Case 4
Ds1(0.5)	0.6326	0.3898	0.4156	0.5569
Ds2(0.5)	0.4537	0.3034	0.2685	0.3660
Ds1(0.1)	0.8419	0.4645	0.5090	0.7273
Ds2(0.1)	0.6314	0.3944	0.3462	0.4971
ISE	309771	64753	32422	115634
$\left\|G_p\right\|_\infty$	0.0811	0.0543	0.0462	0.0641
V1	7071.8	10000	9740	7931
V2	4290.6	6428	5649	4532
Ad	2089.5	1551	3033	3286

Table 1: Results for open loop design

On the other hand, looking at the value of the plant dimensions, it can be seen that case 3 is the most expensive because the volumes and cross section of the settler are larger that the other cases. Clearly, there is a tradeoff between cheaper plants and plants with better disturbance rejection. Depending on the plant requirements, the cost function weights have to be properly tuned. In figures 3 and 4 can be seen the improvement when Integrated Design techniques are applied.

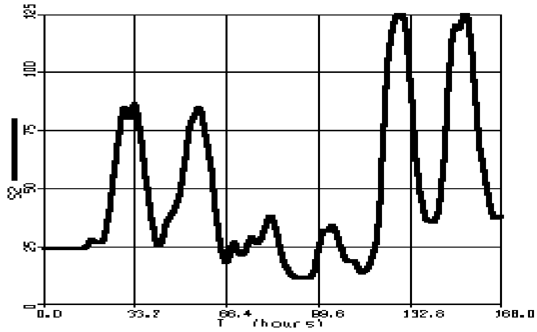

Figure 3: Substrate at the output for the design without controllability constraints (case 1)

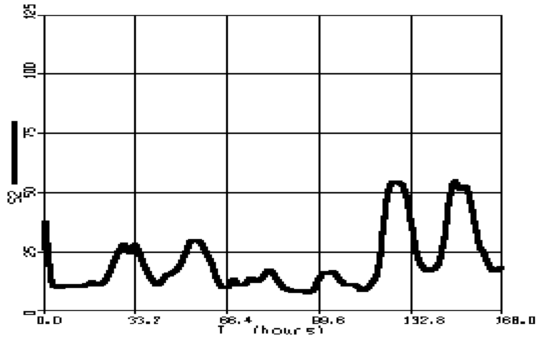

Figure 4: Substrate at the output for the design considering the ISE (case 3)

5.2. Closed loop results

In figure 5 can be seen the results corresponding to case 1, and the parameters of the designed plant are:

V_1= 7220; V_2 = 3993; A_d = 2461
K_p = -5.12; T_i = 7.79

In figure 6 can be seen the results corresponding to case 2. In this case the plant and PI parameters obtained are:

V_1= 7397; V_2 = 5968; A_d = 3535
K_p = -1.86; T_i = 8.76

Although the disturbance rejection is better for some cases of open loop design, plants designed with the PI controller are cheaper because their dimensions (reactors and settler) are smaller, and the output is still within an acceptable range. Here we have also the advantage of designing a tuned PI added to the plant.

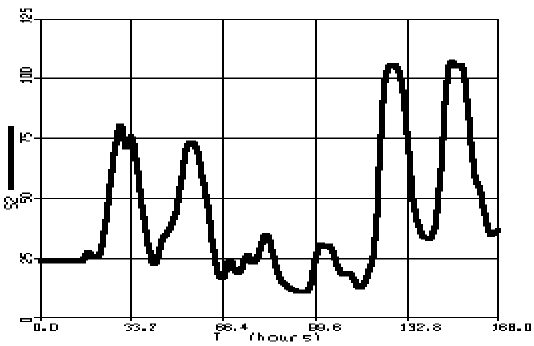

Figure 5: Substrate at the output for the closed loop design case 1.

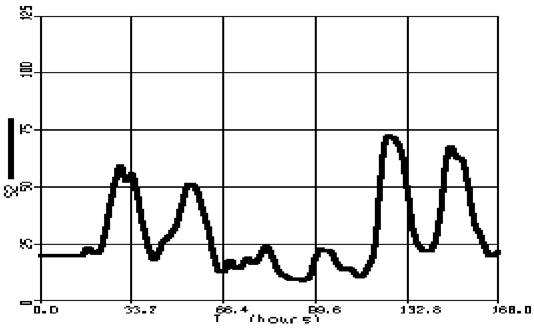

Figure 6: Substrate at the output for the closed loop design case 2

Comparing figures 5 and 6 can be also seen that the fact of considering a more strict constraint over the $\|G_p\|_\infty$, stated as a LMI, produces a solution with better disturbance rejection. In figure 7 the control law for the second close loop design case is presented.

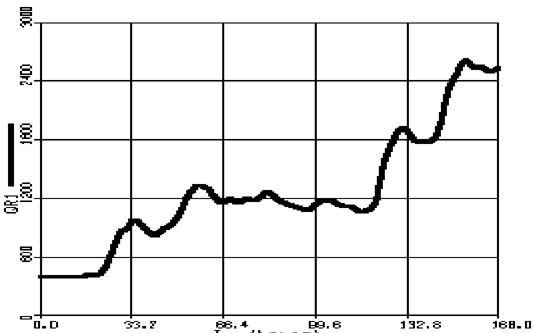

Figure 7: Recycle flow rate for the closed loop design case 2.

6. Conclusions

The design procedure shown in this paper produce better controllable plants that the classical procedure. The responses for open loop design, comparing the design considering only economic costs and the design including controllability, show clearly a better behaviour in the second case. When the PI controller is added to the structure, the designed plant is able to reject disturbances with smaller units that in open loop designs. This is an important result because one can obtain an optimum plant with lower construction costs and good disturbance rejection. Note also that no further PI tuning is needed because the optimization gives also its optimum parameters.

The solved problem guarantees that the non-linear model of the plant is satisfied, as well as the operation and process constraints. The model linearization at every iteration allows solving the problem including LMI constraints in the optimum calculated point, assuring that the final solution satisfies the corresponding constraint.

Acknowledgments

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