LINEAR MATRIZ INEQUALITIES IN INTEGRATED PROCESS DESIGN

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Abstract

This work presents a methodology for process design that allows to include certain control conditions at the design stage. The control constraints may be included for the open or closed loop system and they are described by Linear Matrix Inequalities. The problem reduces to a quadratic programming problem where some of the explicit constraints are no linear and the dynamic constraints, associated to the control requirements are faced as an implicit constraint. In this step, another feasibility problem is solved. Performance requirements such as, stability, s-plane pole location, $\boldsymbol{H_\infty}$ *disturbance rejections are easily included in the approach. In the closed loop design, state feedback control is used. An application to the design of a hydraulic process is featured.*

Keywords: Process design, process optimization, linear matrix inequalities, state feedback control.

1 PROCESS DESIGN

Classical process design looks to determine optimal operating conditions and process units dimensions in order to achieve some predetermine production objective. Process engineers determine the necessary structures, operating conditions and plant physical parameters. The main objective has been generally an economic optimization one. At this stage, the process operating dynamics are of no concern to the designer.

Good system's control properties, strongly related to system's dynamics, such as controllability, have been largely studied by researchers. The first ideas on how to include controllability in the design stage were introduced by Nishida e Ichikawa [11], [12]. Controllability evaluation on steady state systems was presented by Fisher [3]. He describes a systematic procedure to evaluate "a priori" system's controllability in the design stage. Lately, there has been much interest in computing controllability conditions in the design stages. Such interest has grown into the concept of Integrated Design [9], [10], by means of which, plant parameters and its control system are obtained at the design stage.

The objective of this work is to feature a methodology for process design that allows to include certain control conditions at the design stage (integrated design). The control constraints may be included for the open or closed loop system and they are described by Linear Matrix Inequalities. The integrated design ends up being a non-lineal, multi objective optimization problem with economic and control constraints, using LMIs [1].

The benefits of this methodology, are that at the end of the design process a system is obtained that satisfies the economic and steady state operating constraints, and also exhibits good control performance (dynamical constraints).

2 OPEN LOOP DESIGN (PROCESS DESIGN)

2.1 LMI CONDITIONS

LMIs are particularly suited to formulate a number of, analysis and design, control system performance criteria [8], [4]. Among them we may cite: asymptotic stability conditions, pole location (time profile response), disturbance rejection. In this work we will use all of them. For disturbance rejection we will consider the $\boldsymbol{H_\infty}$ norm and for pole location we will consider LMI regions.

2.1.1 Asymptotic stability

For the autonomous system given by:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{t})\boldsymbol{x} \tag{1}$$

the asymptotic stability condition may be determine by the existence of a solution to the following LMI [5]:

$$\begin{pmatrix} \boldsymbol{P} & 0 \\ 0 & -\boldsymbol{P}\boldsymbol{A}^T - \boldsymbol{A}\boldsymbol{P} \end{pmatrix} > 0 \tag{2}$$

that is, system (1) is stable iff there exists a solution $\boldsymbol{P}$ to LMI (2).

2.1.2 H_∞ disturbance rejection.

Let:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{B}_1\boldsymbol{w} \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \end{cases} \tag{3}$$

where $\boldsymbol{x} \in \boldsymbol{R}^n$ is the state vector, $\boldsymbol{u} \in \boldsymbol{R}^m$ is the control input, $\boldsymbol{w}$ is the disturbance and $\boldsymbol{y} \in \boldsymbol{R}^p$ is the measurable output. $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{B_1}$, $\boldsymbol{C}$ are constant matrices with appropriate dimensions. Let $\boldsymbol{G_d}$ be the transfer function from the disturbance to the output, that is:

$$G_d(s) = \frac{Y(s)}{W(s)} \quad (4)$$

We will say that system (3) features a γ_d disturbance rejection [4], if:

$$\|G_d(s)\|_\infty < \gamma_d \quad (5)$$

for a given scalar $\gamma_d > 0$. Such condition may be verified by the existence of a matrix $P > 0$ such that the following LMI is feasible:

$$\begin{pmatrix} AP + PA^T + \gamma_d^{-2} B_1 B_1^T & PC^T \\ CP & -I \end{pmatrix} < 0 \quad (6)$$

2.1.3 LMI Regions

A number of (convex) s-plane regions may be depicted by satisfaction of LMI conditions [2]. Among them, we particularly focus on the regions we describe below.

✓ s-Plane region to the left of -α_o (α_o>0)

It is well known that the location of the poles of a system in the left half plane determines the speed of response. The deeper into that plane, the faster the response. Let us now consider the s-plane region of figure 1.

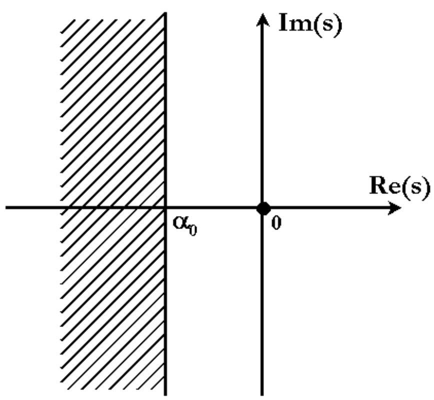

Figure 1: Semi left half plane. Imaginary axis displaced by α_0

System (1) locates all its poles in the region depicted in figure 1 [2], if there exists matrix P>0, such that:

$$AP + PA^T + 2\alpha_0 P < 0 \quad (7)$$

✓ Cone centered at the origin, with angle θ

This region assures damping on the time response of the system. The region is depicted in figure 2. Regarding system (1), all its poles (eigenvalues of the A matrix) are located within the region depicted in figure 2, iff there exists $P > 0$, such that the following LMI is satisfied [2].

$$\begin{pmatrix} \operatorname{sen}\theta\left(AP + PA^T\right) & \cos\theta\left(AP - PA^T\right) \\ \cos\theta\left(-AP + PA^T\right) & \operatorname{sen}\theta\left(AP + PA^T\right) \end{pmatrix} < 0 \quad (8)$$

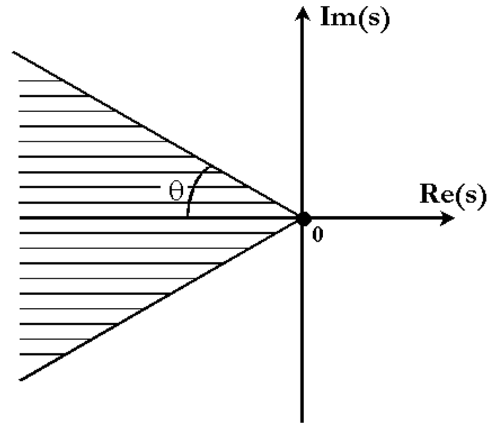

Figure 2: LMI cone region, with angle θ

3 CLOSED LOOP DESIGN (PROCESS AND CONTROL SYSTEM DESIGN)

3.1 LMI CONDITIONS

The LMIs condition using state feedback control are given by the substitution of the control law $u = -Kx$ into the dynamic matrix of the linear system. All closed loop condition can be expressed as the follows LMIs.

3.1.1 Asymptotic stability

$$\begin{pmatrix} P & 0 \\ 0 & -AP - PA^T - BR - R^T B^T \end{pmatrix} > 0 \quad (9)$$

3.1.2 H_∞ disturbance rejection

$$\begin{pmatrix} AP + PA^T + BR + R^T B^T + \gamma_d^{-2} B_1 B_1^T & PC^T \\ CP & -I \end{pmatrix} < 0 \quad (10)$$

3.1.3 LMI Regions

✓ s-Plane region to the left of -α_o (α_o>0)

$$AP + PA^T + BR + R^T B^T + 2\alpha_0 P < 0 \quad (11)$$

✓ Cone centered at the origin, with angle θ

$$\begin{pmatrix} \operatorname{sen}\theta\left(AP + PA^T + BR + R^T B^T\right) & \cos\theta\left(AP - PA^T + BR - R^T B^T\right) \\ \cos\theta\left(-AP + PA^T - BR + R^T B^T\right) & \operatorname{sen}\theta\left(AP + PA^T + BR + R^T B^T\right) \end{pmatrix} < 0 \quad (12)$$

All, convex constraints in $P > 0$ and R, where de gain of the state feedback is given by:

$$K = RP^{-1} \quad (13)$$

4 THE INTEGRAL DESIGN ALGORITHM

In order to include the dynamical constraints, that will be described by LMIs, in the process design stage, and given that in general, processes are non linear, it is required to perform a linearization, since LMI conditions are formulated for linear processes. The integral design algorithm is as follows:

❶ Define the process design optimization problem, considering operating conditions and explicit linear and non-linear constraints.

❷ At each iteration, the decision variable candidates (best solution to the moment) are used to compute a linear model of the system.

❸ With the linear system (***A,B,C***) the performance conditions (stability, pole locations, disturbance rejection) are verified by means of the appropriate LMIs. The existence (or not) of a solution is then submitted to the optimization problem formulated in step 1, in order for it to keep on looking for the optimal solution

❹ The process goes on until an optimum value x_{opt} is attained. Observe that once an optimum is reached, all conditions, static and dynamic, are satisfied.

This algorithm is implemented using standard numerical tools. In our case by means of the Matlab optimization and LMI toolboxes.

5 HYDRAULIC SYSTEM

5.1 PROCESS DESCRIPTION

To illustrate our proposal, we have chosen a dynamical system composed of two interconnected tanks, as shown in figure 3. There is an incoming flow (q_{in}) to reservoir 1, the control signal u(t), and another incoming flow (f_d(t)) to tank 2, regard as disturbance. Valves v_1 and v_2 may be set to a fixed value between 0 and 100%. The final control objective is to command level 2 by manipulating q_{in}.

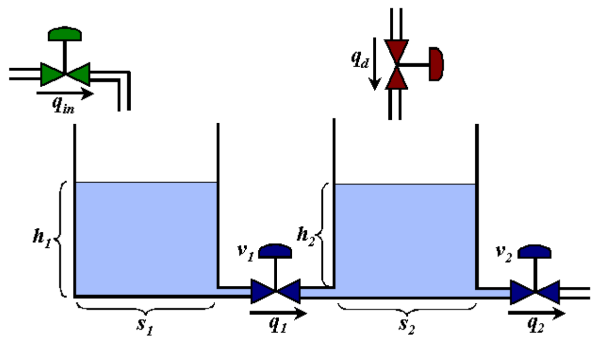

Figure 3: Hydraulic System

5.2 PROCESS MODEL

A simple mass balance yields:

$$S_1\frac{dh_1}{dt} = q_{in} - q_1, \quad S_2\frac{dh_2}{dt} = q_1 + q_d - q_2$$

where

$$q_1 = k_1 a_1\sqrt{h_1 - h_2}\ ;\ \ q_2 = k_2 a_2\sqrt{h_2}\ ;\ \ q_d = f_d/8\ ;\ \ q_{in} = u(t)$$

k_1, k_2 are the valves constants and a_1, a_2 the valves openings. f_d is the disturbance and q_d is the effect of the disturbance on tank 2. S_1 and S_2 are reservoir's areas 1 and 2, h_1 and h_2 are the levels.

Simple manipulations yield:

$$S_1\frac{dh_1}{dt} = q_{in} - k_1 a_1\sqrt{h_1 - h_2}$$

$$S_2\frac{dh_2}{dt} = k_1 a_1\sqrt{h_1 - h_2} + \frac{f_d}{8} - k_2 a_2\sqrt{h_2}$$

5.3 OPERATING CONSTRAINTS

In most processes conditions such as flow delivery or residence time are common. We have chosen to impose these types of constraints into our system. The residence time constraints are depicted by:

Tank 1:

$$\frac{V_1}{q_1} = \frac{S_1 h_1}{k_1 a_1\sqrt{h_1 - h_2}} \geq Tr_1 \Rightarrow Tr_1 k_1 a_1\sqrt{h_1 - h_2} - s_1 h_1 \leq 0$$

Tank 2:

$$\frac{V_2}{q_2} = \frac{S_2 h_2}{k_2 a_2\sqrt{h_2}} \geq Tr_2 \Rightarrow Tr_2 k_2 a_2 - s_2\sqrt{h_2} \leq 0$$

where Tr_1 and Tr_2 the residence times.

Remark: we also impose that $h_1 > h_2$.

6 SYSTEM'S OPTIMIZATION

The objective function and constraints are:

✓ **For the non-linear optimization problem:**

The objective functions are given by:

$$f(x) = \eta_1 (R_1)^2 + \eta_2 (R_2)^2 + \eta_3 (h_1)^2 + \eta_4 (h_2)^2 + \eta_5 (S_1)^2 + \eta_6 (S_2)^2$$

for open loop case and

$$f(x) = \eta_1 (R_1)^2 + \eta_2 (R_2)^2 + \eta_3 (h_1)^2 + \eta_4 (h_2)^2$$

for closed loop case, because S_1=S_2=2, and the constraints:

1.- The steady state operating point is an equilibrium point (or as closed to it as possible). It translates into:

$$\textit{Residue 1} = R_1 = q_{in} - k_1 a_1\sqrt{h_1 - h_2}$$

$$\textit{Residue 2} = R_2 = k_1 a_1\sqrt{h_1 - h_2} + \frac{f_d}{8} - k_2 a_2\sqrt{h_2}$$

and:

$$R_1, R_2 \rightarrow 0$$

2.- Heights of the reservoirs, as small as possible, hence h_1 and h_2 must be minimized. Normally, there is a safety level, to avoid dangerous conditions. Without loss of generality we simply impose that 1.5h_i i=1,2 we less than some safety level, that is:

$$H_1 = 1{,}5h_1 \quad \text{con} \quad h_1 \rightarrow h_{1opt}$$
$$H_2 = 1{,}5h_2 \quad \text{con} \quad h_2 \rightarrow h_{2opt}$$

3.- Tank areas should be minimized in open loop case. In closed loop design $S_1 = S_2 = 2$.

$$S_1 \rightarrow S_{1opt} \qquad S_2 \rightarrow S_{2opt}$$

4.- Valves openings are limited to:

$$0 < a_1 , a_2 < 1$$

✓ **For the dynamical constraints (LMI feasibility conditions):**

We impose, either alone or together, as a multi objective feasibility problem: Asymptotic stability, LMI region pole location (Semi left half plane, Cone) and H_∞ disturbance rejection.

7 OPEN LOOP RESULTS

7.1 NON-LINEAR DESIGN

In the first case we followed a classical approach, that is, no control constraints (dynamical constraints) were included in the design problem. Numerical results are featured in table 1.

Residue R_1	Residue R_2
0.00000000517148	-0.00000001055143

Height h_1	Height h_2
2.84561394923148	1.78873102431188

Area S_1	Area S_2
6.67730673214957	3.83704425258219

Valve opening a_1	Valve opening a_2
0.97271710245128	0.74770017362659

Table 1: Classical approach.

In order to evaluate the performance characteristics of the system obtained, first a step input is submitted to the non-linear system model in t=0 and t=200. The response is shown in figure 4. Then an impulse disturbance is submitted to the same system. The response is shown in figure 5.

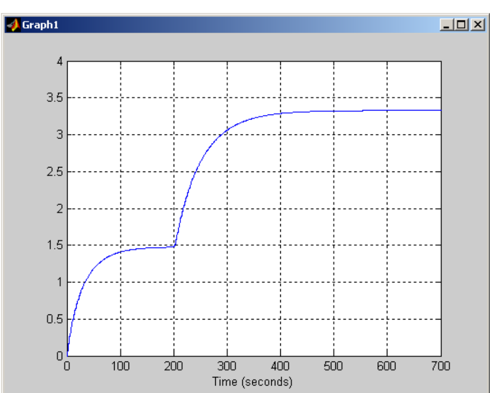

Figure 4: Step response

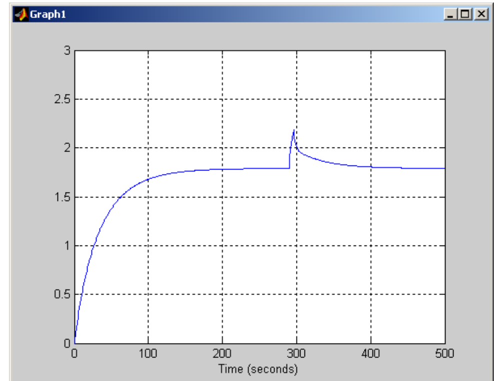

Figure 5: Disturbance rejection

7.2 NON LINEAR DESIGN WITH POLE PLACEMENT AND DISTURBANCE REJECTION

In this case, we want all the poles of the system to the left of α_0=-0.035 and within a cone centered at the origin with angle θ = pi/7. We also impose that the H_∞ norm of the transfer function from disturbance to output be $\leq \gamma_d$, and $\gamma_d = 0.5625$. Table 2 gathers the numerical results.

Residue R_1	Residue R_2
-0.00000000747697	-0.00000000358803

Height h_1	Height h_2
2.09093442060715	1.03348267484108

Area S_1	Area S_2
4.84045960447927	3.97083708543862

Valve opening a_1	Valve opening a_2
0.97245548126551	0.98366769921824

Table 2: Optimization results with pole location and disturbance rejection.

Again, the results were evaluated under the same conditions and the step response is shown in figure 6 and pulse rejection in figure 7.

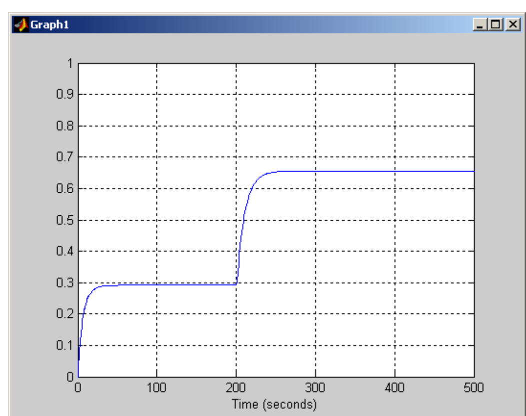

Figure 6: Step response

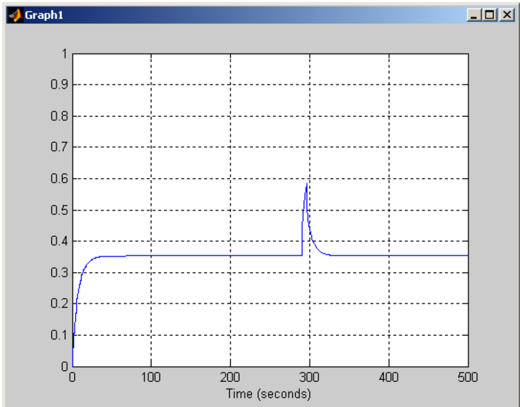

Figure 7: Disturbance rejection

8 CLOSED LOOP RESULTS

8.1 NON-LINEAR DESIGN WITH ASYMPTOTIC STABILITY

In this section we design not only the plant but also its associated control system, all of it simultaneously. In the first case we only impose that the closed loop system be stable.

The results are shown in table 3

Residue R_1	Residue R_2
0.00000001149432	0.00000005947075

Height h_1	Height h_2
10.46522936860677	9.46499858290133

Valve opening a_1	Valve opening a_2
0.99988460977494	0.32504214636676

Gain K_1	Gain K_2
-0.55087064937711	-2.01946098625478

Table 3: Optimization results with asymptotic stability

and the time response for pulse rejection is presented in figure 8.

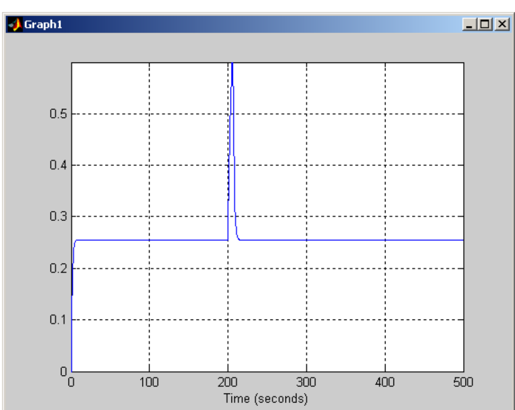

Figure 8: Pulse rejection

Figure 9 shows the control signal.

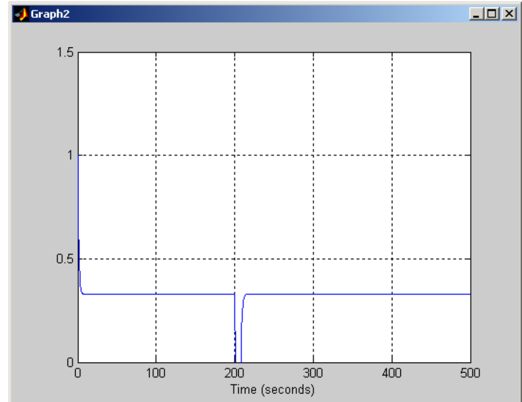

Figure 9: Control signal

8.2 NON-LINEAR DESIGN WITH STABILITY AND DISTURBANCE REJECTION

In this case we want that the closed loop system be stable and the H_∞ norm of the transfer function from disturbance to output be $\leq \gamma_d$, and $\gamma_d = 0.5$. The numerical results are included in table 4.

Residue R_1	Residue R_2
-0.00000000407289	-0.00000000371341

Height h_1	Height h_2
3.35192429306033	2.19317323744678

Valve opening a_1	Apertura a_2
0.92897693491001	0.32504214636676

Gain K_1	Gain K_2
-0.81176280537093	-3.08878252208647

Table 4: Optimization results with asymptotic stability and disturbance rejection

The time response for pulse rejection and the control signal are featured in figure 10 and 11. We remark that in all simulations shown in this work, we used the non-linear model to simulate the time responses of the system.

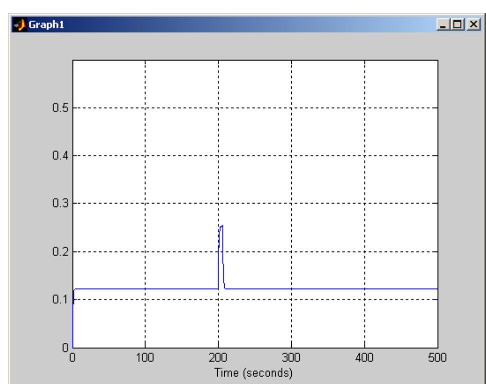

Figure 10: Pulse rejection

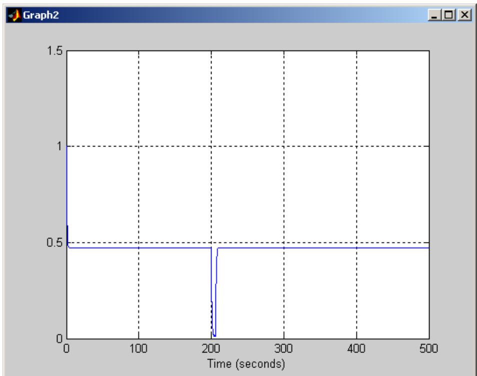

Figure 11: Control signal

9 CONCLUSIONS

The procedure presented in this work allows to verify some dynamic conditions of the system, at the design stage, besides all other static considerations normally taken on process design. Here, we have dealt particularly with stability, pole location, and disturbance rejection, some other performance criteria might equally be included (***H_2***, ***l_1***, etc.). For doing so, we have made use of linear matrix inequalities to formulate the control constraints. By means of an example, we have illustrated the approach and the results expose the significant improvement in the resulting process design when the dynamic constraints are considered explicitly in the formulation. We remark that in all cases, the axes had to be re scaled in order to accommodate properly the results. When further dynamic considerations were taken into account better system profiles were always found. Furthermore, our approach does not impose limits to the classical design approach, but rather incorporates other, implicit, constraints into the non-linear programming search of an optimum. At the end, we end up with a process that satisfies the static constraints (operating equilibrium point, etc.) and the dynamic constraints (stability for instance). The design criteria may be applied to the system open loop, in which case we would be looking for plants with smooth behaviors.

The approach to obtain a process and its control system (closed loop design) presented in this work allows to obtain even better processes with its control system. In the closed loop example, the plant has smaller dimensions and exhibits much better disturbance rejections. The approach may be extended to cope with uncertain systems. In which case we could guarantee certain performance despite the linearization step.

ACKNOWLEDGES

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